Developing Algebraic Thinking through Problem-Solving Activities

NCTM 2017 San Antonio, Texas Session #608

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Using Playing Cards

Let's play the game 'Salute' 

- Need three players on the team 
- Deck of cards 
- Paper to write problem
The students were introduced to a system of equations.

**HOW MUCH IS EACH SYMBOL WORTH?**

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<thead>
<tr>
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<th>32</th>
</tr>
</thead>
<tbody>
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<td>29</td>
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<td>25</td>
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<td>33</td>
<td>40</td>
<td>37</td>
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How much is each symbol worth?

Sun? _______   Cloud? _______   Flower? _______

Explain your reasoning.

What is the cost of each fish?

Goldfish? ____________  Beta? ____________  Clown Fish? ____________

Explain your reasoning.

### How Much Does Each Whale Weigh in Tons?

<table>
<thead>
<tr>
<th>SUM</th>
<th>Right Whale</th>
<th>Gray Whale</th>
<th>Blue Whale</th>
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<tbody>
<tr>
<td>330</td>
<td></td>
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<td>170</td>
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<td>375</td>
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<td>295</td>
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**What is the weight of each whale?**

Right Whale? ______  Gray Whale? ______  Blue Whale? ______

**Explain your reasoning.**

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Which is worth more, a SMILE or a FROWN?

Figure This! The costs of combinations of frowns, smiles, and neutral faces are shown. How much is a smile worth?

Hint: Find a way to combine two of the rows or columns that have something in common.

Reasoning about unknowns is essential in studying equations. Economists, nurses, chemists, and engineers all use equations in their work.

What is the value of each face?

Frown? ________  Smile? ________  Neutral face? ________

Explain your reasoning.

Two of Everything

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
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Rule__________________
What would you choose?

Circle your choice:

- Choice A: 100 coins each day for 10 days
- Choice B: 5 coins and a magical pot that doubled the coins each day for 10 days

Justify your reasoning:

Final thought:

After working through the problem, would you still make the same choice? Why or why not?

Birthday Party Patterns

Tom wants to arrange the tables for his birthday party in the shape of a T, for Tom. He is trying to figure out how many tables he will need for different sizes of T’s.

1. Without drawing or building, describe how to make the next arrangement of tables.

2. Use colored tiles to build arrangements 1–5. Explain how your arrangements show your answer to question 1.

3. Use your answers to questions 1 and 2 to complete the table to the right.

4. Without drawing or counting tables, how many tables will Tom need for arrangements 10, 15, and 50? ________________________

On the reverse side of this page, explain how you found your answers.
## Tiling a Patio

<table>
<thead>
<tr>
<th>Patio Number</th>
<th>Number of Brown Tiles</th>
<th>Number of White Tiles</th>
<th>Total Number of Brown and White tiles</th>
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</table>

### Graph

### Equation

### Table

### Solution
Crossing the River Problem

Scenario

Eight adults and two children need to cross a river. A small boat is available that can hold one adult, or one or two children. Everyone can row the boat. How many one-way trips does it take for them all to cross the river?
INTERNET RESOURCES

❖ Scales and Balance

http://nlvm.usu.edu/en/nav/frames_asid_324_g_3_t_2.html

❖ Pan Balance Shapes

http://illuminations.nctm.org/Activity.aspx?id=3531

❖ Function Machine:

http://www.shodor.org/interactivate/activities/FunctionMachine/

❖ Function Machine Math Playground

http://www.mathplayground.com/functionmachine.html

❖ Stop that Creature!

http://pbskids.org/cyberchase/media/games/functions/
Free Apps for the iPad

• **Visual Algebra Puzzles**
  Create your own algebra puzzles then try to solve them! This easy to use, educational tool was designed to work together with Shuttle Mission Math, an algebraic reasoning game in the app store. Puzzles can be solved with at least one of the following visual strategies: Scale Up, Scale Down (multiply or divide).


• **Shuttle Mission Math**
  Shuttle Mission Math is a mathematical puzzle game that makes algebraic thinking both visual and interactive. The goal is to find the weight of each space creature and assemble a team for the next shuttle mission.


• **Algebra Champ**
  Game like environment for solving linear equations

Bibliography


