Learning Mathematics:

Perspectives from Developmental Psychology

Overview:

- Becoming numerate
- What we know about learning
- Some current research
- Conclusions

To be numerate:

- Children need to think logically
  - Examples:
    - Counting -- Counting requires understanding core principles (one to one correspondence, constant order, cardinality)
    - Transitivity -- A > B, B > C, then A > C

To be numerate:

- Children need to learn conventional systems
  - Examples:
    - Counting in different languages
    - Systems of representation such as graphs, charts, tables, equations

To be numerate: Number systems

- Chinese has a very strict base-10 system
- English is less uniform (eleven, twelve)
- Japanese uses different terms for counting different things

To be numerate:

- Children need to use their mathematical thinking meaningfully and appropriately in situations
  - Example
    - Understanding conceptually the situation a problem represents so that you can choose the proper procedure

To be numerate:

- Measuring two boards to see if you have cut them to the same length
  - You need to understand that if A = B and B = C, then A = C
  - You need to understand our system of measure and the relationships between units
  - You need to know which tool (ruler, tape measure) to use and which units or measures are appropriate (inches, feet, length, area)

Learning: Piaget

- The child actively constructs knowledge from experience in the world
- Learning involves a tug-of-war between **assimilation** and **accommodation**
- New understanding is based on previous understanding

Learning: Vygotsky

- Children learn by engaging in meaningful activity

- At a given time, there are some tasks a child can do only with support from another individual

- Cognitive development is supported through interactions with more capable individuals

Learning: Vygotsky

- The **zone of proximal development** (ZPD) is the range of tasks a student cannot do independently, but can do with help and guidance.
- Students learn the most by trying tasks within the ZPD.
- **Scaffolding** is help that allows a student to complete a task they are not able to do independently.

Learning: Vygotsky

A 6 year old has lost a toy and asks her father for help finding it. The father asks her where she last saw the toy, the child says, “I can’t remember.” He asks a series of questions -- “Did you have it in your room? Outside? Next door?” To each question, the child answers, “No.”When he says, “in the car?” the child says, “I think so” and finds the toy.

Who remembered?

Learning: Vygotsky

- We need to consider two aspects of development
  - Actual developmental level: The things students can do independently
  - Level of potential development: The things students can do with the help of a more competent individual

Principles of Learning:

- Learning involves relating new information to prior knowledge
- Students actively construct their knowledge
- Social interaction is essential for cognitive development

Examples from research:

- Patterns to look for:
  - Students' prior conceptions of mathematical concepts and procedures will shape their new understanding
  - Students often have trouble linking mathematical conventions to their understanding of mathematical concepts

Informal knowledge of math:

- Counting
  - Ability to carry out counting procedures
  - Understanding of underlying concepts
- Knowledge of relative quantity
- Ability to carry out simple addition and subtraction
  - Often use counting as the solution strategy

Counting:

- Methods of studying of children’s counting
  - Observed young children counting objects
  - Asked children to identify errors in counting

- Children understand the three basic principles
  - One-to-one correspondence
  - Constant order
  - Cardinality

The transition to school:

I: How much is 3 plus 4.
S: Six.
I: Uh-huh, how did you know that?
S: I was like thinking and counting.
I: Thinking and counting at the same time? Can you do that out loud for me? How do you do 3 plus 4?
S: I had three in my head. And I had four in my head. So, I had three plus four… (she uses her fingers to count)…three, four, five, six, seven…. Seven.

The transition to school:

I: Can you write this for me: 3 plus 4 equals 7.
[Toby writes 3 + 4 = 7]
I: OK, can you write 5 equals 2 + 3?
S: [writes 5 = 2 + 3] I hope….yeah, you could…
I: You could?
S: I think….
I: Can you write it that way?
S: Yeah, I just wrote it!
I: You just…(they laugh together). You just wrote it, but is it right to do it that way? [Toby shakes her head]
I: It's not right. What's wrong with it?
S: Because this should be over here and this should be over here.

The transition to school:

I: Can you tell me, what does the plus mean?
S: I'm not sure. I don't know.
I: I mean, does plus tell you to do something… what is it all about? [A pause. Toby shrugs.]
Not sure? What about equals? What does equals mean?
S: It tells you, um, I'm not sure about this….I think it tells you three plus four, three plus four, so it's telling you, that, um, I think, the, um, the end is coming up. The end.
I: What do you mean the end is coming up?
S: Like, if you have equals, and so you have seven, then. So, if you do three plus four equals seven, that would be right.
I: That would be right, so equal means something is coming up…like the answer.

The transition to Algebra:

- Because of experiences with arithmetic, students have a hard time accepting an algebraic expression as an answer.

- Arithmetic is often oriented toward producing answers (a single number)

- In algebra, $x + 7$ describes the operations and the result (the answer)

Buggy algorithms:

12	18	26
-4	-6	- 8
12	12	22

307	856	606	308	835
-182	- 699	- 568	-287	-217
285	157	168	181	618

Buggy algorithms:

I: I'll tell you some numbers and you tell me what comes next. Like, if I said, 1,2,… you would say…
S: Three
I: OK. 57, 58…
S: [jumps in quickly with]: 59
I: OK, 68, 69
S: [hesitates a little]: 70.
I: 84,85
S: 86
I: 87,88
S: 89
I: 89
S: Eighty-tennnn…

Buggy algorithms:

Expression	Incorrect translation
$3x \quad (x = 2)$	32
$(x + y)^2$	$x^2 + y^2$
$\dfrac{x}{y + z}$	$\dfrac{x}{y} + \dfrac{x}{z}$
$x \cdot (yz)$	$xy \cdot xz$

Word problems:

- Students often solve an arithmetic word problem without writing an equation, or they translate the words of the problem directly into mathematical symbols.

- They do not consider the situation that the problem represents.

Word Problems:

At Kroger, butter costs 65 cents per stick. This is 2 cents less per stick than butter at Randall's. If you need to buy 4 sticks of butter, how much will you pay at Randall's?

Common error:

4 (65 - 2)

Correct approach:

4 (65 + 2)

Nearly all students who got the word problem wrong were able to generate the correct answer to the simple computational problem 4 (65 + 2).

Word problems:

There are 6 times as many students as teachers at this school. Use S for the number of students and T for the number of teachers.

Common error: $6S = T$

Word problems:

Source of errors:

1) Syntactic approach: translate the words literally into symbols
2) Static comparison: 6S represents a static group of students, T represents a static group of teachers. The = sign does not represent a mathematical relationship, it just separates the two groups.

Correct interpretation:

S and T are quantities NOT static groups. An operation must be performed to make the two quantities equal.

Could be written: $S = 6T$ or $S/6 = T$

We have seen that:

- Students actively try to make sense of the world and construct their own understanding.
- Students' prior knowledge shapes their new understandings. (This sometimes leads to <u>mis</u>understandings.)
- Students struggle with linking mathematical concepts to mathematical conventions.

Implications for Instruction:

- Students need opportunities to explore concrete situations with guidance.
- We need to help students link their understanding of concepts to their use of mathematical conventions (and procedures).
- Assessments that provide a window into what students understand and **how** they are thinking are essential.