3-D: The Foundation for Developing Geometric Thinking

Dr. Jackie Sack
RUSMP

Spring Networking Conference

February 24, 2007

Does it make sense to begin with 2-D figures?

- Rectilinearity or straightness?
- Flatness?
- Parallelism?
- Right angles?
- Symmetry?
- Circles?
- Similarity?

What skills are needed?

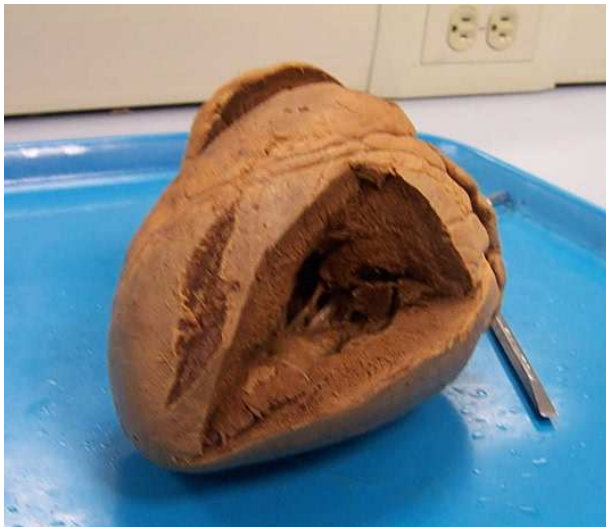
- Turn, shrink and deform 2-D and 3-D objects.
- Analyze and draw perspective views, count component parts and describe attributes that may not be visible but can be inferred.
- Physically and mentally change the position, orientation, and size of objects in systematic ways as understandings about congruence, similarity and transformations develop.

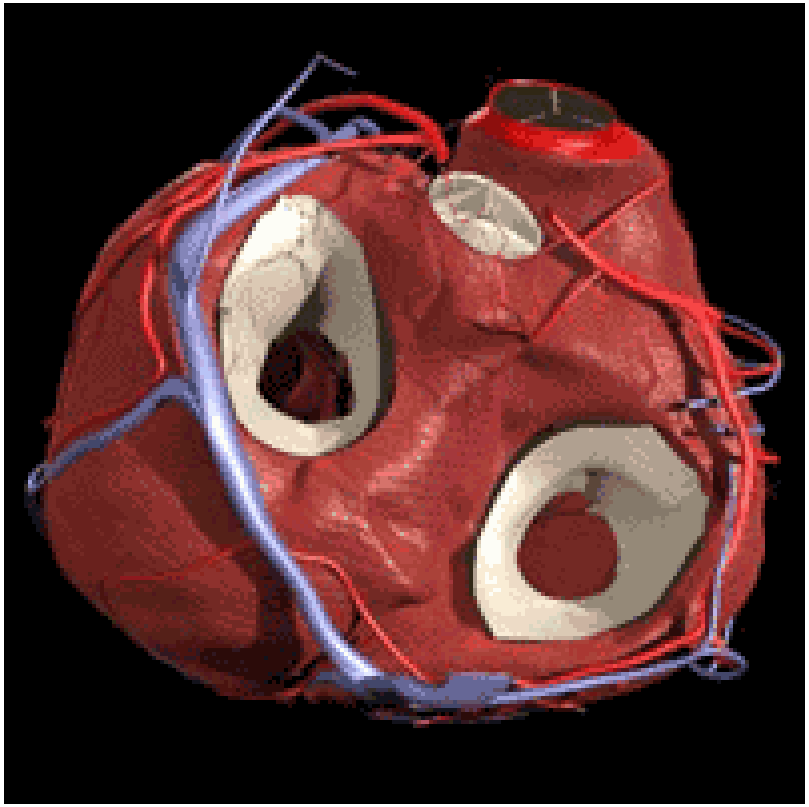
(NCTM, 2000)

TEKS

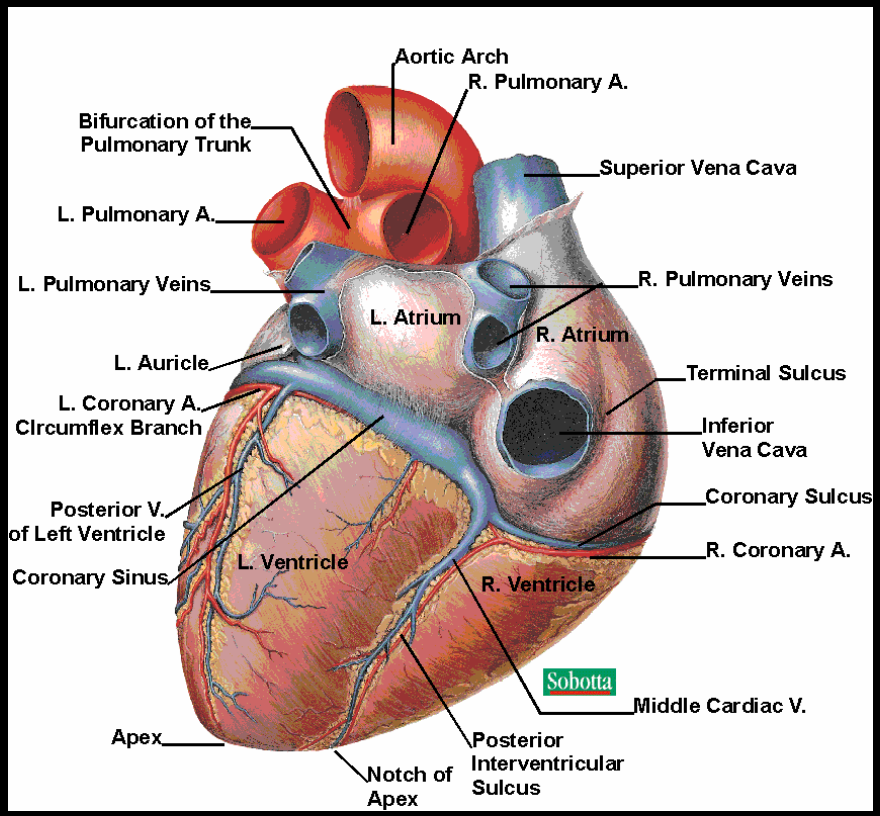
Later…

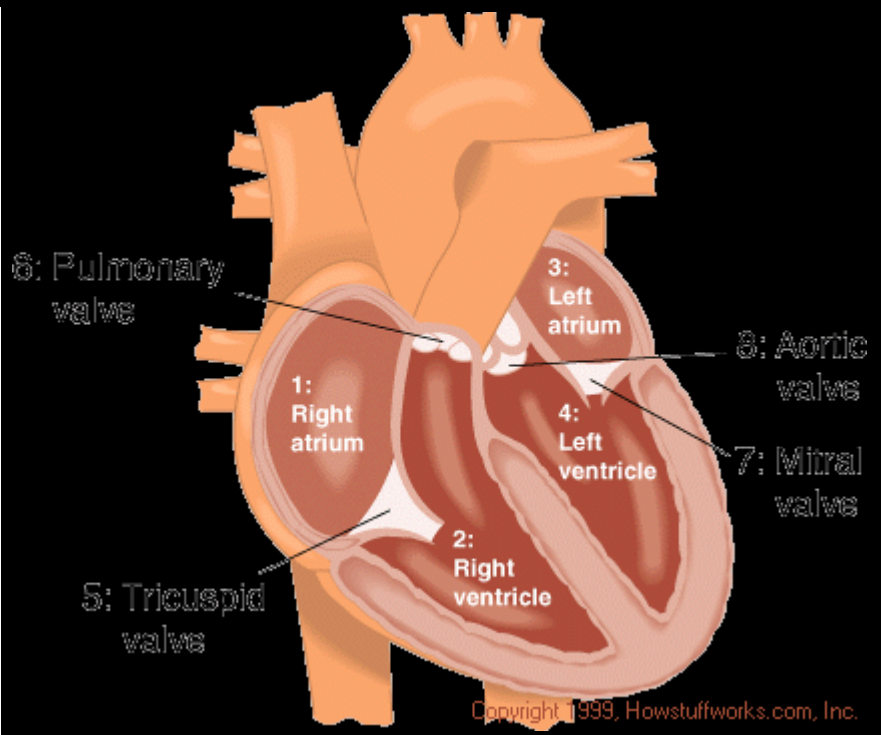
3-D Models



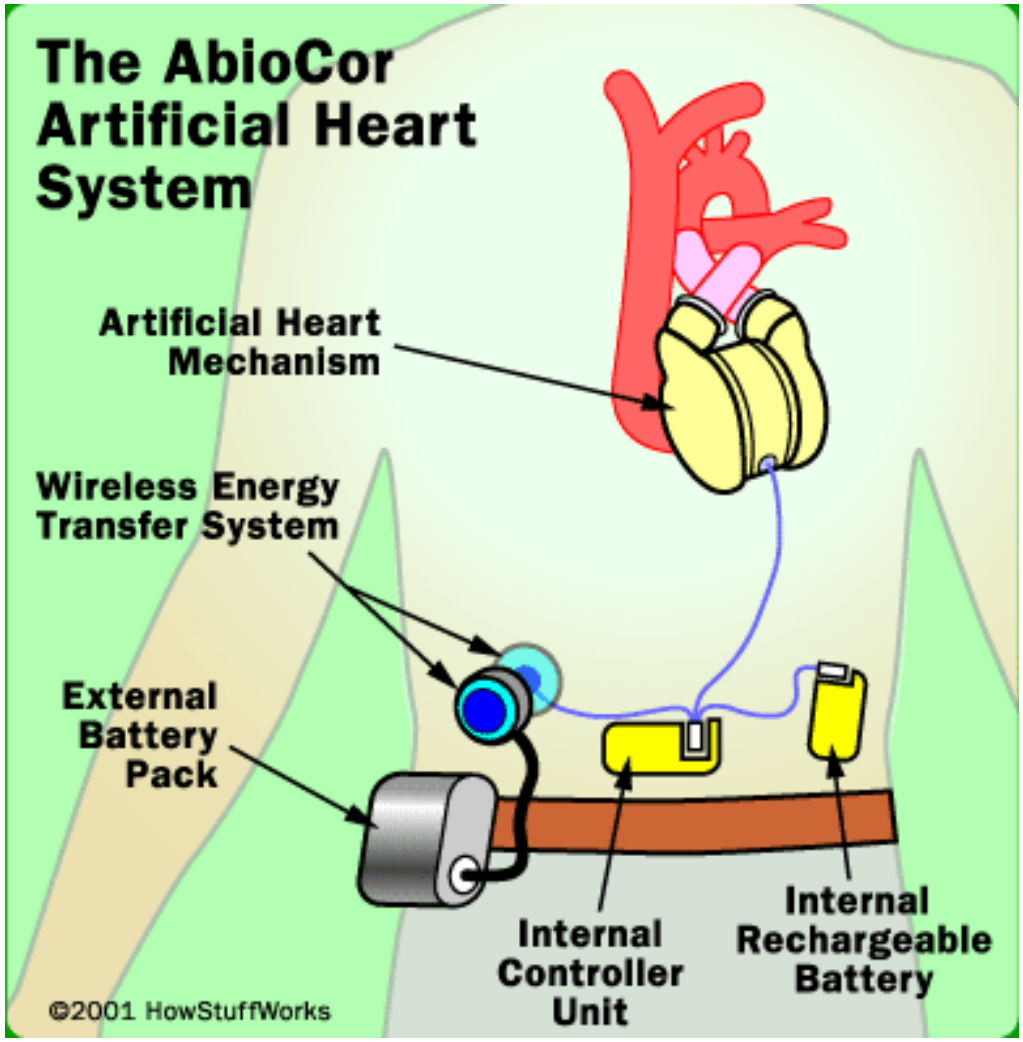


Conventional-Graphic Models

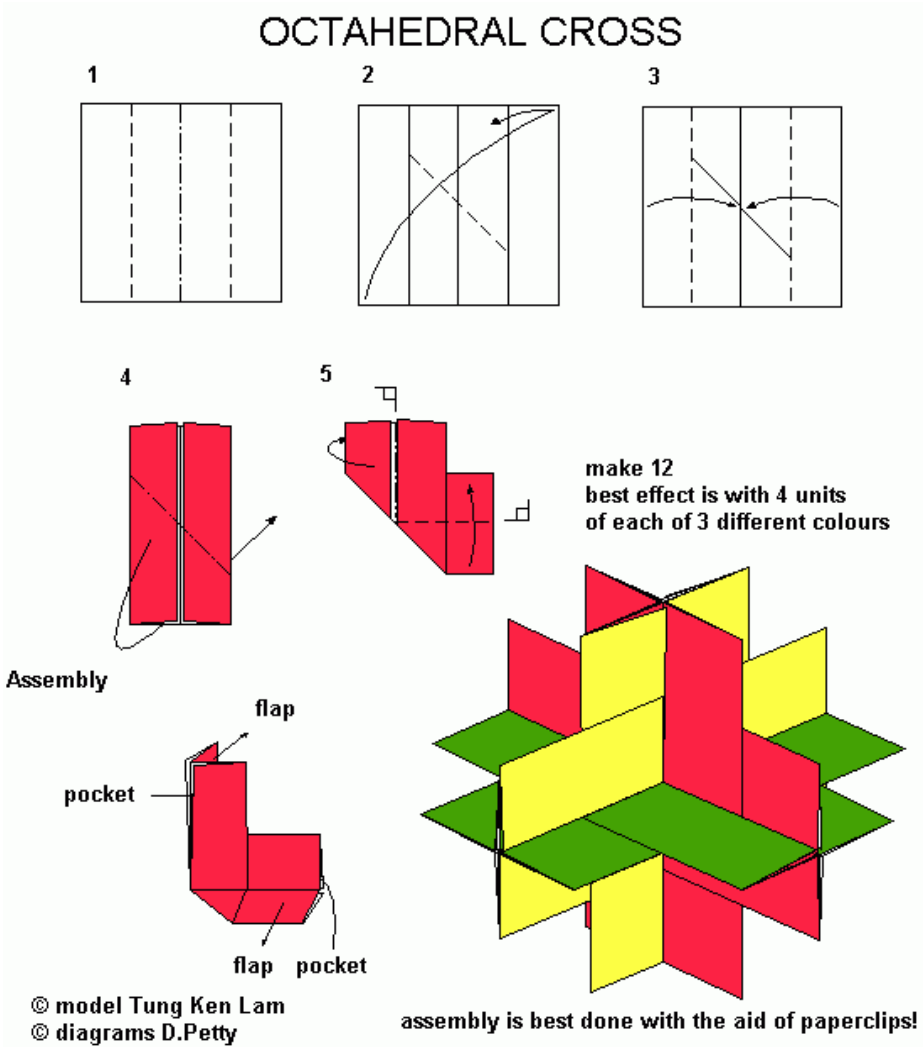




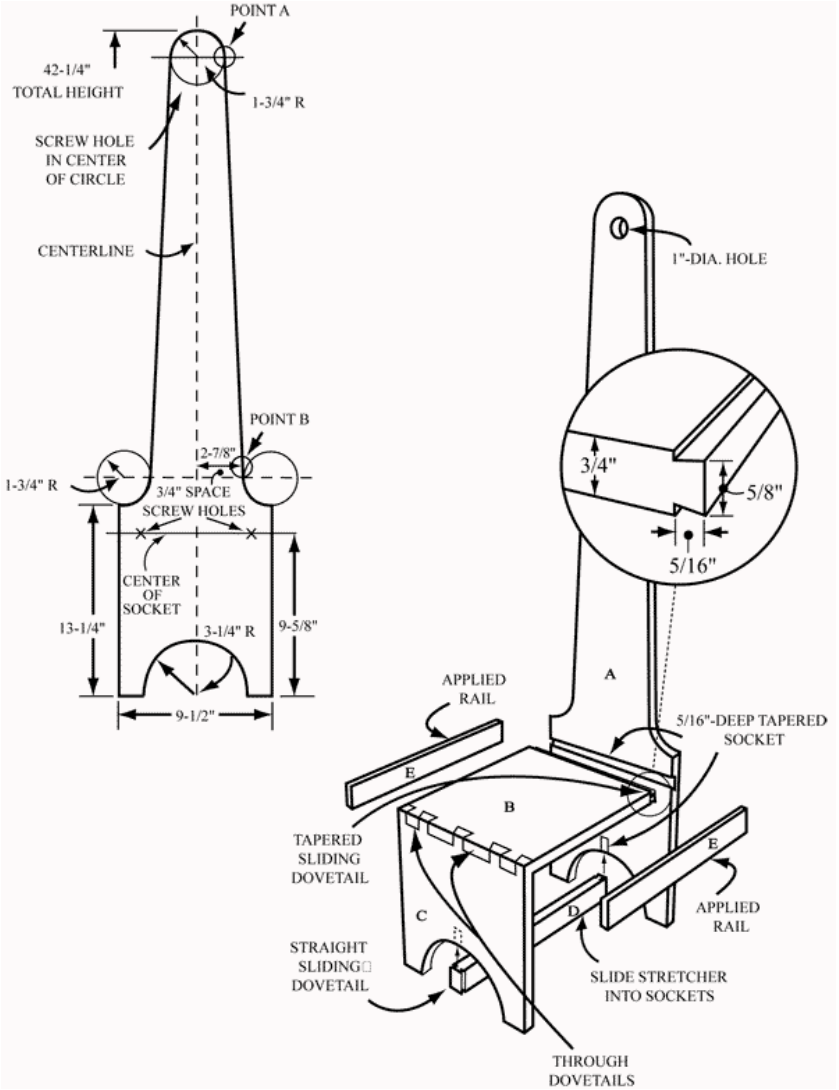
Conventional-Graphic Models: Functional Diagrams



Conventional-Graphic Models: Assembly Diagrams

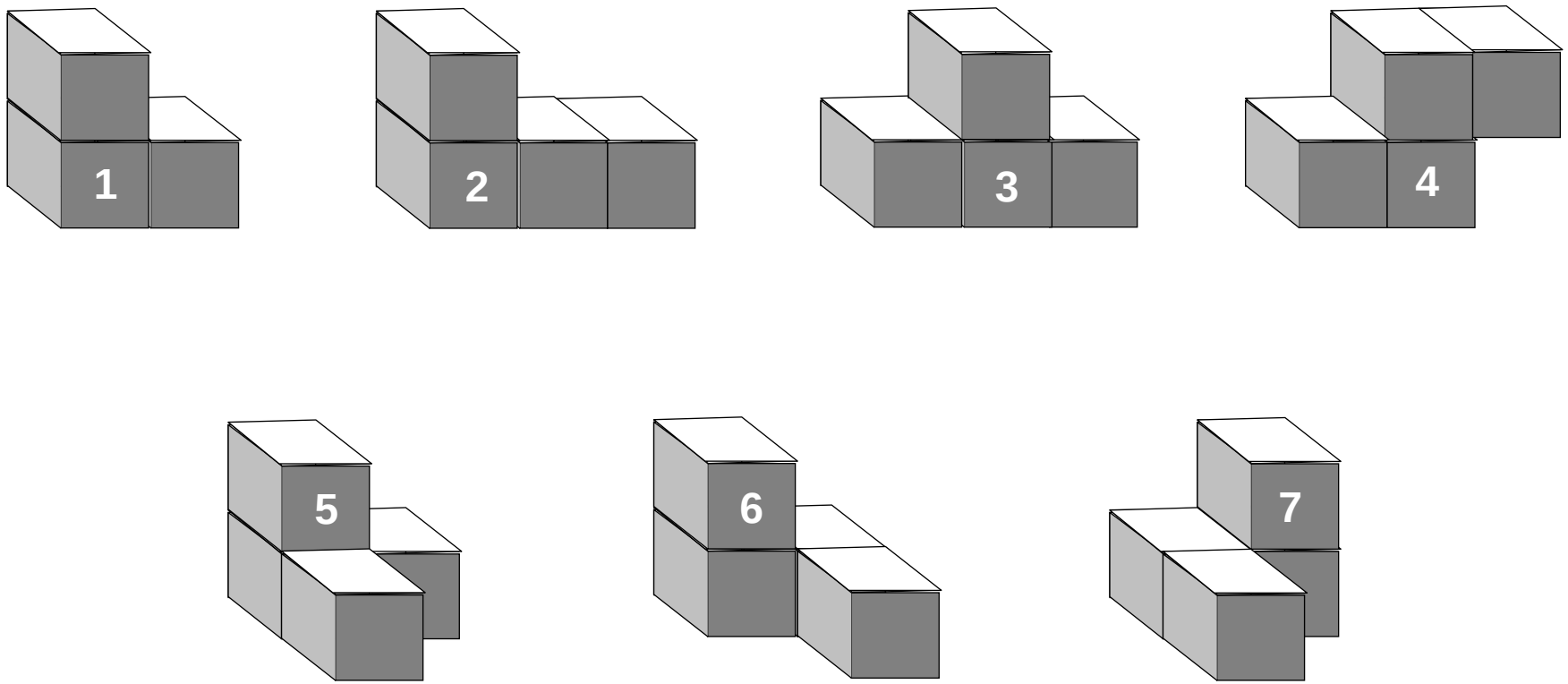


Conventional-Graphic Models: Structural Diagrams



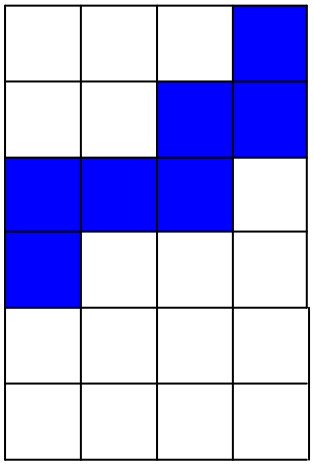
Intervention Program

Soma Pieces

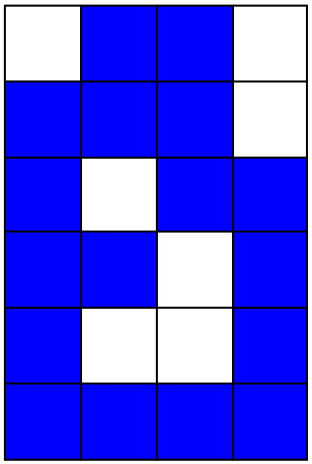


Three visual modes

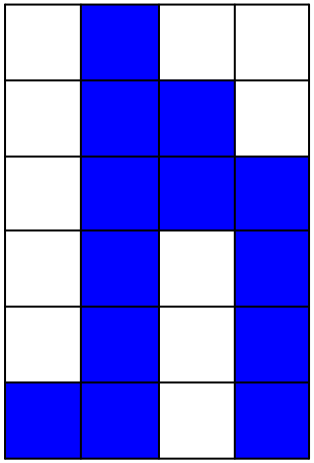
- Full-scale or scaled-down models of objects
- Conventional-graphic models
- Semiotic models



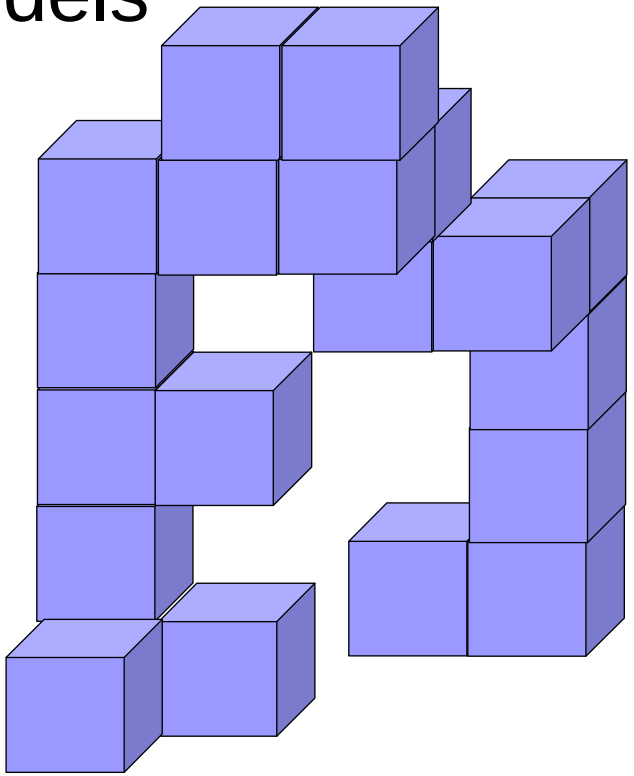
Top View



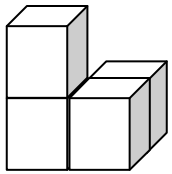
Front View



Side View



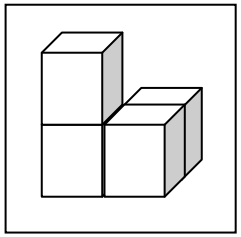
Framework for 3-Dimensional Visualization



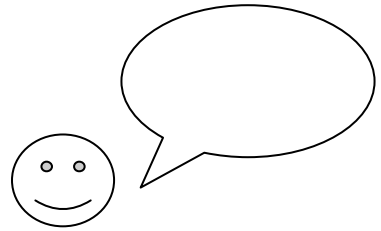
3-DIMENSIONAL MODEL

REBUILD IT

CONVENTIONAL GRAPHIC REPRESENTATION OF THE 3-D MODEL



VERBAL DESCRIPTION OF THE 3-D MODEL (oral or written)

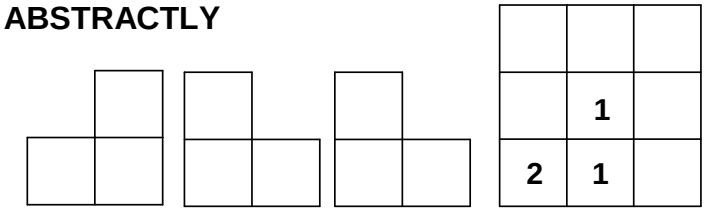


DRAW OR RECOGNIZE IT IN A PICTURE

TALK ABOUT IT

REPRESENT IT ABSTRACTLY

SEMIOTIC OR ABSTRACT REPRESENTATION OF THE 3-D MODEL



top **front** **side**

3-Dimensional Model Stimulus

Which piece?

Can you rebuild it
using loose cubes?

3-Dimensional Model Stimulus

Can you make
this figure using
two Soma
pieces?

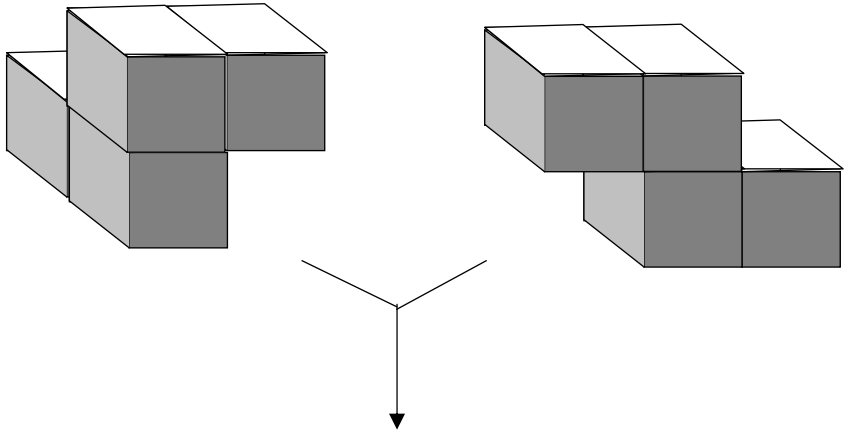
Rebuild it using loose cubes.

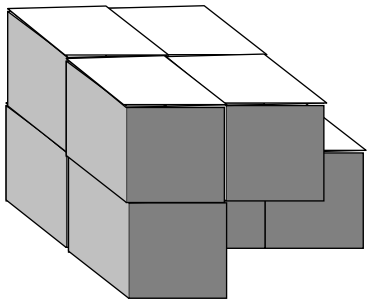
Draw it.

Explain how to build it.

2-D Conventional Graphic Model

Show how these two Soma pieces can be combined to create this figure.





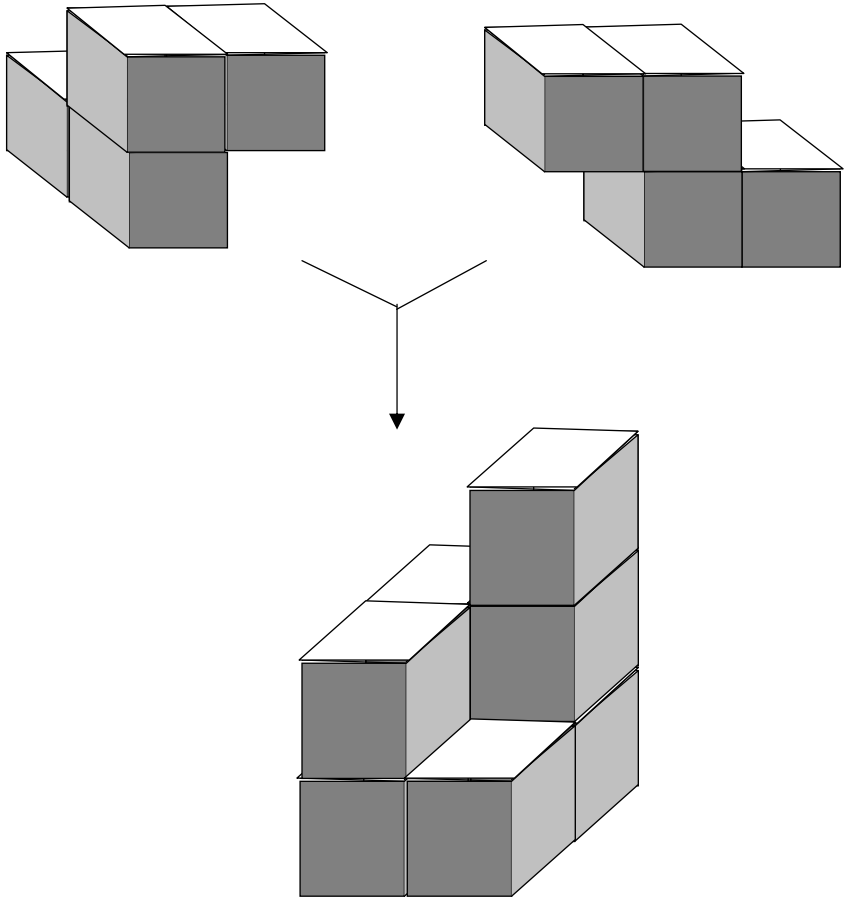
Rebuild it using loose cubes.

Draw it.

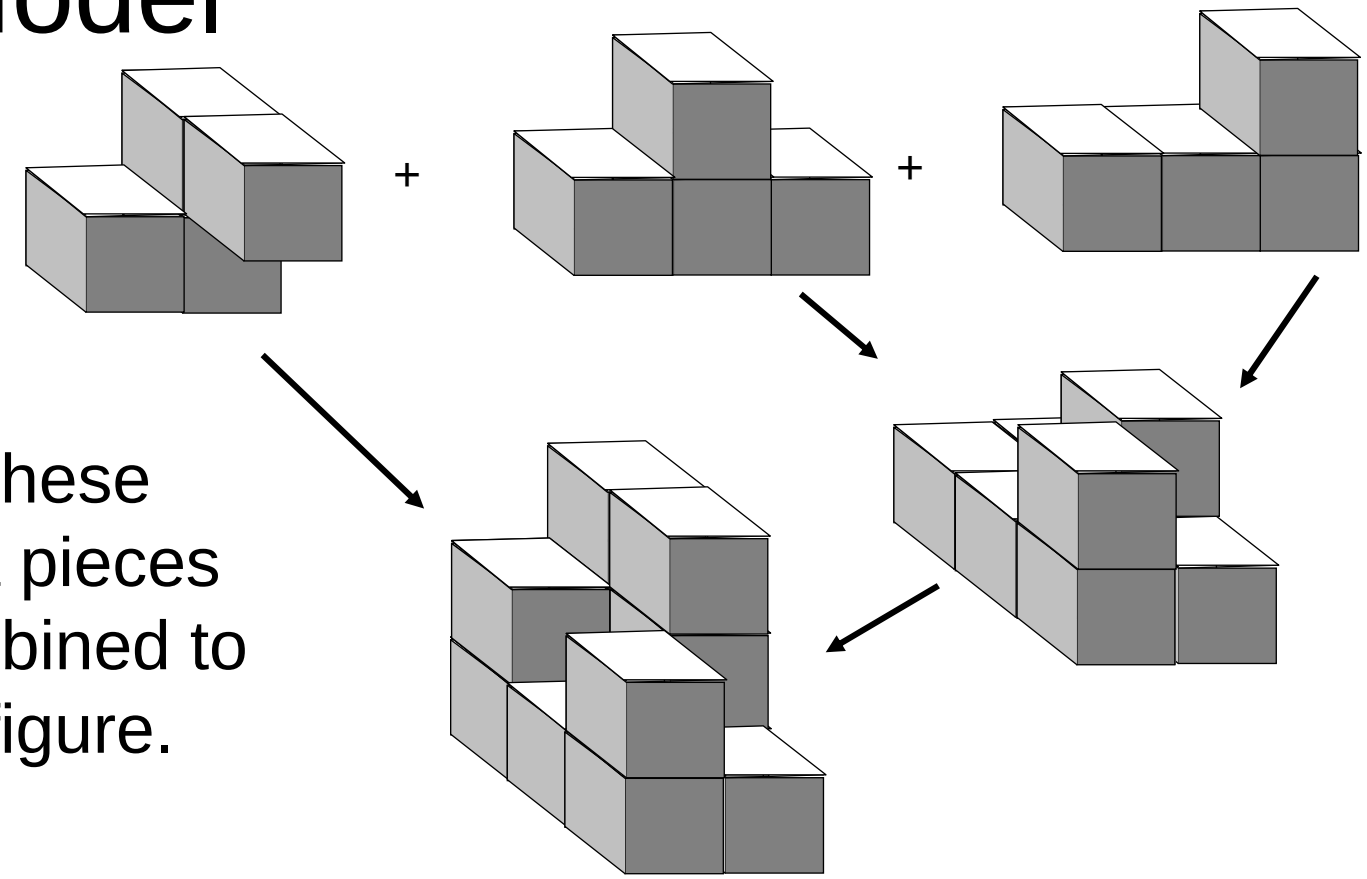
Explain how to build it.

2-D Conventional Graphic Model

Show how these two Soma pieces can be combined to create this figure.

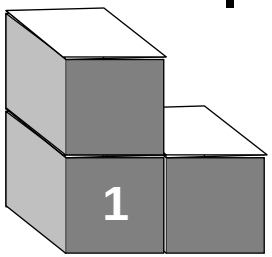


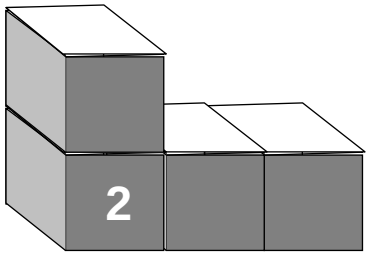
2-D Conventional Graphic Model

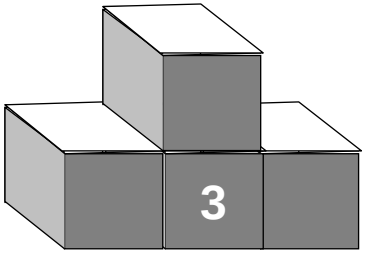


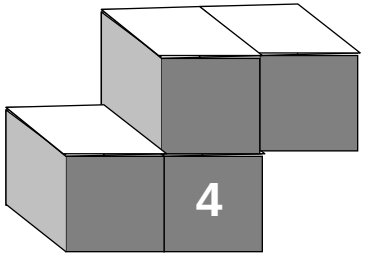
Show how these three Soma pieces can be combined to create this figure.

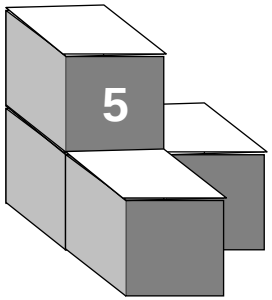
2-D Conventional Graphic Model



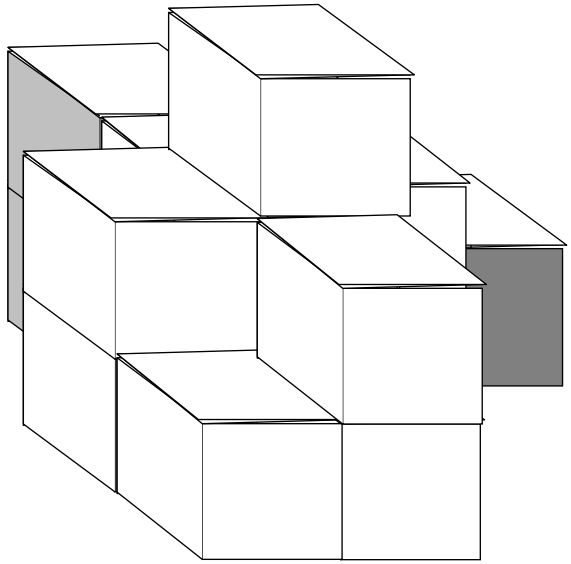


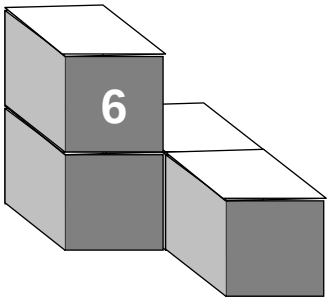


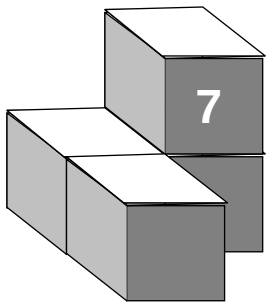




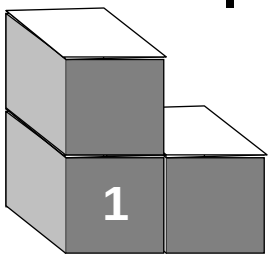
Which two Soma pieces were combined to create this figure?

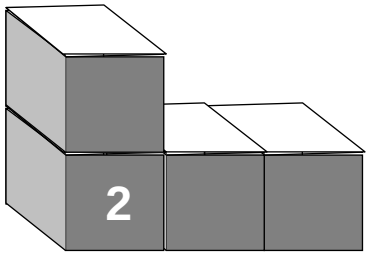


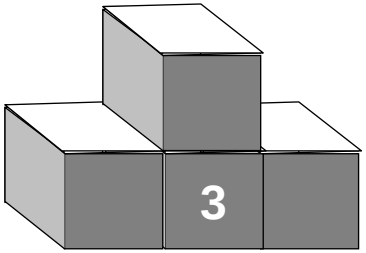


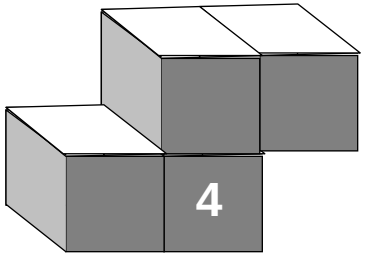


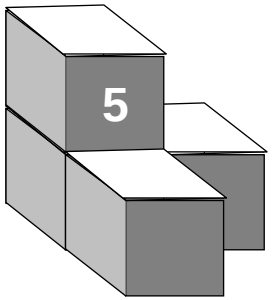
2-D Conventional Graphic Model

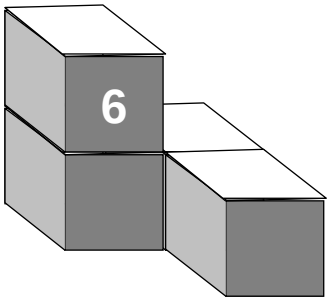


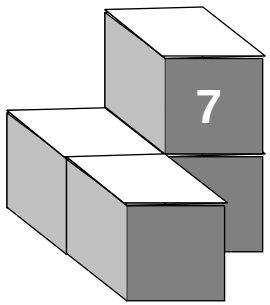




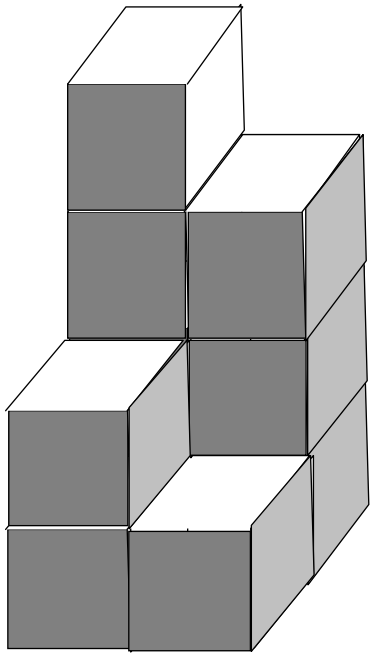






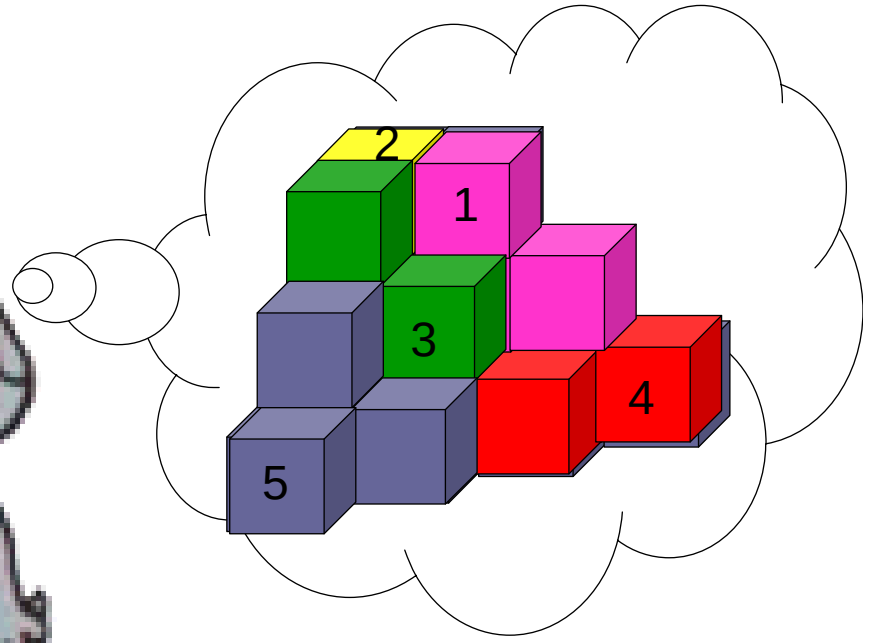


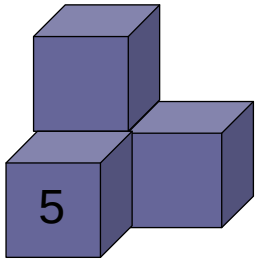
Which two Soma pieces were combined to create this figure?

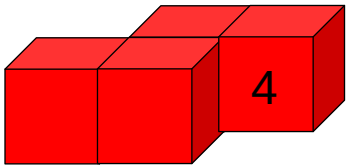


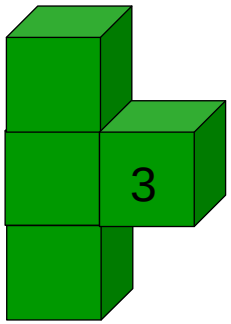
Describe it verbally

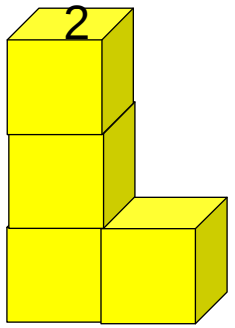
Use Soma pieces 1, 2, 3, 4 and 5.
5 and 4 go on the lower front.
Stand 3 behind 5, three cubes tall; and 2 next to 3 with its short leg on the ground pointing toward the front, next to 4.
1 goes on top of 2 and 4.

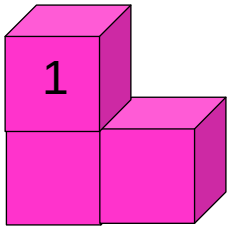




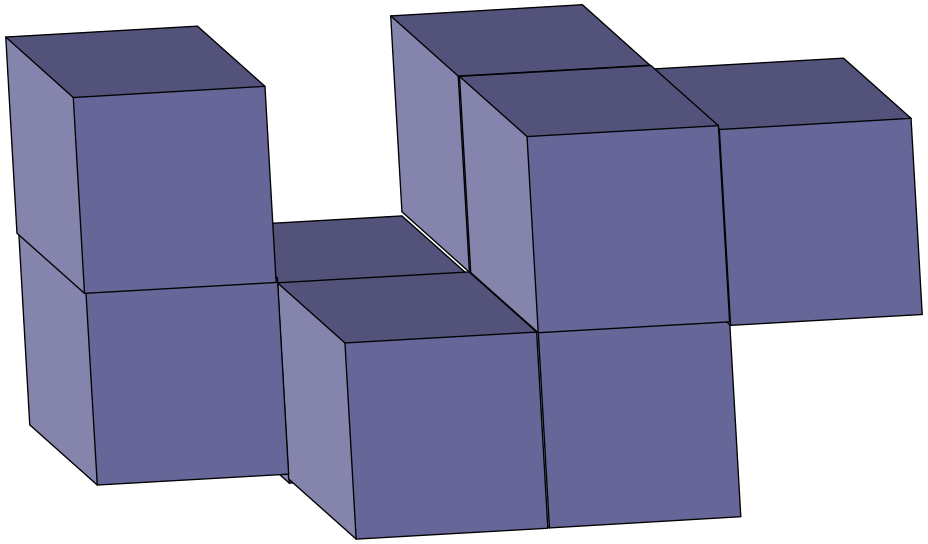


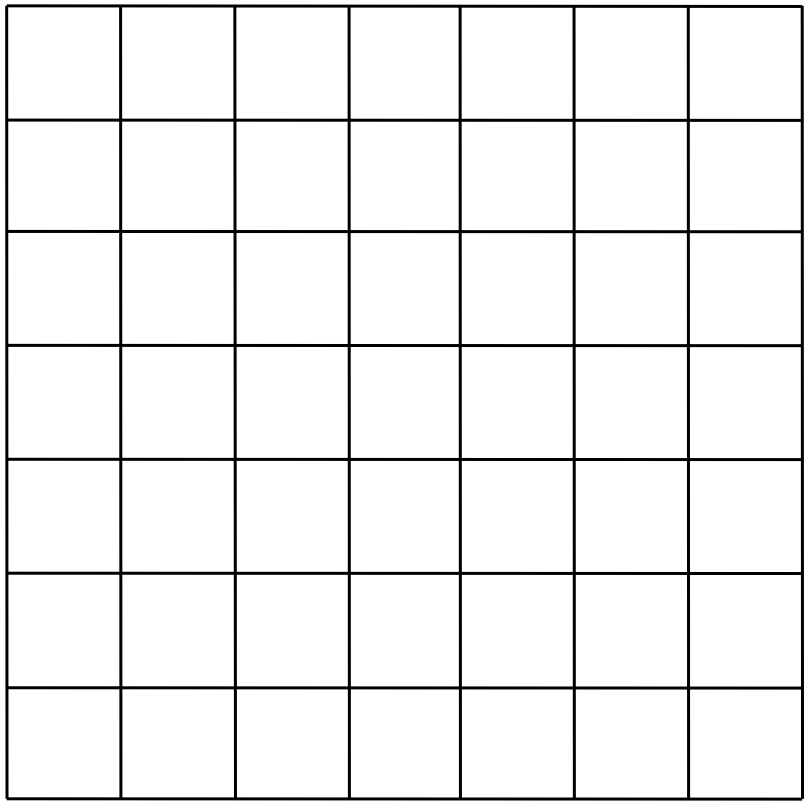




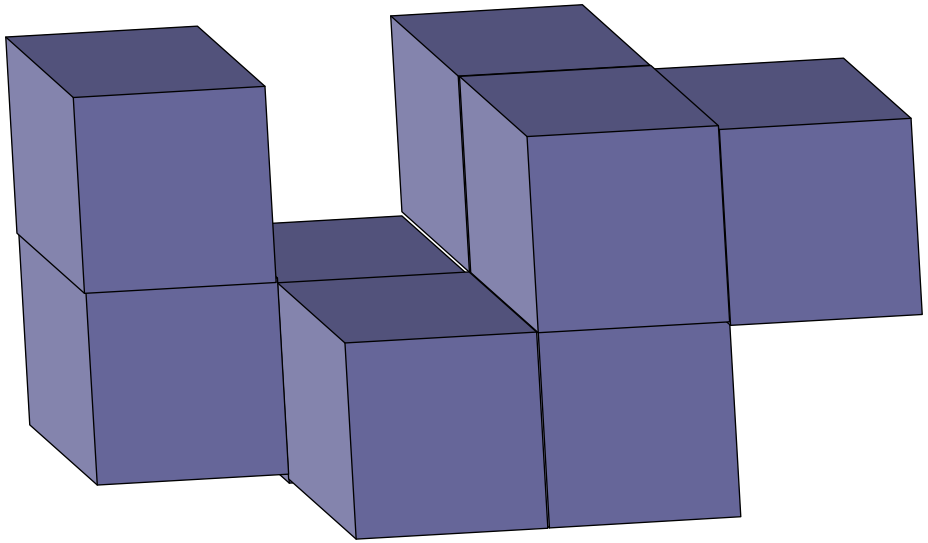


Represent the figure abstractly





Represent the figure abstractly

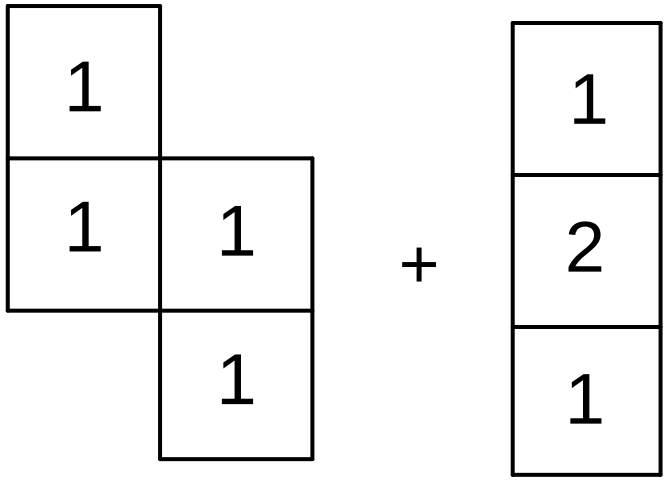


6	6		
	6	5	
6		5	
		5	5

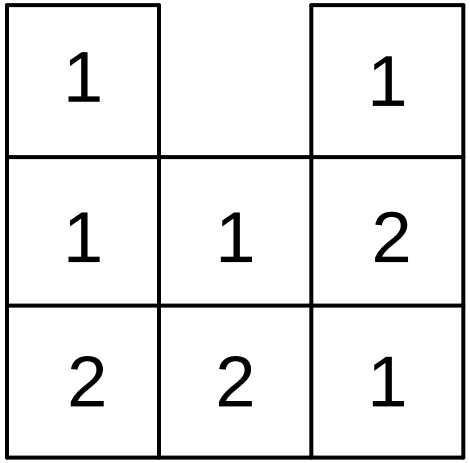
Lower level

Upper level

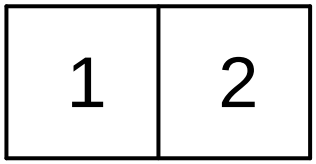
Represent the figure abstractly

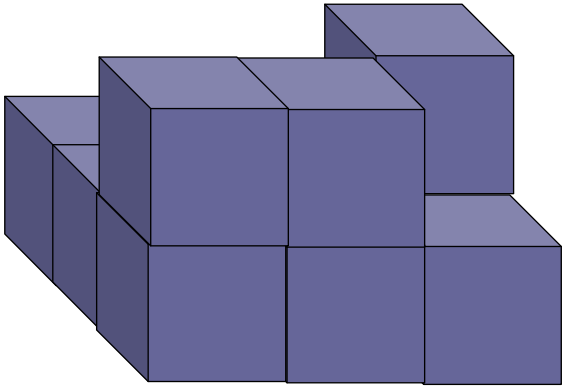


+



+





Represent the figure abstractly

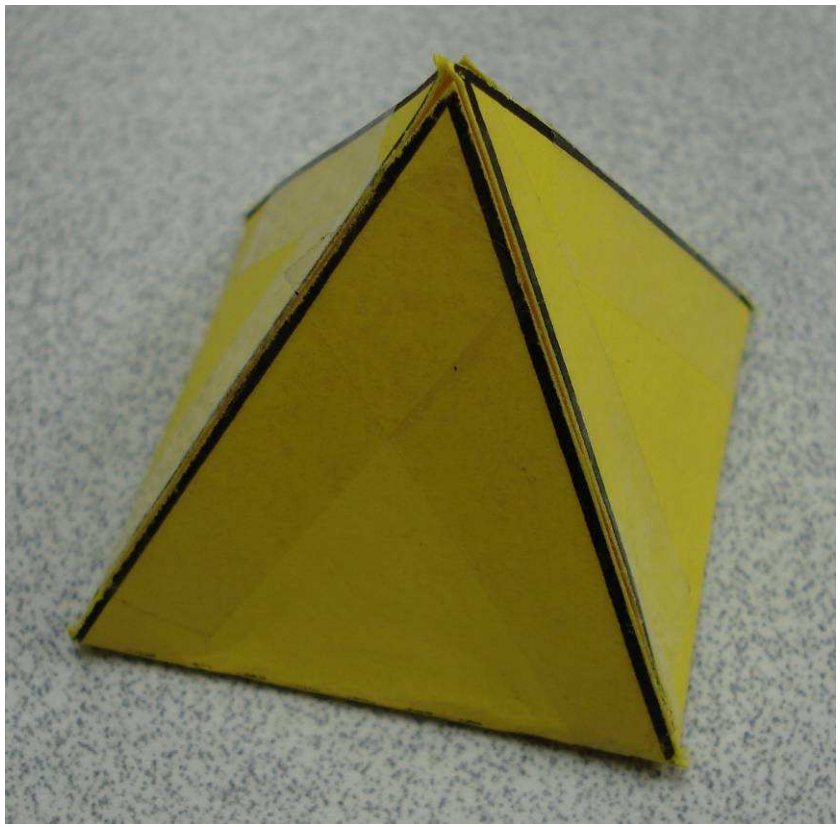
- How many and which Soma pieces do you need to build this figure?

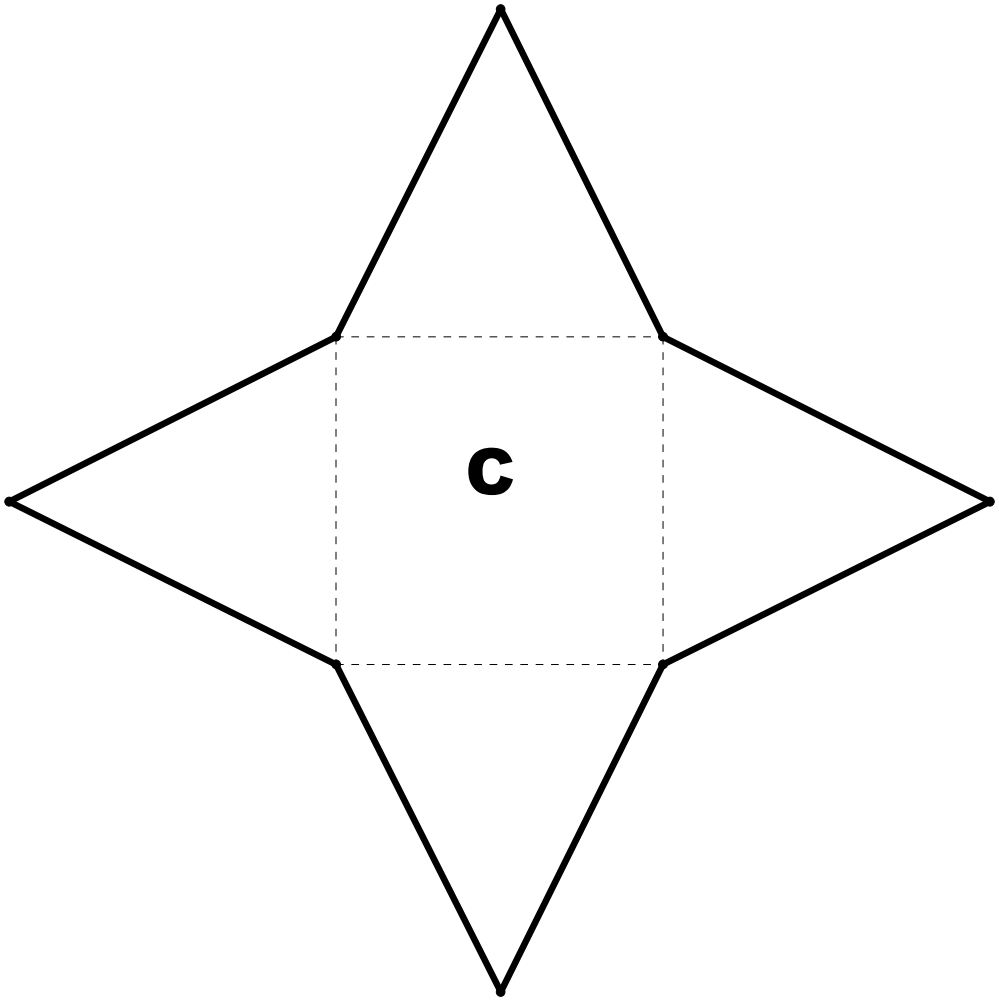
- Build the figure.

1	1	2
2	1	1
1	1	2

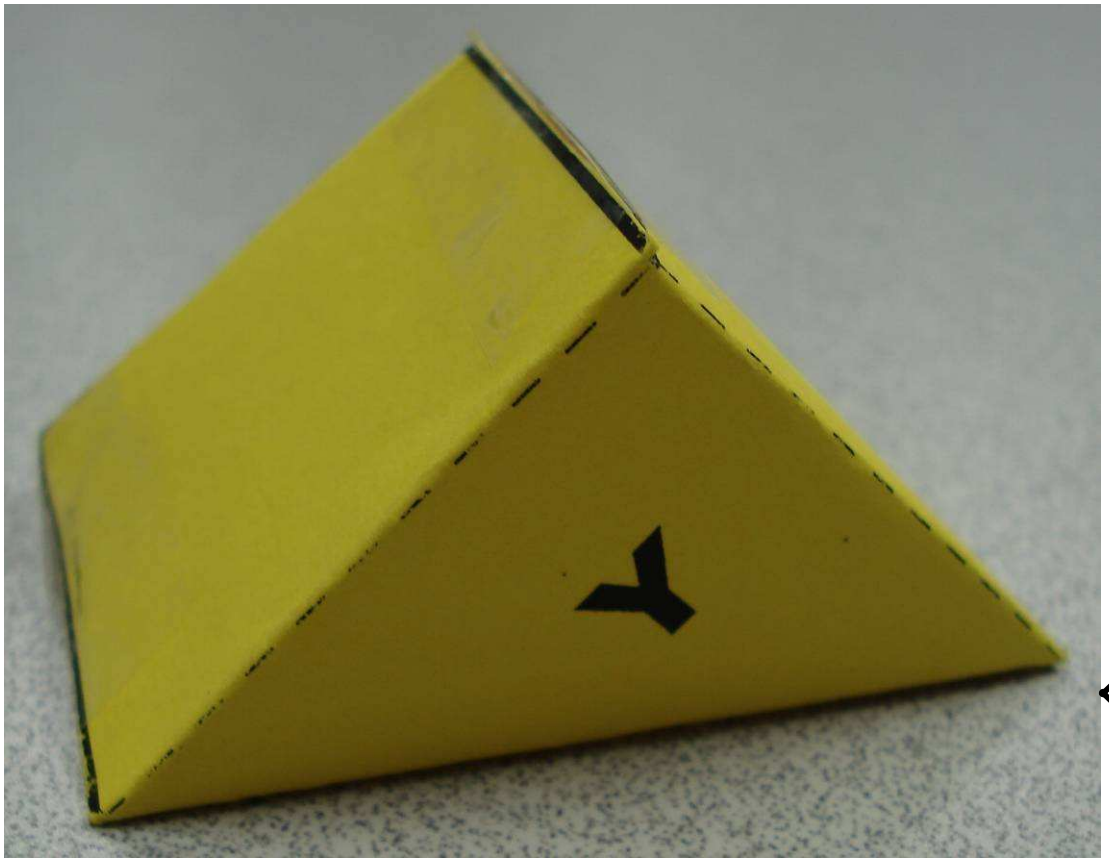
Beyond cubes…

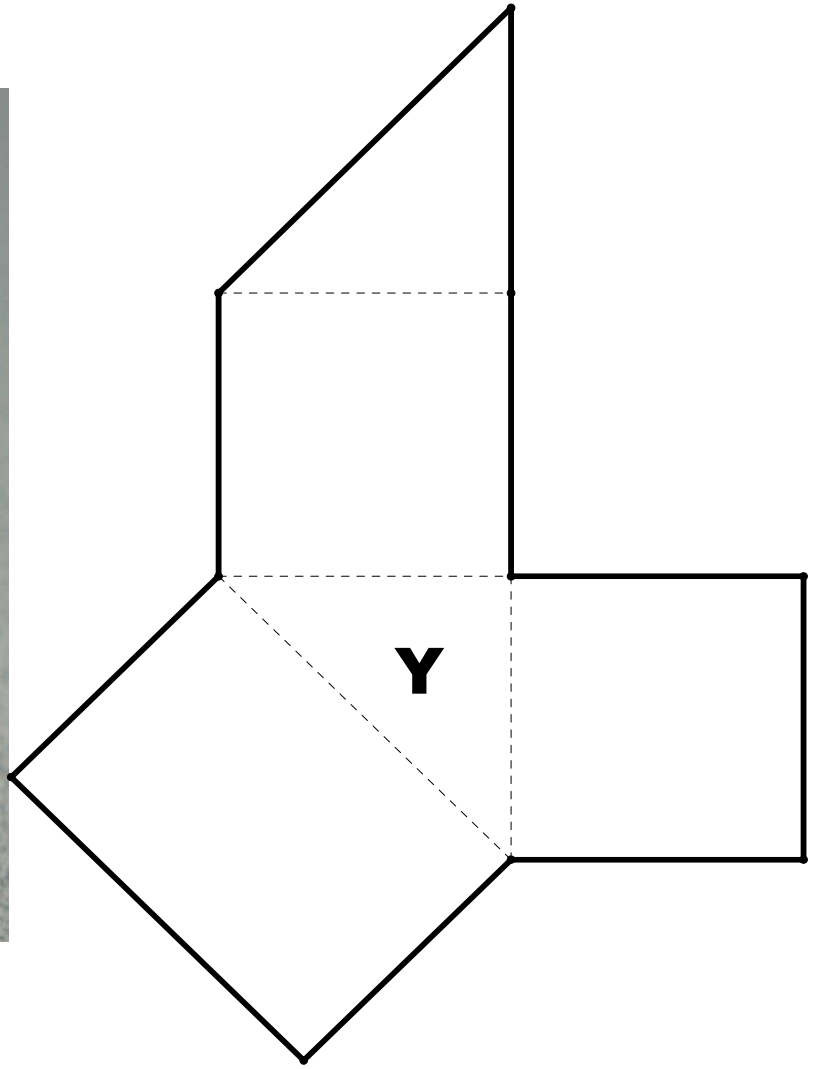
Describe the figure's net



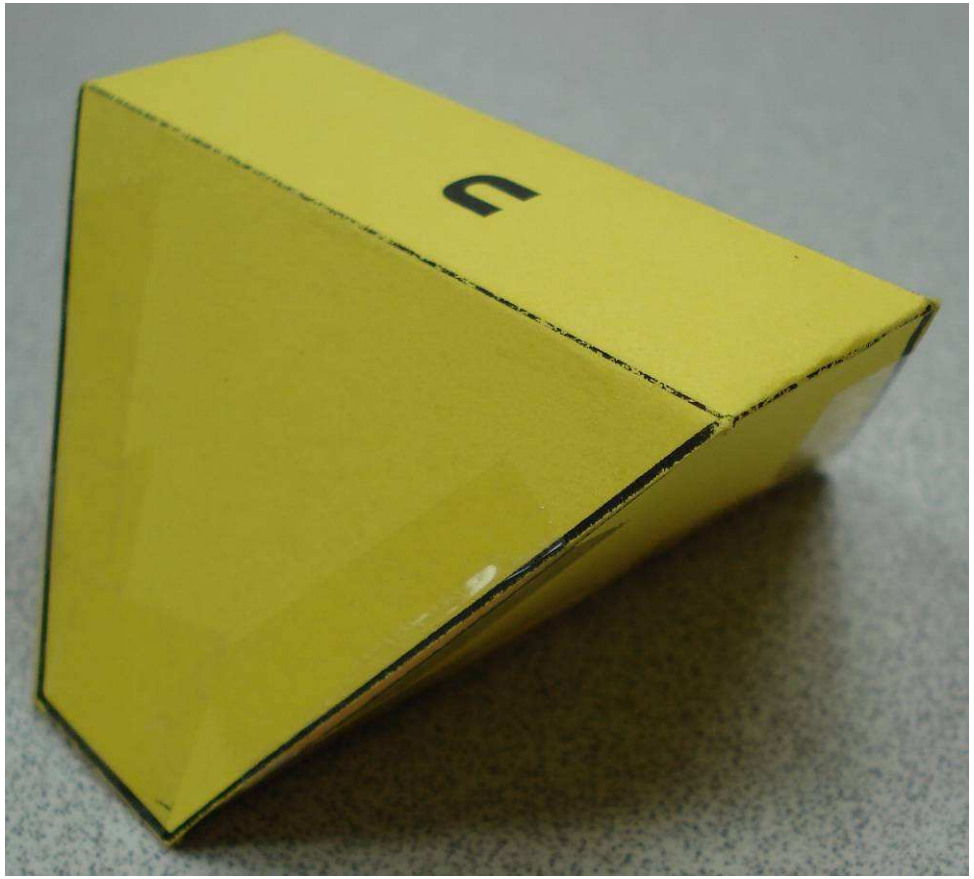


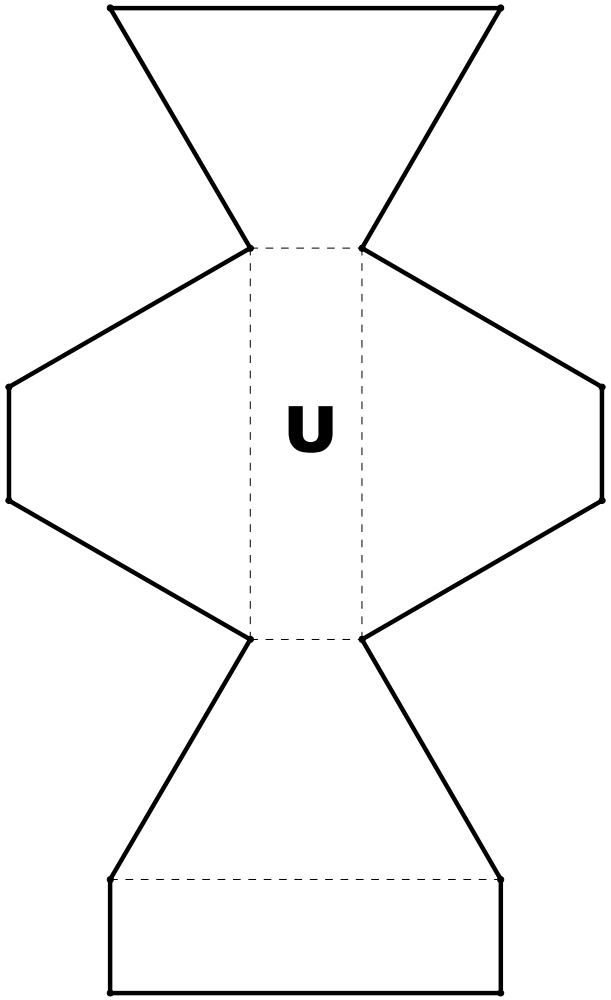
Describe the 3-D figure



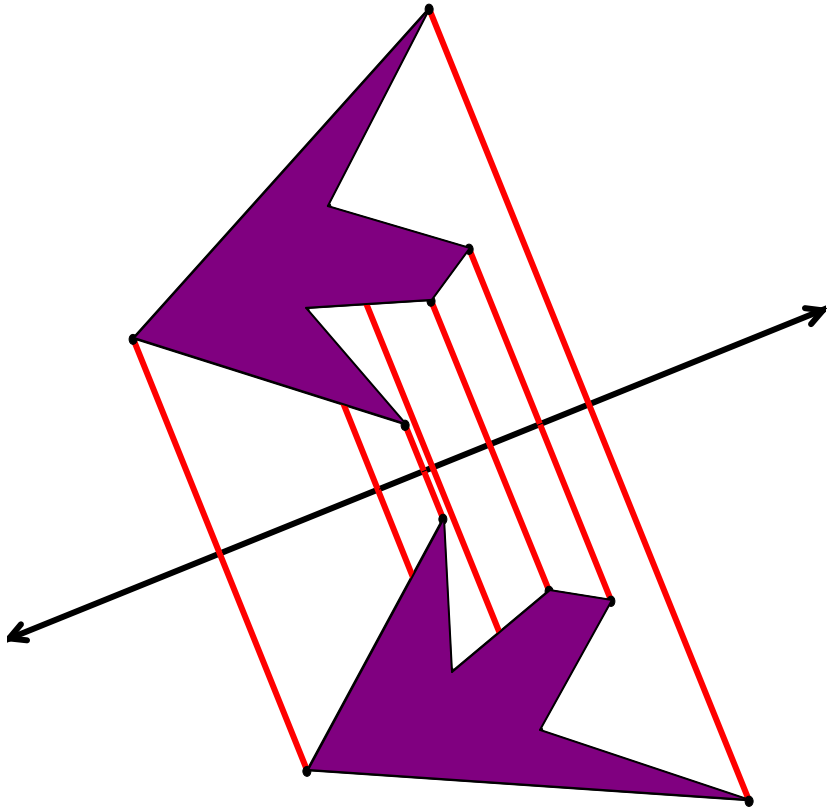


Describe the 3-D figure

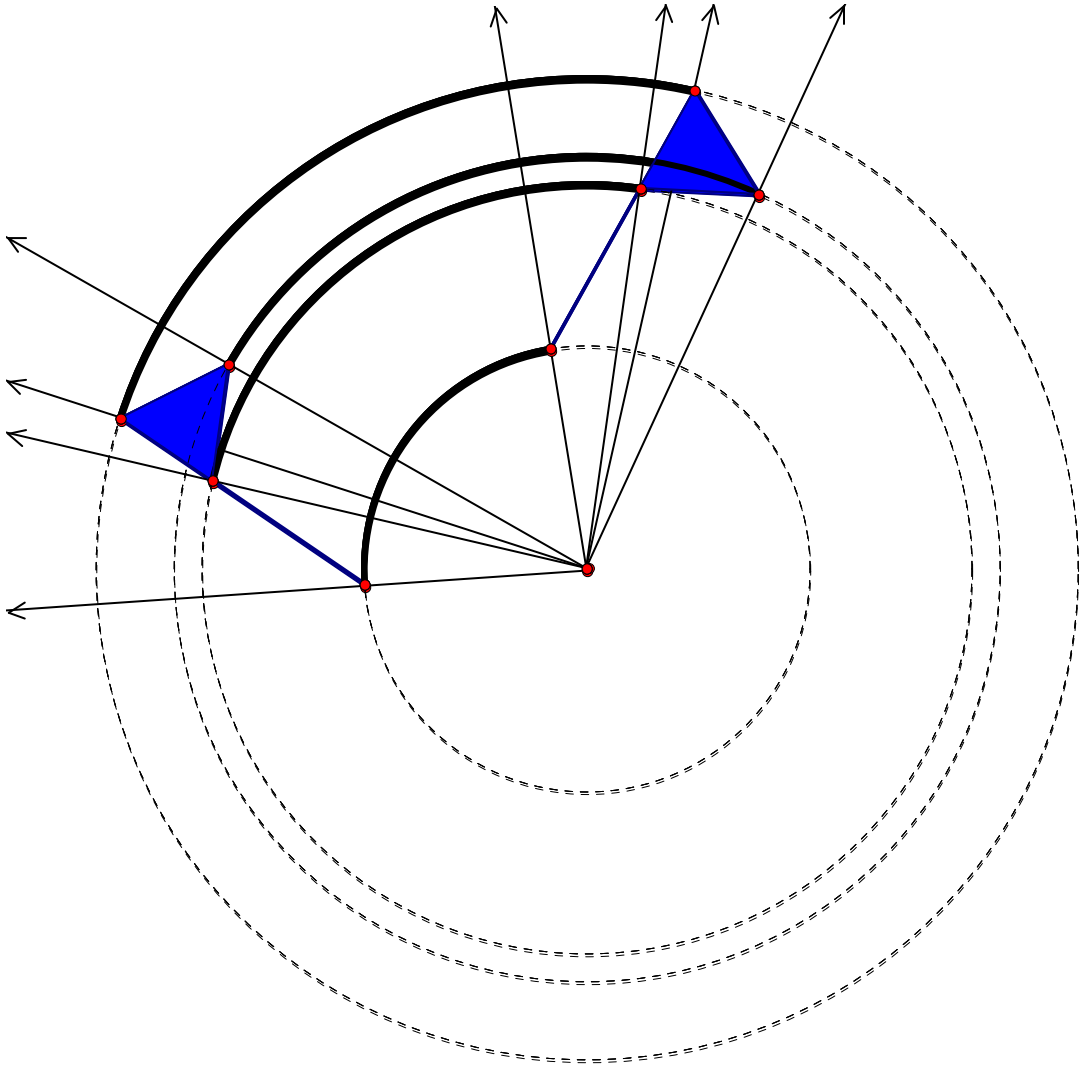




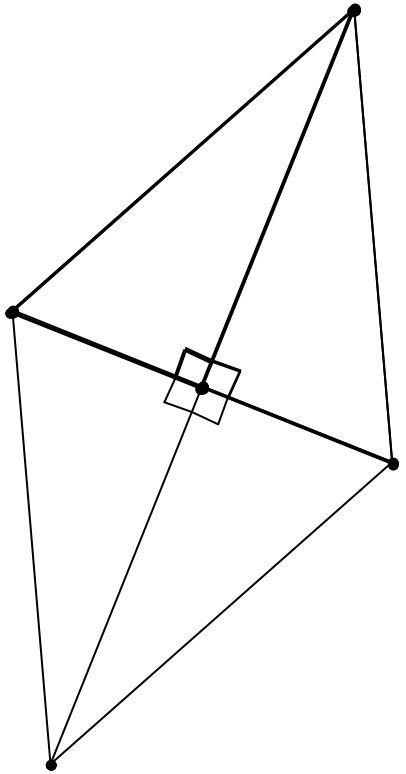
2-D Implications: Reflections



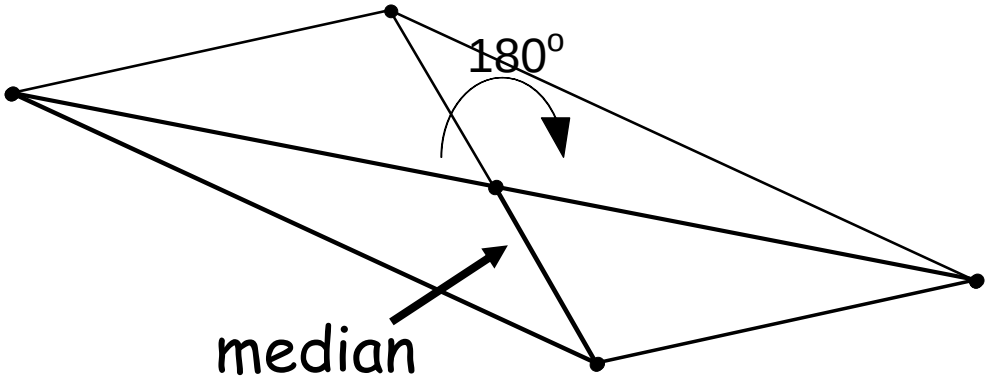
2-D Implications: Rotations



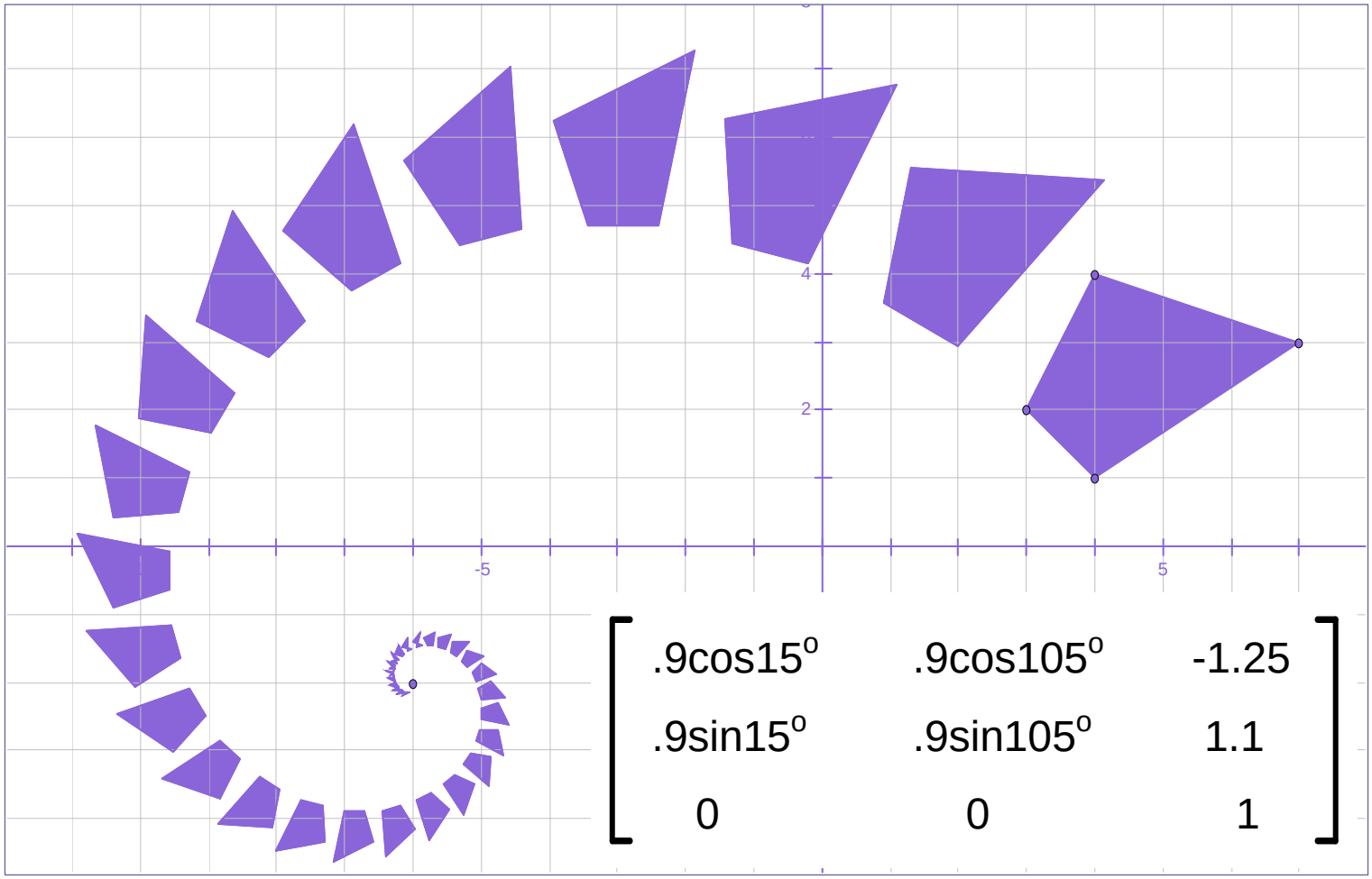
Transformations: 2-D Geometry



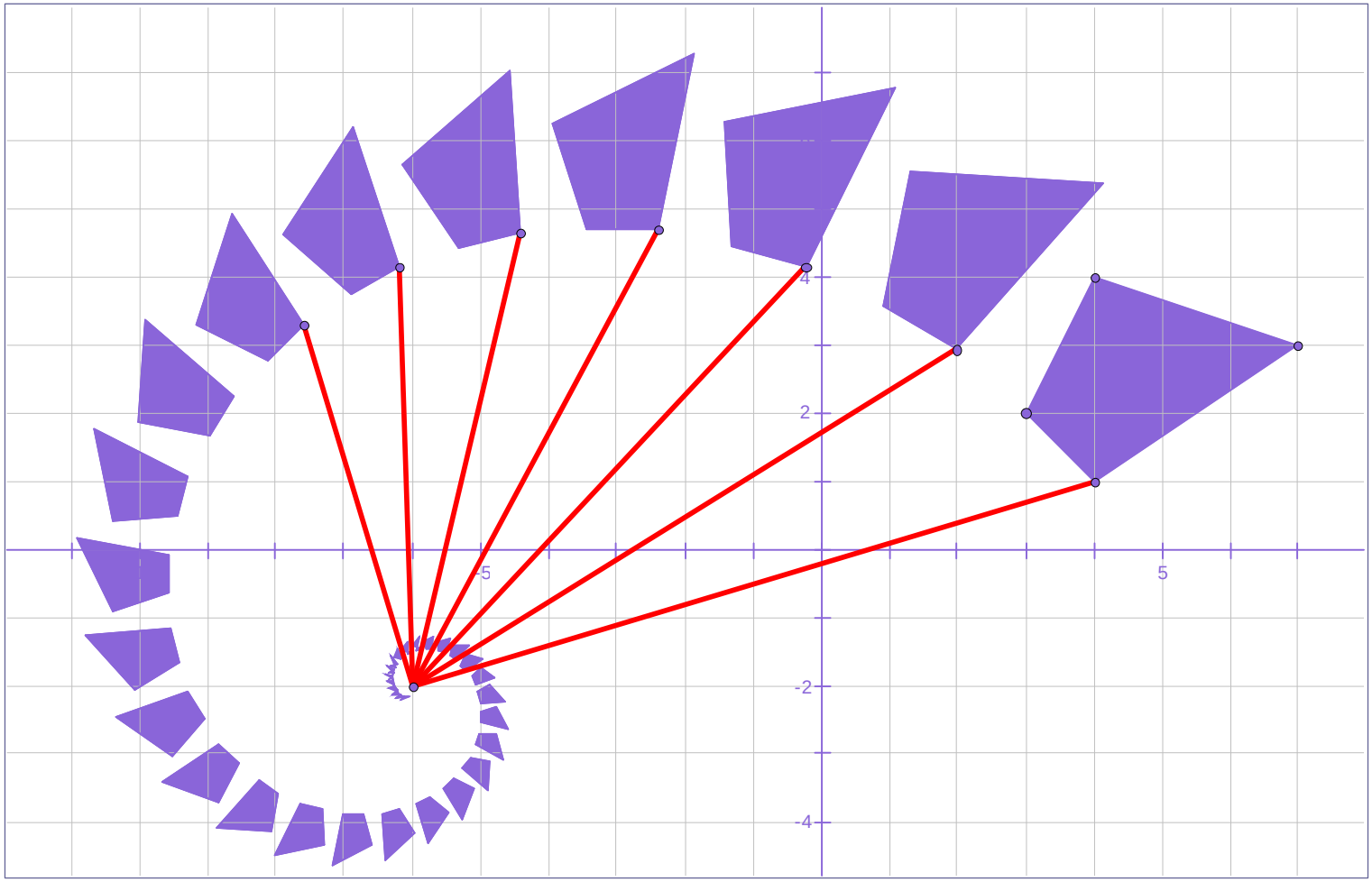
Transformations: 2-D Geometry



Transformations: Pre-Calculus – Calculus



Transformations: Back to Geometry



TEKS: Grade 1

(1.6) **Geometry and spatial reasoning.** The student uses attributes to identify two- and three-dimensional geometric figures. The student compares and contrasts two- and three-dimensional geometric figures or both.

The student is expected to:

- (A) describe and identify two-dimensional geometric figures, including circles, triangles, rectangles, and squares (a special type of rectangle);
- **(B) describe and identify three-dimensional geometric figures, including spheres, rectangular prisms (including cubes), cylinders, and cones;**
- **(C) describe and identify two- and three-dimensional geometric figures in order to sort them according to a given attribute using informal and formal language;** and
- (D) use concrete models to combine two-dimensional geometric figures to make new geometric figures.

TEKS: Grade 2

(2.7) **Geometry and spatial reasoning.** The student uses attributes to identify two- and three-dimensional geometric figures. The student compares and contrasts two- and three-dimensional geometric figures or both.

The student is expected to:

- **(A) describe attributes (the number of vertices, faces, edges, sides) of two- and three-dimensional geometric figures such as circles, polygons, spheres, cones, cylinders, prisms, and pyramids, etc.;**
- **(B) use attributes to describe how 2 two-dimensional figures or 2 three-dimensional geometric figures are alike or different; and**
- (C) cut two-dimensional geometric figures apart and identify the new geometric figures formed.

TEKS: Grade 3

(3.8) **Geometry and spatial reasoning.** The student uses formal geometric vocabulary.

The student is expected to identify, classify, and describe two- and three-dimensional geometric figures by their attributes. The student compares two- dimensional figures, three-dimensional figures, or both by their attributes using formal geometry vocabulary.

(3.9) **Geometry and spatial reasoning.** The student recognizes congruence and symmetry.

The student is expected to:

- (A) identify congruent two-dimensional figures;
- (B) create two-dimensional figures with lines of symmetry using concrete models and technology; and
- (C) identify lines of symmetry in two-dimensional geometric figures.

TEKS: Grade 4

(4.8) **Geometry and spatial reasoning.** The student identifies and describes attributes of geometric figures using formal geometric language.

The student is expected to:

- (A) identify and describe right, acute, and obtuse angles;
- (B) identify and describe parallel and intersecting (including perpendicular) lines using concrete objects and pictorial models; and
- **(C) use essential attributes to define two- and three-dimensional geometric figures.**

(4.9) **Geometry and spatial reasoning.** The student connects transformations to congruence and symmetry.

The student is expected to:

- (A) demonstrate translations, reflections, and rotations using concrete models;
- (B) use translations, reflections, and rotations to verify that two shapes are congruent; and
- (C) use reflections to verify that a shape has symmetry.

TEKS: Grade 5

(5.7) **Geometry and spatial reasoning.** The student generates geometric definitions using critical attributes.

The student is expected to identify essential attributes including parallel, perpendicular, and congruent parts of two- and three-dimensional geometric figures.

(5.8) **Geometry and spatial reasoning.** The student models transformations.

The student is expected to:

- (A) sketch the results of translations, rotations, and reflections on a Quadrant I coordinate grid; and
- (B) identify the transformation that generates one figure from the other when given two congruent figures on a Quadrant I coordinate grid.

TEKS: Grade 6

(6.6) **Geometry and spatial reasoning.** The student uses geometric vocabulary to describe angles, polygons, and circles.

The student is expected to:

- (A) use angle measurements to classify angles as acute, obtuse, or right;
- (B) identify relationships involving angles in triangles and quadrilaterals; and
- (C) describe the relationship between radius, diameter, and circumference of a circle.

TEKS: Grade 7

(7.6) **Geometry and spatial reasoning.** The student compares and classifies two- and three-dimensional figures using geometric vocabulary and properties.

The student is expected to:

- (A) use angle measurements to classify pairs of angles as complementary or supplementary;
- (B) use properties to classify triangles and quadrilaterals;
- **(C) use properties to classify three-dimensional figures, including pyramids, cones, prisms, and cylinders; and**
- (D) use critical attributes to define similarity.

(7.7) **Geometry and spatial reasoning.** The student uses coordinate geometry to describe location on a plane.

The student is expected to:

- (A) locate and name points on a coordinate plane using ordered pairs of integers; and
- (B) graph reflections across the horizontal or vertical axis and graph translations on a coordinate plane.

(7.8) **Geometry and spatial reasoning.** The student uses geometry to model and describe the physical world.

- The student is expected to:
- **(A) sketch three-dimensional figures when given the top, side, and front views;**
- **(B) make a net (two-dimensional model) of the surface area of a three-dimensional figure; and**
- (C) use geometric concepts and properties to solve problems in fields such as art and architecture.

TEKS: Grade 8

(8.6) **Geometry and spatial reasoning.** The student uses transformational geometry to develop spatial sense.

The student is expected to:

- (A) generate similar figures using dilations including enlargements and reductions; and
- (B) graph dilations, reflections, and translations on a coordinate plane.

(8.7) **Geometry and spatial reasoning.** The student uses geometry to model and describe the physical world.

The student is expected to:

- **(A) draw three-dimensional figures from different perspectives;**
- (B) use geometric concepts and properties to solve problems in fields such as art and architecture;