ALGEBRAIC REASONING = SUCCESSFUL PROBLEM SOLVING

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Why algebraic reasoning?

“Algebraic reasoning pervades all of mathematics and is essential for making mathematics useful in daily life.”

(Van de Walle, 2004)
What is algebraic reasoning?

“Algebraic thinking or reasoning involves forming generalizations from experiences with number and computation, formalizing these ideas with the use of a meaningful symbol system, and exploring the concepts of pattern and function.”

(Van de Walle, 2004)
Algebraic Reasoning includes:

- Pictorial, graphic, and verbal descriptions
- Numeric representations

How would I use algebraic reasoning in problem solving?
Algebraic Reasoning

- Generalization from arithmetic
- Meaningful use of symbols
- Study of patterns and functions
Generalization from Arithmetic
Developing Arithmetic in the Elementary Grades

- The separation of arithmetic and algebra deprives students of powerful ways of thinking about mathematics.

- Fundamental properties that children use in calculating are the basis for most of the symbolic manipulation in algebra.
Let’s play the game *Salute*

- Three players on a team
- Deck of cards
- Paper to record (optional)
Using Playing Cards to Form Equations

Figure 1

Four problems involving playing cards

\[ \square + 7 = 10 \quad (x + 7 = 10) \]

\[ \square + \square = 8 + 6 \quad (2x = 8 + 6) \]

The first two cards are the same.

\[ \square + \square + \square = \square + \square \quad (3x = 2y) \]

The first three cards are the same, and the last two cards are the same.

\[ \square + \square + \square + \square = \square + \square + \square + \square \]

All eight cards must be different.
Using Playing Cards to Form Equations
Generalization from Arithmetic: Another Example
Systems of Equations

Objectives of the investigation:

Students will:

• develop their ability to reason with and represent using variables;

• move away from random guess-and-check to a more logical approach for finding values for variables in a system of equations; and

• understand various approaches to solving the same problem.
Questions to ask students

• Tell me what you were thinking.

• Did you solve this in a different way?

• How do you know this is true?

• Does this always work?
The students were introduced to a system of equations.

**HOW MUCH IS EACH SYMBOL WORTH?**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cloud</td>
<td>32</td>
</tr>
<tr>
<td>Sun</td>
<td>29</td>
</tr>
<tr>
<td>Daisy</td>
<td>25</td>
</tr>
<tr>
<td>Cloud</td>
<td>24</td>
</tr>
<tr>
<td>Sun</td>
<td>33</td>
</tr>
<tr>
<td>Sun</td>
<td>40</td>
</tr>
<tr>
<td>Sun</td>
<td>37</td>
</tr>
</tbody>
</table>
(a) The second activity used symbols familiar to the students.

**HOW MUCH DOES EACH FISH COST?**

<table>
<thead>
<tr>
<th>SUM</th>
<th>Goldfish</th>
<th>Beta</th>
<th>Clown Fish</th>
</tr>
</thead>
<tbody>
<tr>
<td>$6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$12</td>
<td></td>
<td></td>
<td></td>
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<td>$19</td>
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<td>$16</td>
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<td>$21</td>
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</table>
### WHAT IS THE LENGTH OF EACH SHARK IN FEET?

<table>
<thead>
<tr>
<th>SUM</th>
<th>Tiger Shark</th>
<th>Bull Shark</th>
<th>Hammerhead Shark</th>
</tr>
</thead>
<tbody>
<tr>
<td>52</td>
<td><img src="image1" alt="Tiger Shark" /></td>
<td><img src="image2" alt="Bull Shark" /></td>
<td><img src="image3" alt="Hammerhead Shark" /></td>
</tr>
<tr>
<td>37</td>
<td><img src="image1" alt="Tiger Shark" /></td>
<td><img src="image2" alt="Bull Shark" /></td>
<td><img src="image3" alt="Hammerhead Shark" /></td>
</tr>
<tr>
<td>41</td>
<td><img src="image1" alt="Tiger Shark" /></td>
<td><img src="image2" alt="Bull Shark" /></td>
<td><img src="image3" alt="Hammerhead Shark" /></td>
</tr>
<tr>
<td>34</td>
<td><img src="image1" alt="Tiger Shark" /></td>
<td><img src="image2" alt="Bull Shark" /></td>
<td><img src="image3" alt="Hammerhead Shark" /></td>
</tr>
<tr>
<td>35</td>
<td><img src="image1" alt="Tiger Shark" /></td>
<td><img src="image2" alt="Bull Shark" /></td>
<td><img src="image3" alt="Hammerhead Shark" /></td>
</tr>
<tr>
<td>31</td>
<td><img src="image1" alt="Tiger Shark" /></td>
<td><img src="image2" alt="Bull Shark" /></td>
<td><img src="image3" alt="Hammerhead Shark" /></td>
</tr>
<tr>
<td>30</td>
<td><img src="image1" alt="Tiger Shark" /></td>
<td><img src="image2" alt="Bull Shark" /></td>
<td><img src="image3" alt="Hammerhead Shark" /></td>
</tr>
</tbody>
</table>
Figure This! The costs of combinations of frowns, smiles, and neutral faces are shown. How much is a smile worth?

**Hint:** Find a way to combine two of the rows or columns that have something in common.

Reasoning about unknowns is essential in studying equations. Economists, nurses, chemists, and engineers all use equations in their work.
Make up your own chart

<p>| | | |</p>
<table>
<thead>
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</table>
Meaningful Use of Symbols
Two of Everything
by Lily Toy Hong
**Two of Everything**

- Read the book.

- Act out the story using a magical pot.

- Develop a table of values using input and output.

- Utilize pattern found from the table to generalize a rule verbally and use symbols.
Activity Sheet 1

What would you choose?

❖ Choice A: 100 coins each day for 10 days

❖ Choice B: 5 coins and a magic pot that doubled the coins each day for 10 days

Work it out!
Activity Sheet 2

Square Numbers

Pattern I see: sides are the same.

1x1  2x2  3x3  4x4
Meaningful Use of Symbols
The Dinner Table Problem

Scenario

Susan is preparing for a dinner party. She has seven square tables that can seat one person on each side. How many people can she invite if the tables will be placed end-to-end making one long row?
## Dinner Table Problem

<table>
<thead>
<tr>
<th>Dinner Tables</th>
<th>Show How</th>
<th>Number of People</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td><img src="image" alt="Diagram" /></td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td><img src="image" alt="Diagram" /></td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Finding Patterns and Functions
The Tiling a Patio problem

Alfredo Gomez is designing square patios. Each patio has a square garden area in the center. Alfredo uses brown tiles to represent the soil of the garden. Around each garden, he designs a border of white tiles. The pictures show the three smallest square patios that he can design with brown tiles for the garden and white tiles for the border.
## Tiling a Patio

<table>
<thead>
<tr>
<th>Patio Number</th>
<th>Number of Brown Tiles</th>
<th>Number of White Tiles</th>
<th>Total Number of Brown and White Tiles</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>12</td>
<td>16</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

It is now time for GETS!
Crossing the River Problem

Students will:

• develop their ability to reason with and represent with variables;

• move away from random guess-and-check to a more logical approach for finding values for variables in a system of equations; and

• understand various approaches to solving the same problem.
Crossing the River Problem

Scenario

Eight adults and two children need to cross a river. A small boat is available that can hold one adult, or one or two children. Everyone can row the boat. How many one-way trips does it take for them all to cross the river?

Let’s act the story out with:

- one adult and two children
- two adults and two children
Crossing the River Problem

Scenario

Eight adults and two children need to cross a river. A small boat is available that can hold one adult, or one or two children. Everyone can row the boat.

How many one-way trips does it take for them all to cross the river?
Crossing the River Problem

Extension

Can you describe how to solve this problem with two children and any number of adults?
Internet Resources

- **Scales and Balance**
  [http://nlvm.usu.edu/en/nav/frames_asid_324_g_3_t_2.html](http://nlvm.usu.edu/en/nav/frames_asid_324_g_3_t_2.html)

- **Pan Balance Shapes**
  [http://illuminations.nctm.org/ActivityDetail.aspx?id=33](http://illuminations.nctm.org/ActivityDetail.aspx?id=33)

- **Function Machine**:  
  [http://nlvm.usu.edu/en/nav/frames_asid_191_g_3_t_2.html?from=category_g_3_t_2.html](http://nlvm.usu.edu/en/nav/frames_asid_191_g_3_t_2.html?from=category_g_3_t_2.html)

- **Function Machine Math Playground**  
  [www.mathplayground.com/functionmachine.html](http://www.mathplayground.com/functionmachine.html)

- **Stop that Creature!**  
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