Triangles

Areas of triangles

in different geometries

The geometries which are

complete
(lines go on forever)

homogeneous
(geometry is the same at each point and in each direction)

and two-dimensional

are classified by the points on the number line

For each geometry
there is a real number R
(called its curvature).
For each real number R
there is a unique geometry.

The R-geometry

for a positive number R

is the geometry of the sphere

of radius $1/\sqrt{R}$

The 0-geometry
is Euclidean geometry
(the geometry of the sphere
of *infinite* radius)

The R-geometry

for a negative number R

is the geometry

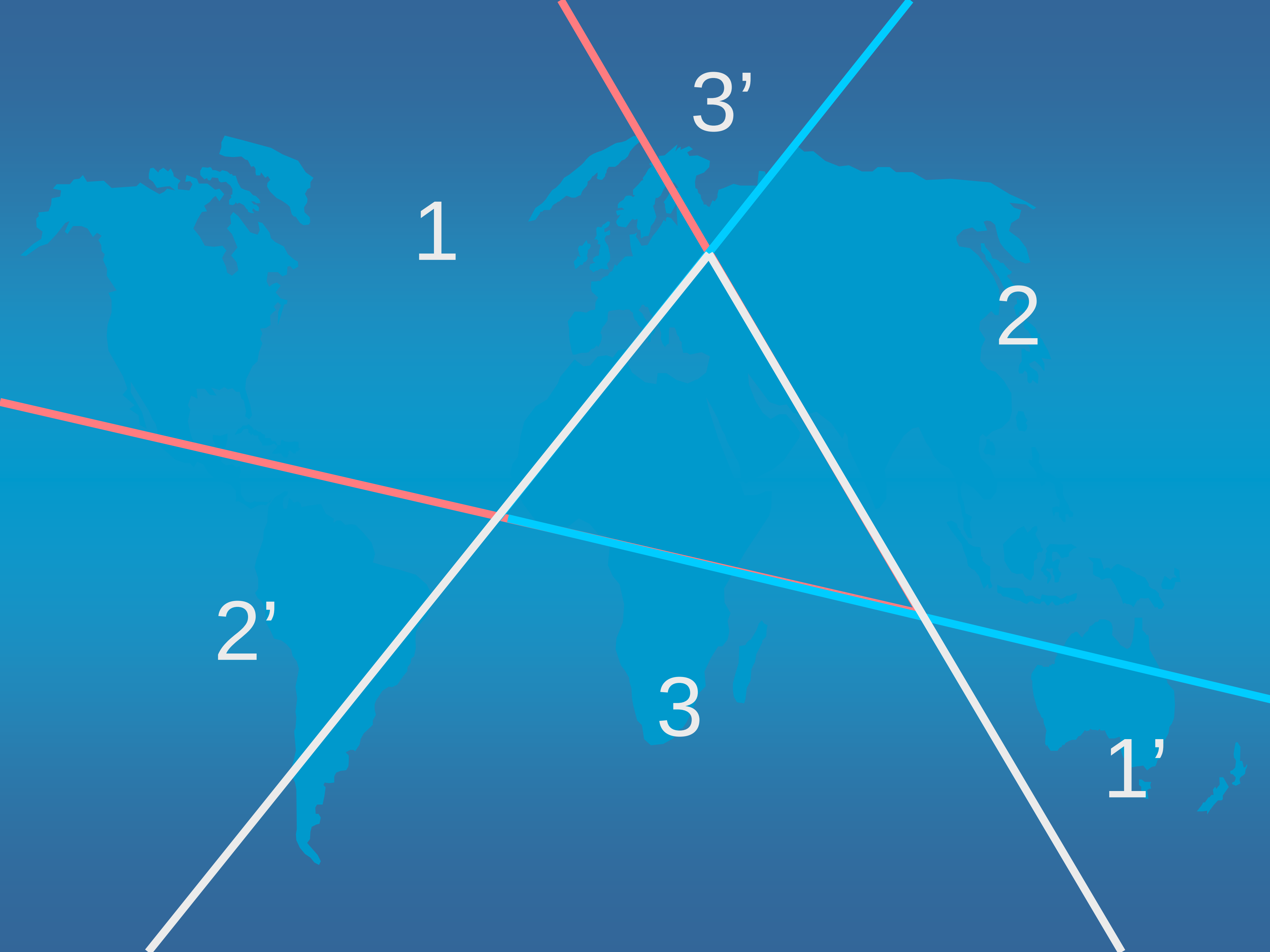
of the (imaginary) $\frac{1}{\sqrt{R}}$

sphere of radius $1/\sqrt{R}$

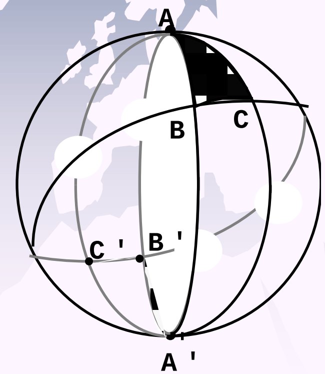
Complete homogeneous 2-dimensional geometries

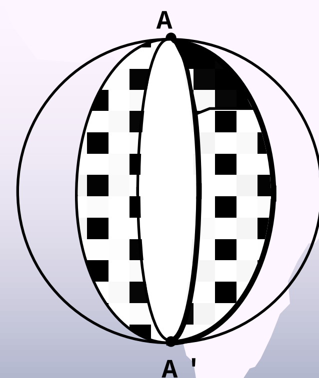
- R > 0: We've experienced one of these
  (we like to think we live on it)
  but it doesn't really exist
- R = 0: We used to think we lived in it
  but it exists only in our minds
  (and in 8-th and 9-th grade classrooms)
- R < 0: We can hardly imagine it
  (it took 3000 years)
  but it's the only one that's real!!!

Triple triangle covers



3’
1
2
2’
3
1’


A
B
C
C '
B '
A '


A
A '

Slice 1 + Slice 2 + Slice 3

+ Slice 1' + Slice 2' + Slice 3'

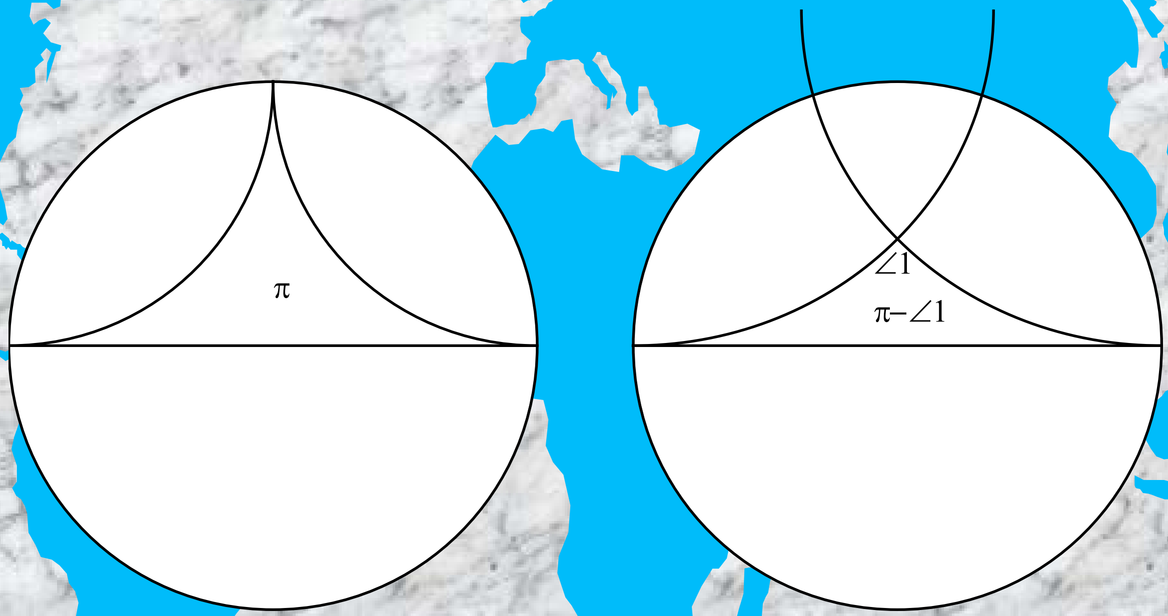
$= 4\pi + \square 2(\text{Area } \Delta)$

$\square 2(\angle 1 + \angle 2 + \angle 3)$

$= 4\pi + 4(\text{Area } \Delta)$

$$(\angle 1 + \angle 2 + \angle 3) - \pi$$

$$= (\text{Area } \Delta)$$



π
∠1
π−∠1

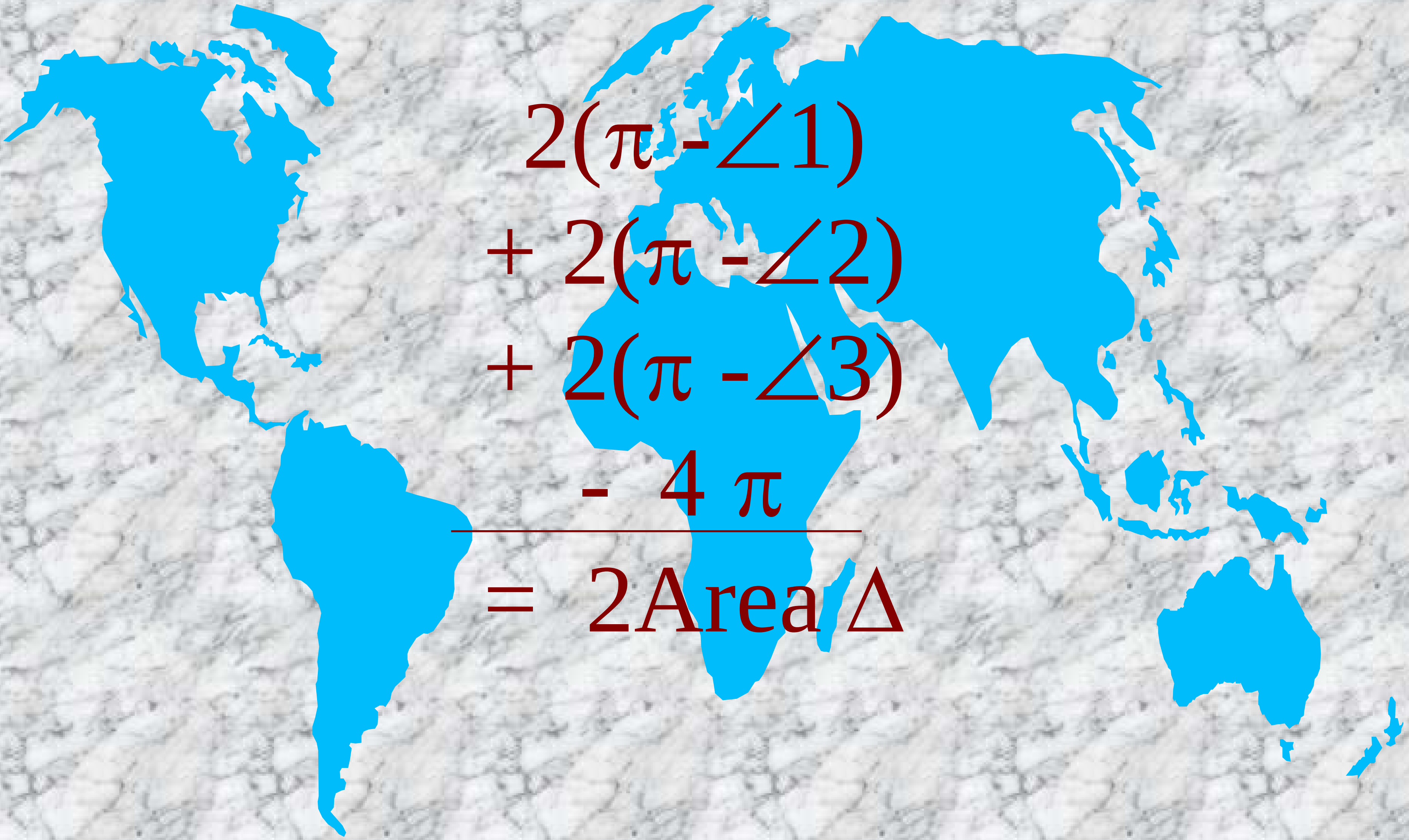

3’
1
2
2’
3
1’

$$\text{Slice } 1 + \text{Slice } 2 + \text{Slice } 3$$

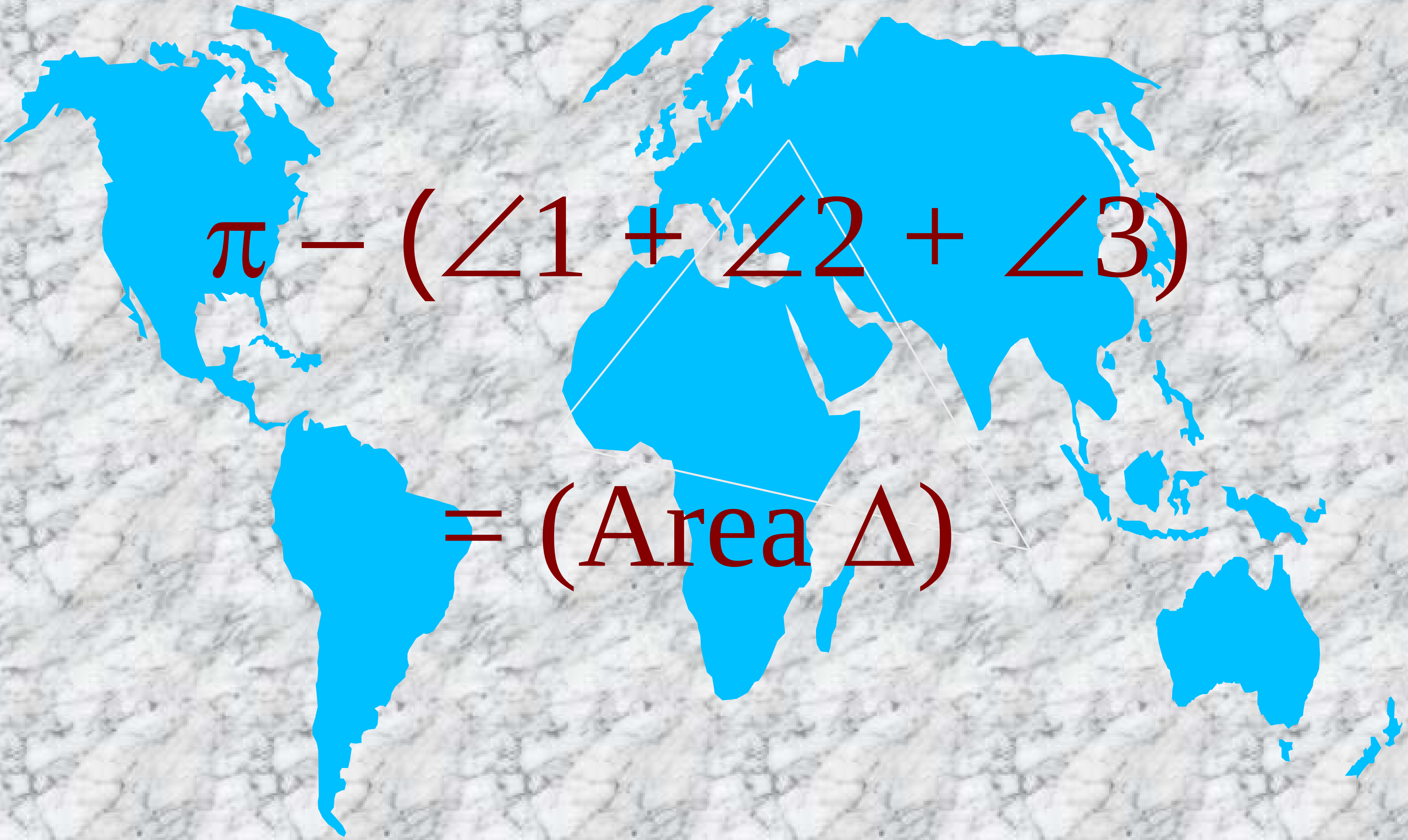
$$+ \text{Slice } 1' + \text{Slice } 2' + \text{Slice } 3'$$

$$- \; 4\pi$$

$$= \; 2\text{Area } \Delta$$



$$
\begin{aligned}
& 2(\pi - \angle 1) \\
+ \; & 2(\pi - \angle 2) \\
+ \; & 2(\pi - \angle 3) \\
- \; & 4\pi \\
\hline
= \; & 2 \text{Area} \, \Delta
\end{aligned}
$$

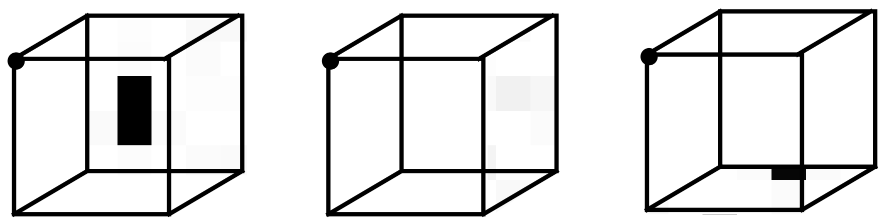


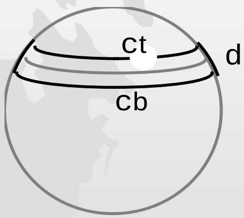
$$\pi - (\angle 1 + \angle 2 + \angle 3) = (\text{Area } \Delta)$$

Higher dimensions

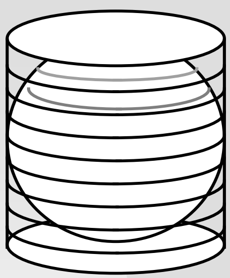
Spheres in higher dimensions

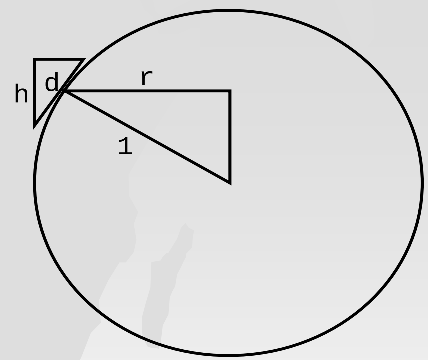
$$m(B(n+1;r)) = r/(n+1)\ m(S(n;r$$





m(S(n+1;r)) = 2šr m(B(n;r))





And we end
with a commercial message…

Rationale for the IAS/Park City Mathematics Institute

- Bring different groups of mathematics professionals together, each for their own professional self-interest:

  --mathematics research (special topic each year)

  --graduate summer school (graduate students can learn a field with the best)

  --superb group of undergrads gets an introduction to a career in mathematics)

Rationale for the IAS/Park City Mathematics Institute

- Bring different groups of mathematics professionals together, each for their own professional self-interest:

  --high school teachers (math learning, reflection on practice, becoming a resource)

  --math education researchers (new paradigm: collaborative research with content specialists)

  --undergraduate teaching faculty (reflection on practice, math learning, becoming a resource)

Rationale for the IAS/Park City Mathematics Institute

- Bring different groups of mathematics professionals together, nationally and internationally (learning, quality control)
- Each groups pursues their own professional development (highest possible standards: challenge to excel)
- Some exploration of shared purpose (based on enlightened self-interest)
- Evolving mutual understanding and mutual support (a political necessity--also a good thing)

http://www.ias.edu/parkcity