

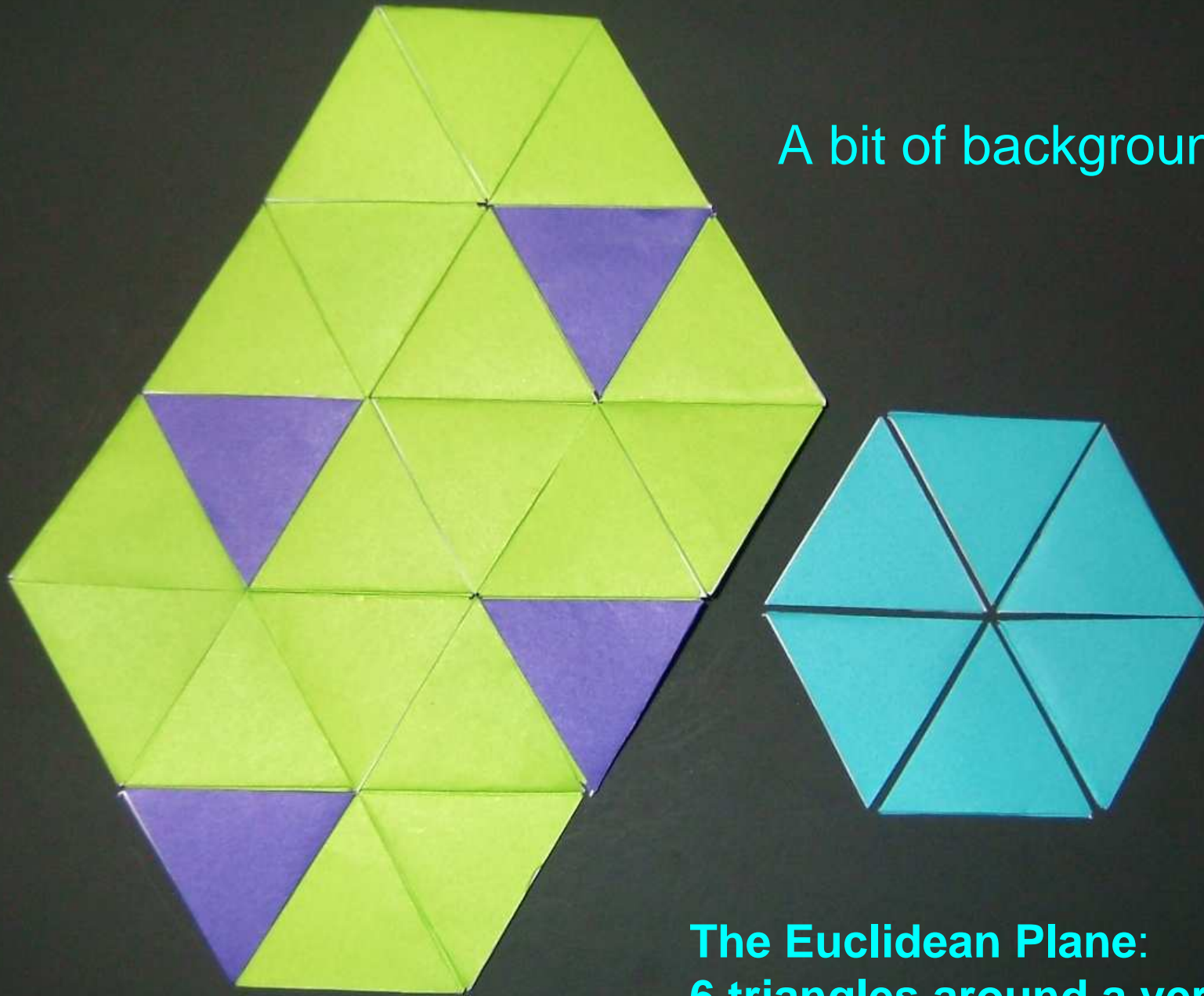
The Intersection of Origami and Non-Euclidean Geometries

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A bit of background...



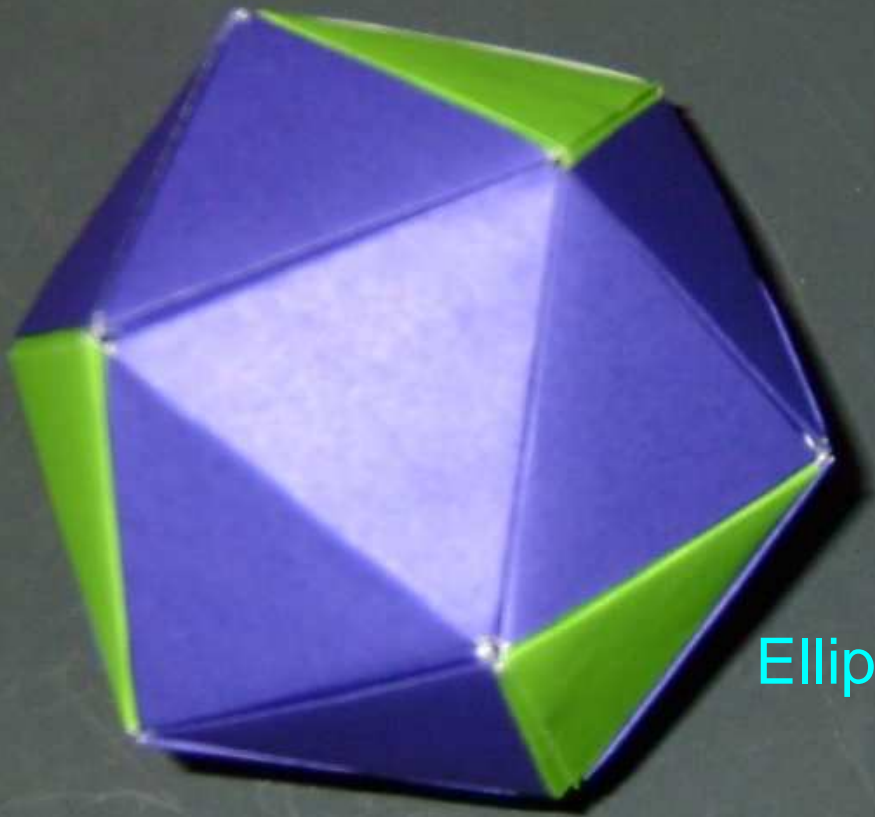
**The Euclidean Plane:
6 triangles around a vertex**

Origami Instructions

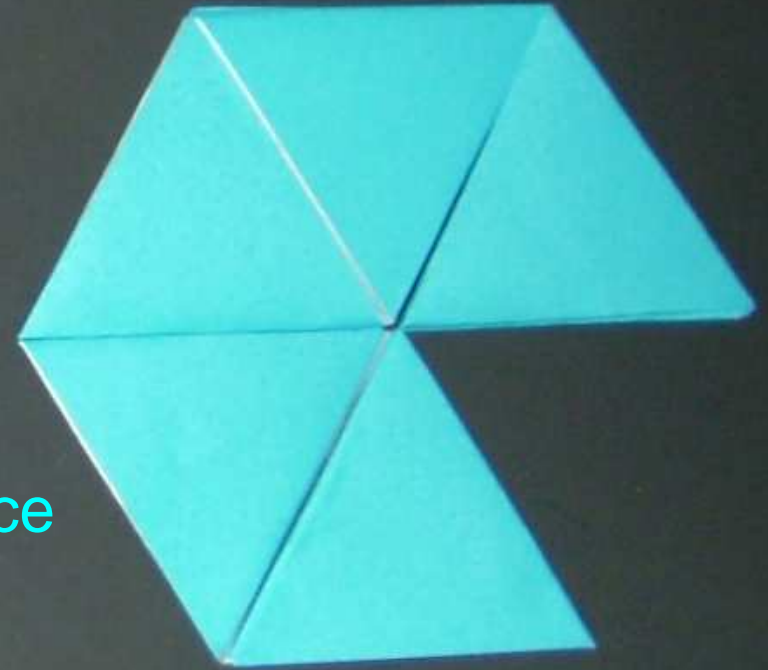
- **Individually, make six to seven origami triangles.**
- **In groups, assemble your models according to the instructions on the following page.**
- **Compare your models to those of others around you.**

- 1. Connect five equilateral triangles together so that they meet at a common vertex. Continue the process of connecting exactly five equilateral triangles out from the original vertex until you cannot add more triangles to your model. Make sure only five equilateral triangles meet at each vertex.**
- 2. Connect seven equilateral triangles so that they meet at a common vertex. Continue the process of connecting exactly seven equilateral triangles together out from the original vertex for at least six more vertices. Make sure that seven equilateral triangles meet at each vertex and that at each edge only two triangles meet. Alternately you may cut out and tape individual equilateral triangles together so that seven triangles fit together at a common vertex.**

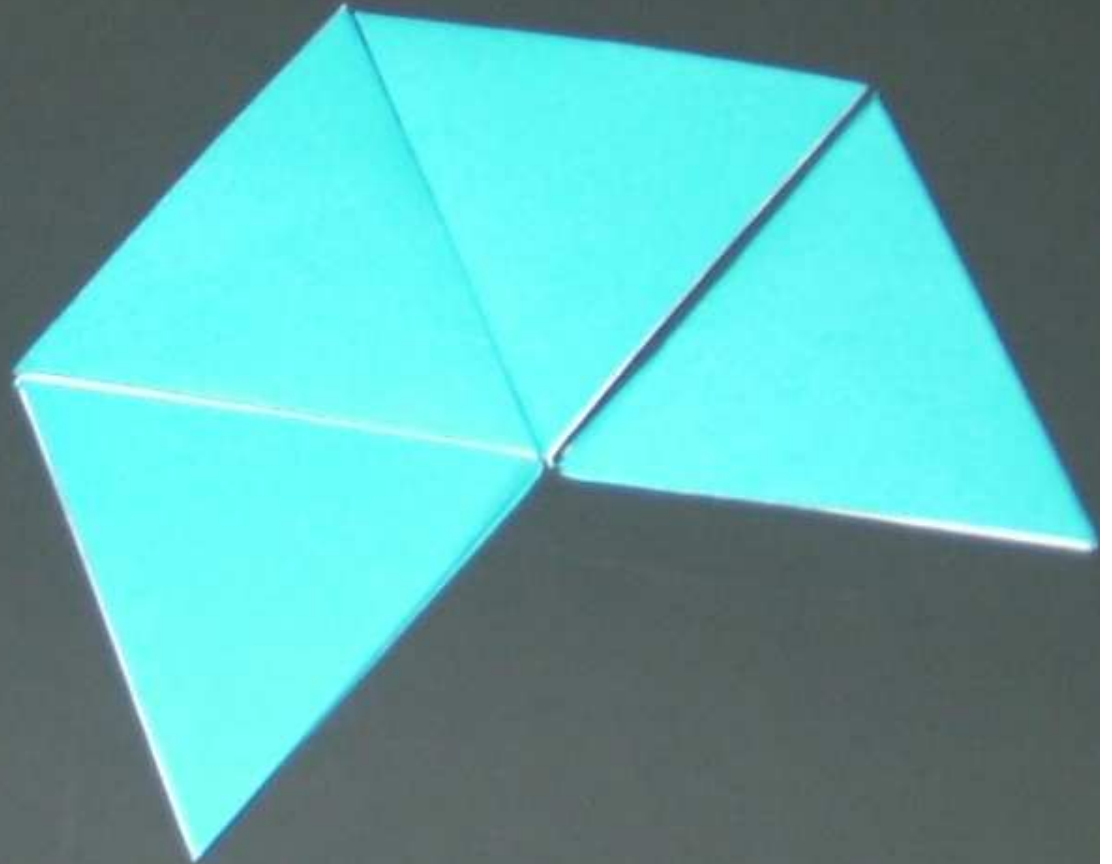
The Icosahedron:
5 triangles around a vertex



Elliptic Space



Elliptic Space

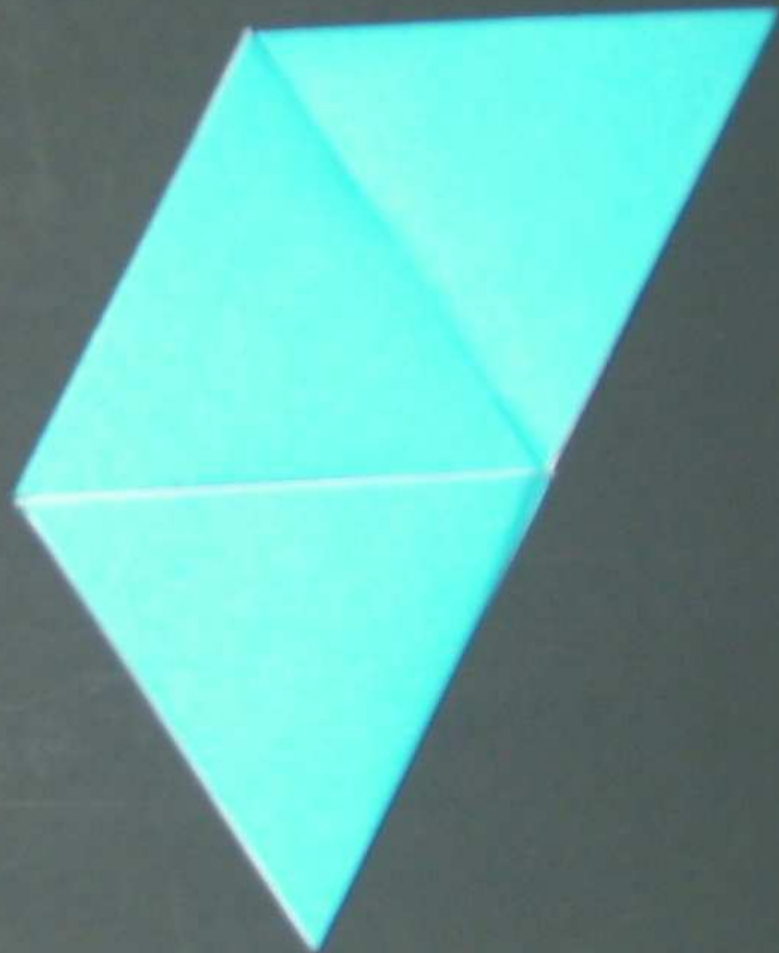


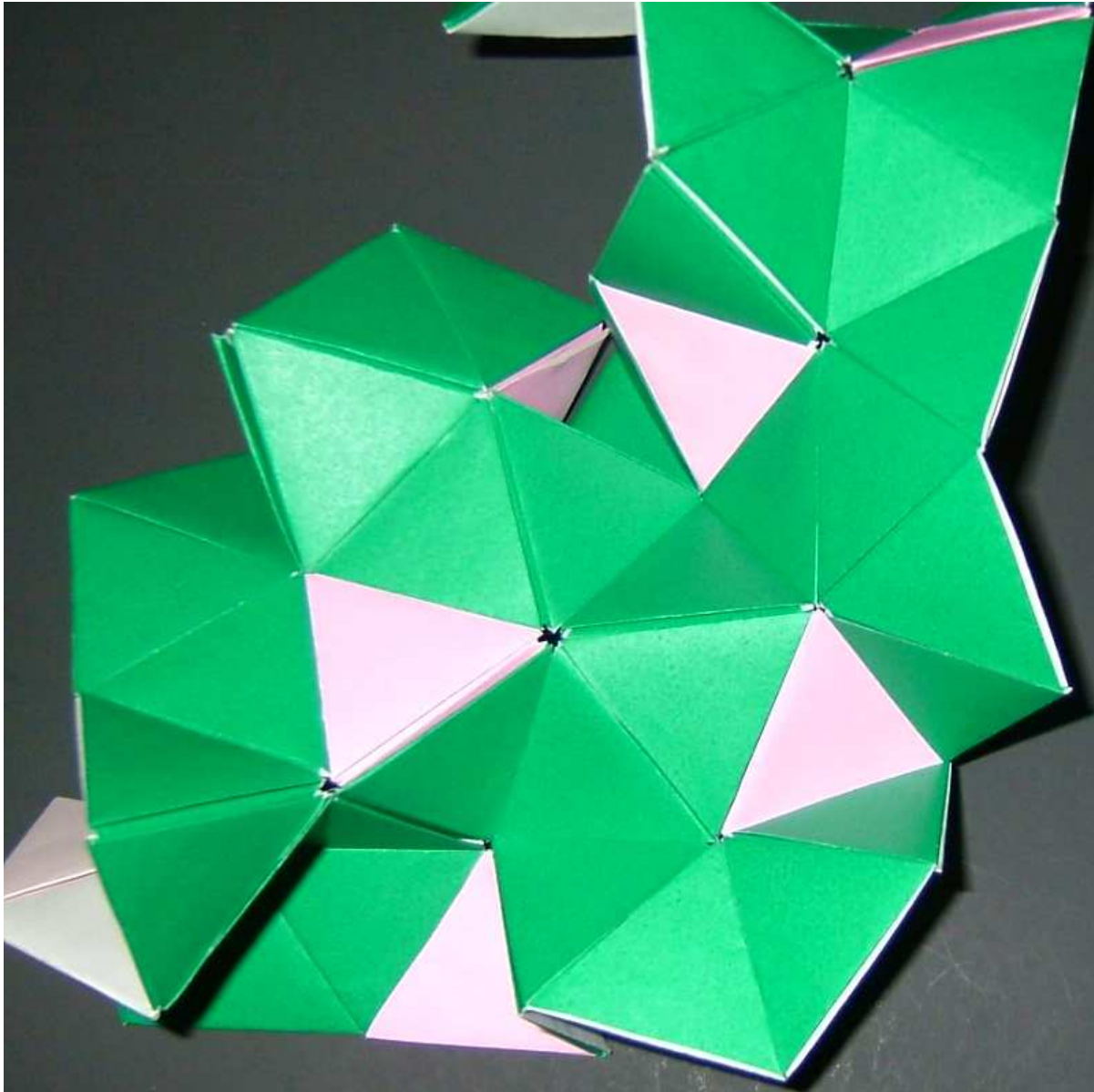
The Octahedron:
4 triangles around a vertex

**The Tetrahedron:
3 triangles around a vertex**

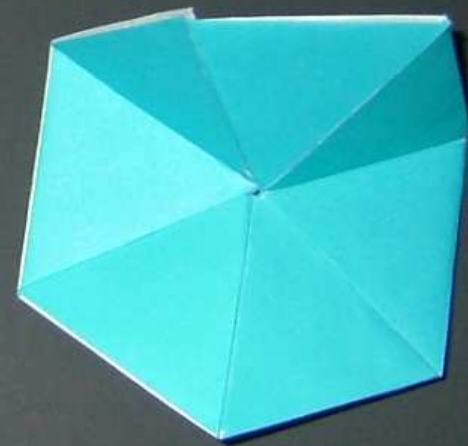


Elliptic Space





Hyperbolic Space:
7 triangles around a
vertex

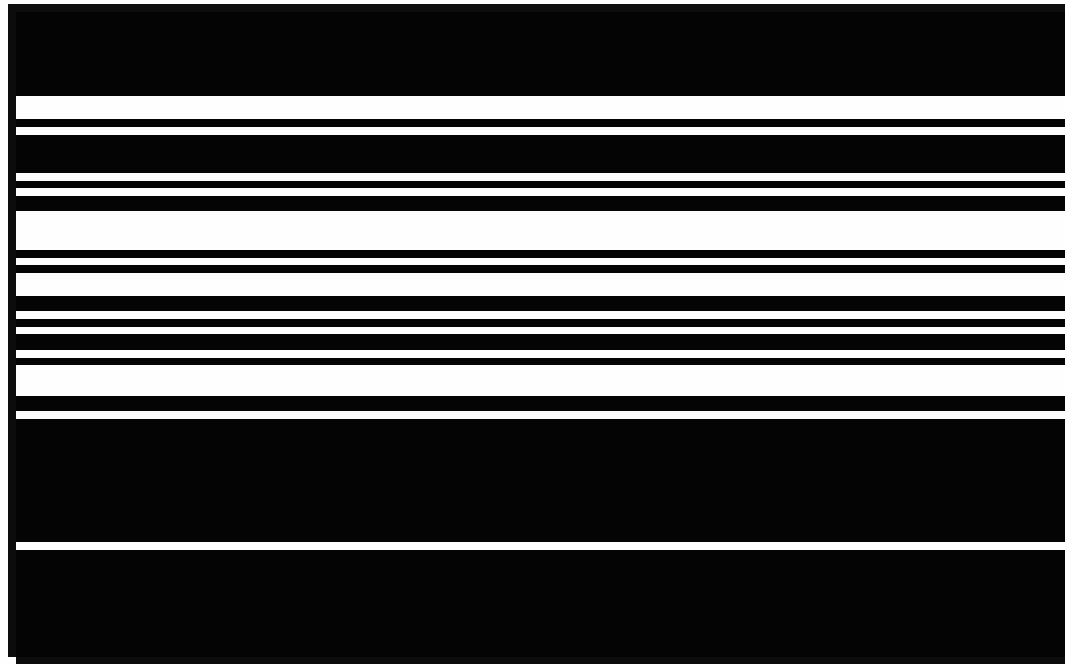


Gaussian Curvature...

- **describes the intrinsic geometry of a surface.**
- **does not change even if the surface is bent without stretching or compressing it.**

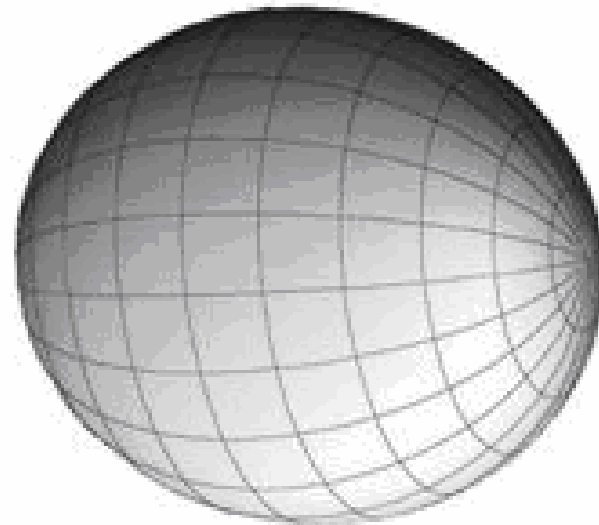
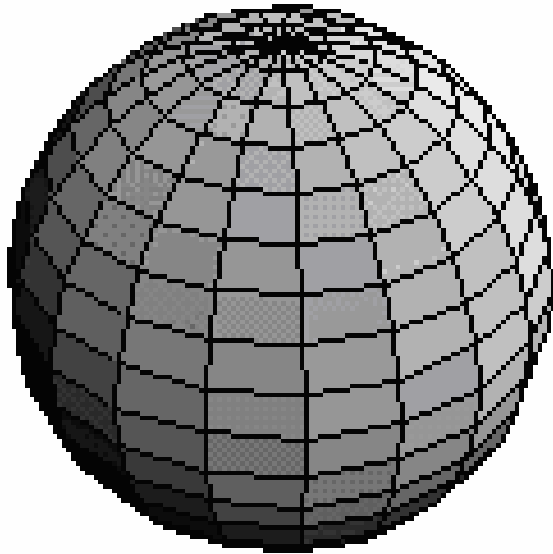
Zero Gaussian Curvature

Euclidean surfaces



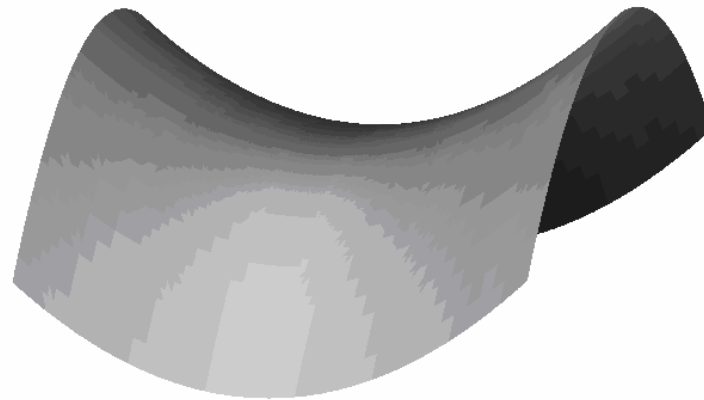
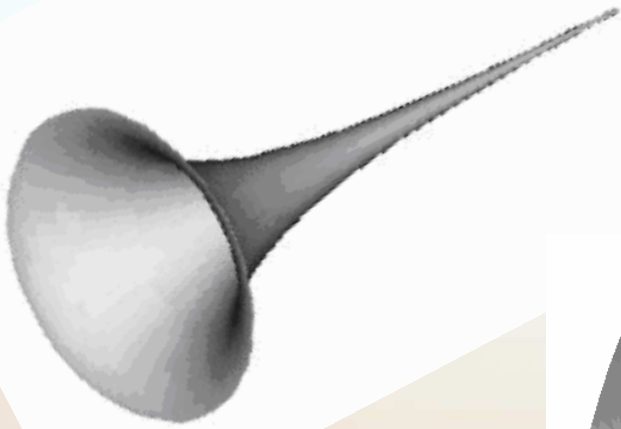
Positive Gaussian Curvature

Elliptic surfaces



Negative Gaussian Curvature

Hyperbolic surfaces



**What is the Gaussian Curvature of a
cylindrical surface
(a cylinder without a top or a bottom) ?**



Euclid's Postulates

- 1. A straight line may be drawn from any point to any other point.**
- 2. The straight line may be produced to any length.**
- 3. Around any point as a center, a circle of any radius may be described.**
- 4. Any two right angles are equal in measure.**

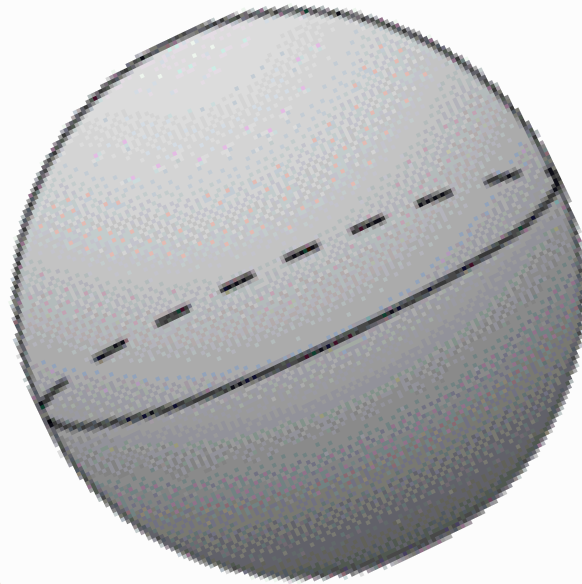
The Fifth Postulate

a.k.a. The Parallel Postulate

5. Given a line and a point not on the line, there is exactly one line through the given point not intersecting the given line.

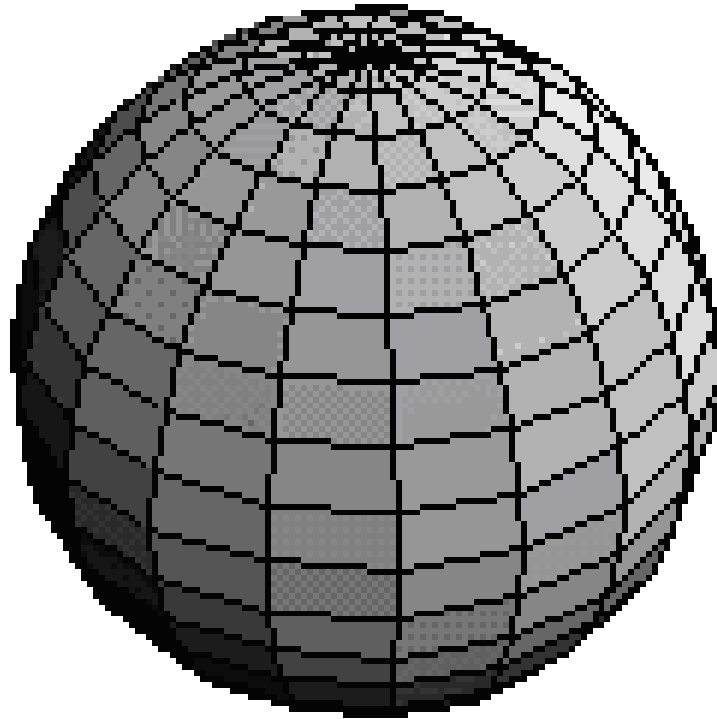
Is this fifth postulate always true?

What are lines in Elliptic Geometry?

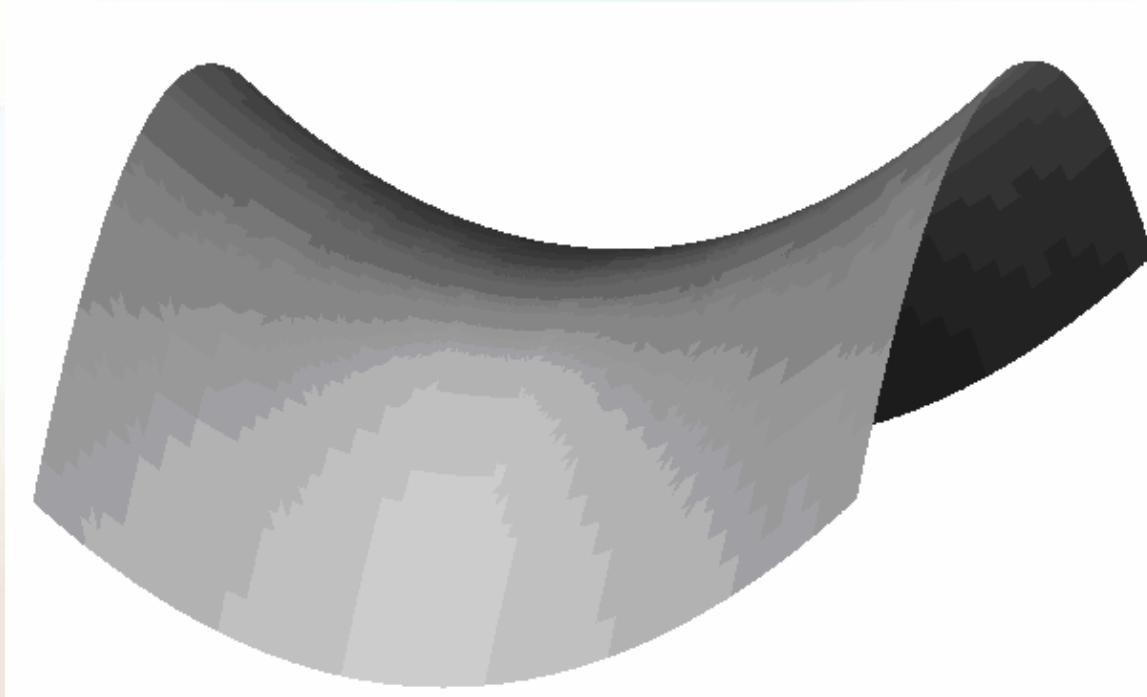


Elliptic Lines

Can they be parallel?

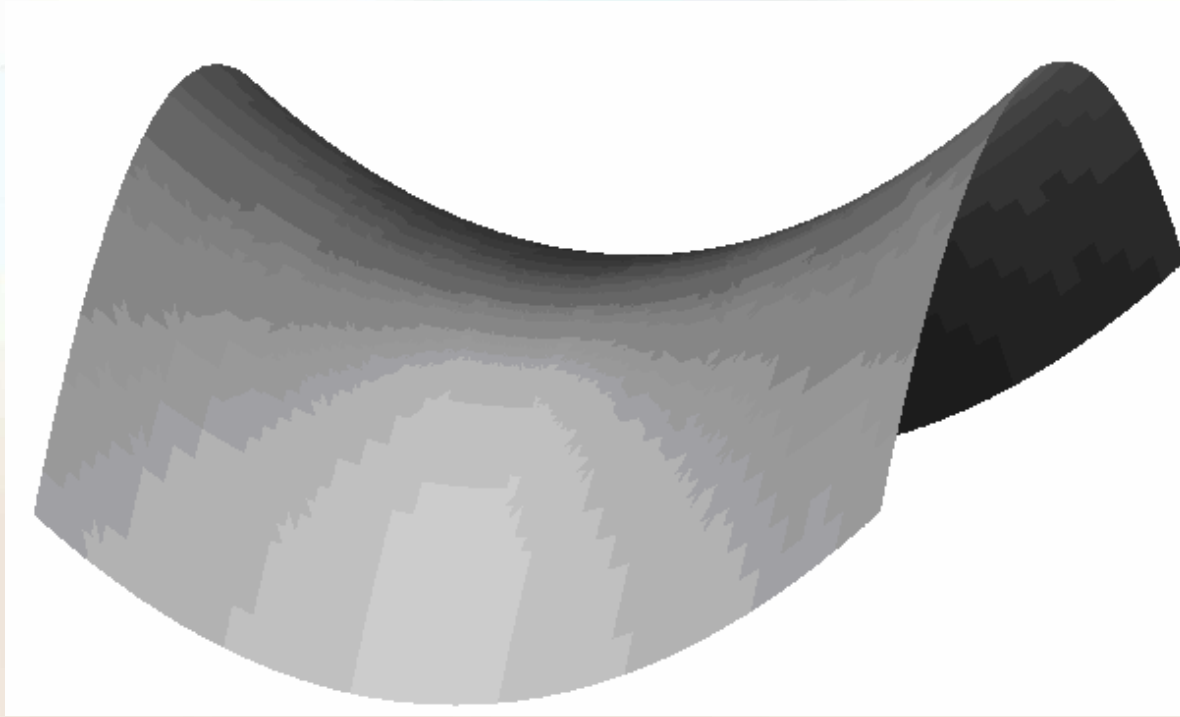


What are lines in Hyperbolic Geometry?



Hyperbolic Lines

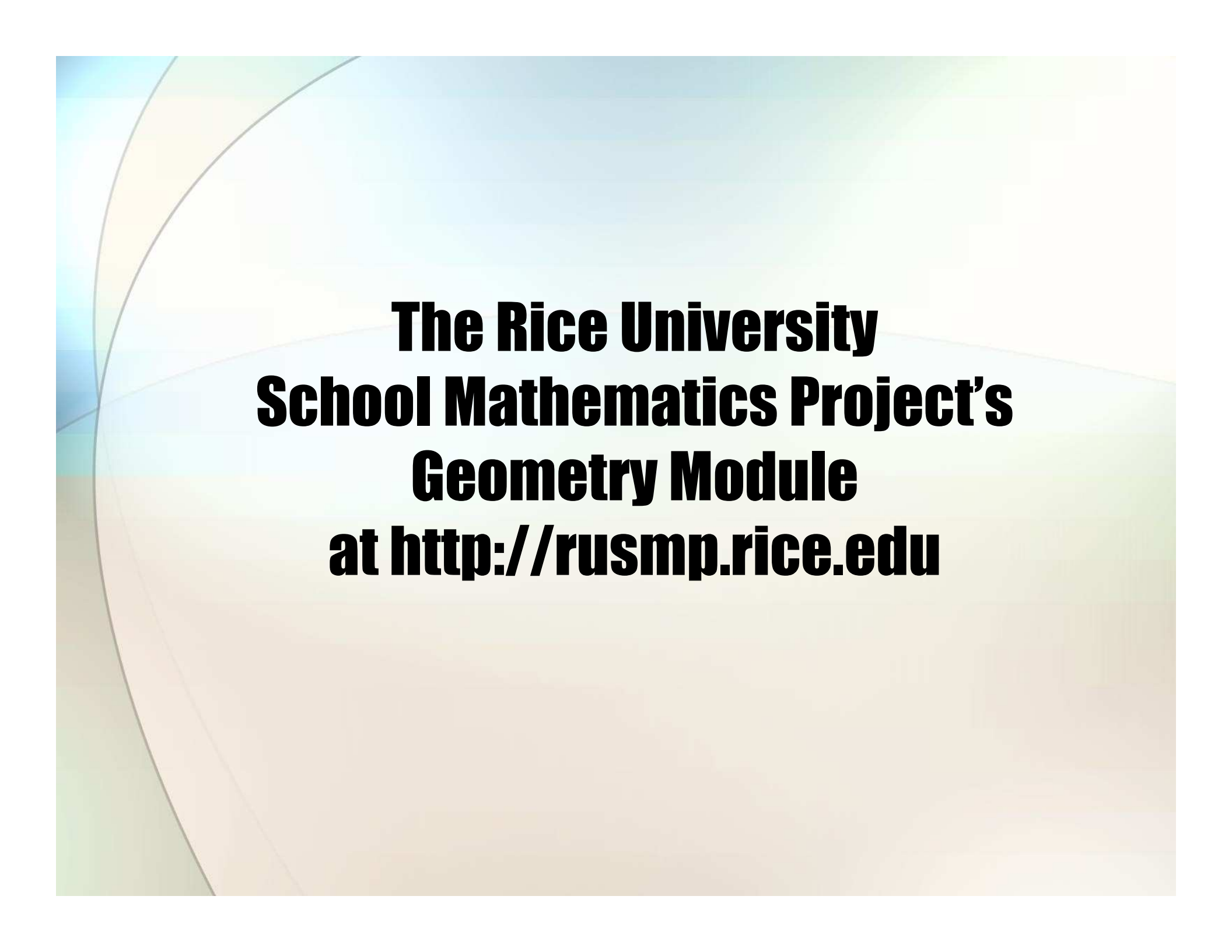
Can they be parallel?



Further Investigations

What is the sum of the measures of the interior angles of a triangle in

- **Euclidean Space?**
- **Elliptic Space?**
- **Hyperbolic Space?**



**The Rice University
School Mathematics Project's
Geometry Module
at <http://rusmp.rice.edu>**