Hands-On Activities for the Algebra 2 Classroom

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CAMT 2015
Thursday, June 25
2:30 pm - 3:30 pm
Room 380A
George R. Brown Convention Center
Houston, Texas

Description

- Hands-on activities for discovery, review, or assessment of the understanding of Algebra 2 topics will be presented.

- Discovery activities provide students opportunities to:
  - recognize the mathematics underlying real-life situations,
  - make connections,
  - construct a deeper understanding of topics, and
  - develop an appreciation of how to look at the world with mathematical eyes.

- Games and puzzles allow students to practice their skills and demonstrate their understanding.

Origin of Logarithms

Problem:
Multiplication and division of multi-digit numbers is tedious, time-consuming, and prone to error

Objective:
Simplify mathematical calculations (multiplication and division) to the level of addition and subtraction

- John Napier (1550 - 1617)
  - Scottish mathematician
  - Devised method that would convert multiplication problems into simpler addition problems for use in astronomy
  - Assembled tables (took 20 years)
  - Algebraic approach published in 1614
    - *Mirifici Logarithmorum Canonis Descriptio (Description of the wonderful canon of logarithms)*
  - Named "logarithm" based on Greek words:
    - *Logos* – meaning ratio (or proportion)
    - *Arithmos* - meaning number

- Joost Burgi (1552 - 1632)
  - Swiss clockmaker/German mathematician
  - Geometric approach published in 1620

- Henry Briggs (1561 - 1630)
  - English mathematician
    - Suggested base 10
  - Combined effort with Napier invented common system of logarithms (common logs) in 1615
    - log(1)=0 and log(10)=1

- William Oughtred (1575 - 1660)
  - English mathematician
  - Realized 2 sliding rules with labels placed in logarithmic scale will physically perform addition of logarithms allowing the result of any multiplication be read off
  - Made slide rule in 1622 by placing 2 logarithmic scales next to each other.
    - Removed need to look up values in a logarithm table by instead requiring values to be aligned in order to perform the multiplication, division and many other operations (depending on the model)
    - Slide rule used in science and engineering until electronic calculators in the 1970s

- John Speidell
  - Mathematics teacher from England
  - Published *New Logarithmes* in 1619 that showed natural logarithms (but omitted decimal points)

- Nicholas Mercator (1620 - 1687)
    - German mathematician
    - First used term "natural logarithm" in the Latin form '*log naturalis*' in *Logarithmo-technica* published in 1668

- Over 100 years later, it was realized that Napier's logarithms are simply the inverse of exponents.

- Leonard Euler (1707 - 1783)
    - Swiss mathematician/physicist
    - Developed today's notation for a logarithm in the late 1700s in terms of bases and exponents
    - Related exponential and logarithmic functions by defining $\log_x y = z$ to hold true when $x^z = y$

Traditional Presentation of Logarithms

When logarithms are presented to students, it is often done in a way that feels remote and abstract with many rules to memorize and foreign notation.

The word "logarithm" is a confusing name for a concept that is actually very simple.

Few students have trouble reading a statement:

$$power_2(32) = 5$$

"The power of 2 that gives the answer 32 is 5."

However, as soon as we write $\log_2(32) = 5$, clarity and transparency is replaced by horror and fear!

Activities

- Super Fun Puzzle
- Bullfighter Riddle
- Logging Time
- Logarithm Maze
- Natural Logarithm Maze
- Log Dominos

Revised Algebra II TEKS

(2) Attributes of functions and their inverses. The student applies mathematical processes to understand that functions have distinct key attributes and understand the relationship between a function and its inverse. The student is expected to:

(A) graph the functions
$f(x)=\sqrt{x}$, $f(x)=\frac{1}{x}$, $f(x)=x^3$, $f(x)=\sqrt[3]{x}$, $f(x)=b^x$, $f(x)=|x|$,
and $f(x) = \log_b(x)$ where b is 2, 10, and e, and, when applicable, analyze the key attributes such as domain, range, intercepts, symmetries, asymptotic behavior, and maximum and minimum given an interval;

(B) graph and write the inverse of a function using notation such as $f^{-1}(x)$;

(C) describe and analyze the relationship between a function and its inverse (quadratic and square root, logarithmic and exponential), including the restriction(s) on domain, which will restrict its range; and

(D) use the composition of two functions, including the necessary restrictions on the domain, to determine if the functions are inverses of each other.

(5) Exponential and logarithmic functions and equations. The student applies mathematical processes to understand that exponential and logarithmic functions can be used to model situations and solve problems. The student is expected to:

(A) determine the effects on the key attributes on the graphs of $f(x) = b^x$ and $f(x) = \log_b(x)$ where b is 2, 10, and e when $f(x)$ is replaced by $af(x)$, $f(x) + d$, and $f(x - c)$ for specific positive and negative real values of a, c, and d;

(B) formulate exponential and logarithmic equations that model real-world situations, including exponential relationships written in recursive notation;

(C) rewrite exponential equations as their corresponding logarithmic equations and logarithmic equations as their corresponding exponential equations;

(D) solve exponential equations of the form $y = ab^x$ where a is a nonzero real number and b is greater than zero and not equal to one and single logarithmic equations having real solutions; and

(E) determine the reasonableness of a solution to a logarithmic equation.

Usefulness of Logarithms

- To find the number of payments on a loan or time to reach an investment goal

- To model natural processes in living systems

- To measure pH or acidity of a chemical solution
  (The pH is the negative logarithm of the concentration of free hydrogen ions.)

- To measure earthquake intensity on the Richter scale.

- To analyze exponential processes. (e.g., cooling of a dead body, growth of bacteria, decay of radioactive isotopes, spread of an epidemic in a population)

- To solve forms of area problems in calculus

1	$\ln e$
$\log_2 6 - \log_2 3$	3
lne^2	$\log_8 16 + \log_8 4$
$\log_4 8 - \log_4 \frac{1}{2}$	4
$2\log_4 8$	$\log_4 32 - \log_4 \frac{1}{8}$
$\ln e$	$\log 10$

$\log 10$	2
$\log_{12} 4 + \log_{12} 3$	$\log_3 81$
$\log 100$	$\ln e^3$
$\log 1000$	$\log_6 4 + \log_6 54$
$2\ln e^2$	$\log 200 + \log 50$
$\log_2 6 - \log_2 3$	$\log_4 8 - \log_4 \frac{1}{2}$

1	$\log_6 4 + \log_6 54$
2	$\log 100$
$\log_8 16 + \log_8 4$	$\log 200 + \log 50$
3	4

$\log_{12} 4 + \log_{12} 3$	$2 \ln e^2$
$2 \ln e^2$	$\ln e^3$
$\log 1000$	$2 \log_4 8$
$\log_3 81$	$\log_4 32 - \log_4 \frac{1}{8}$

What do you call four bullfighters in quicksand?

Instructions: Use the properties of logarithms to simplify each expression.

1. $\log_5(50)$	2. $\log_5(100)$
3. $\log_5(150)$	4. $\log_5(625)$
5. $\log_5\left(\frac{150}{625}\right)$	6. $\log_5\left(\frac{375}{1250}\right)$
7. $\log_5\left(\frac{150}{3125}\right)$	8. $\log_5\left(\frac{4375}{250}\right)$
9. $\log_5\left(\frac{4375}{50}\right)$	10. $\log_5\left(\frac{150}{750}\right)$

K -1	S $-3 + \log_5 6$	A $2 + \log_5 6$	U $2 + \log_5 4$
Q $2 + \log_5 2$	N $2 + \log_5\left(\frac{7}{2}\right)$	O $-1 + \log_5\left(\frac{3}{2}\right)$	I $1 + \log_5\left(\frac{7}{2}\right)$
R $-2 + \log_5 6$	M $1 + \log_5\left(\frac{5}{7}\right)$	W 1.25	T 4

___ ___ ___ ___ ___ ___   ___ ___ ___ ___ ___

1 2 3 4 5 6   7 8 9 10 6

Logarithmic Equations Maze

Directions: Find the solution to each equation to "find the log" and solve the maze. SHOW YOUR WORK!

START: $\log_3 81 = x$

5

$\log_{27} x = \frac{1}{3}$

3

$\log_5 x = 2$

25

$\log_{32} x = \frac{1}{5}$

4 | -4 | 64 | -64 | -25 | 0.1 | 2

$\log_8 x = \frac{1}{3}$

2

$\log_4 x = 3$

12

$\log_9 x = \frac{1}{2}$

3

$\log 0.01 = x$

-2 | -9 | 10 | 6 | -6 | -2 | 10

$\log_{\frac{1}{3}} x = -2$

4

$\log_4 256 = x$

$\frac{1}{9}$

$\log_3 x = -2$

32

$\log_{\frac{1}{5}} x = 2$

9 | $\frac{1}{9}$ | 5 | -9 | 9 | -6 | $\frac{1}{25}$

$\log_{16} x = \frac{1}{4}$

2

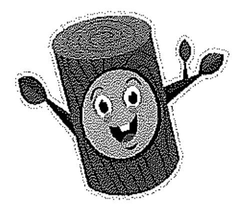
$\log_2 64 = x$

6

$\log_{\sqrt{5}} 5 = x$

2

STOP!



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Logging Time

Directions: Solve each question and place the answer in the indicated row and column of the puzzle. When finished, solve the remaining Sudoku puzzle.

Name ____________________

	A	B	C	D	E	F	G	H	I
1						5			8
2	7				6				
3						8	9	3	
4							3		9
5		5				6	4		2
6			6		1			5	
7	1	2			4	3			
8				1					
9			4	6	8				1

1. Solve for x: $\log_5 25 = x$ **C - 1**

2. Which choice is the expanded form of $\ln[(x-3)(4x+1)]^2$ **I - 8**

   1) $2\ln(x-3)+\ln(4x+1)$ *2)* $2[\ln(x-3)+2\ln(4x+1)]$
   3) $2[\ln(x-3)+\ln(4x+1)]$ *4)* $2[\ln(x-3)-\ln(4x+1)]$

3. Find x: $\log 10^{16} = 2x$ **A - 4**

4. Evaluate: $\ln\left(e^6\right)$ **G - 7**

5. Find x: $\ln 4^4 = \ln 2^x$ **D - 5**

6. Find x to the nearest integer: $e^{6x} = 358{,}700$ **G - 8**

7. Find x to the nearest integer: $x = \log 8000$ **F - 6**

8. Find x: $\log_3(x+2) = 2$ **E - 3**

9. Find x: $\log x = \frac{1}{3}\log 64$ **A - 3**

10. Which choice is equivalent to $y = \log_5 x$? **B - 3**

    1) $x = 5^y$ *2)* $y = 5^x$ *3)* $x = 5\log y$ *4)* $y = \log x^5$

11. Find $f(3)$ when $f(x) = e^{2\ln x}$. **F - 8**

12. Given $f(x) = e^{5x}$, find $\ln(f(1))$. **A - 9**

13. Find x: $\log_4(x+7) = 2$ **B - 6**

14. If $\log(x^3(x-2)^2)$ is expanded to $M\log x + N\log(x-2)$, what is the sum of M and N? **G - 2**

15. Find x to the nearest integer: $3\ln 9x = 12$ **H - 4**

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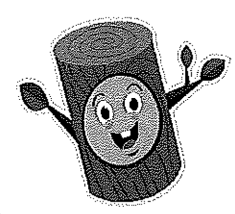
Natural Logarithms Equations Maze

Directions: Find the solution to each equation to "find the log" and solve the maze. SHOW YOUR WORK!

START! $\ln e^x = 6$	4	$\ln x + \ln 3 = 4$	$\frac{e^4}{3}$	$\ln e^{x-2} = 14$	$\ln 10$	$e^{x-2} = 5$
6	$\frac{7}{2}$	$\ln\left(\frac{7}{2}\right)$	$\ln\left(\frac{2}{7}\right)$	16	$2 + ln5$	7
$\ln x + \ln 4x = 2$	$\frac{e^2}{4}$	$4e^x = 14$	$\ln 3$	$e^{\frac{x}{2}} = 4$	$4 \ln 2$	$e^{\ln 3x} = 12$
$\frac{e}{2}$	$\frac{e^3}{2}$	$2 \ln 7$	$\ln 4.5$	$\ln 8$	4	36
$-2 + \ln 2x = 1$	$2e^3$	$41 - e^{2x} = 5$	$\frac{3}{2}$	$e^{2x-3} = 1$	$\ln 3$	$\ln x + \ln 5 = 3$
e^6	$\frac{e^4 + 2}{3}$	$\ln 6$	$\frac{2}{3}$	$\ln 1.5$	$e^{1.5}$	$\frac{e^3}{5}$
$\ln(3x - 2) = 4$	$\ln 20$	$e^{4x} = 9$	$\frac{\ln 3}{2}$	$\ln(x+1)^2 = 2$	$e - 1$	STOP! 

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