Developing Mathematical Problem Solving, Communication, and Reflection through Online Learning Experiences

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Spring Networking Conference
RICE UNIVERSITY
SCHOOL MATHEMATICS PROJECT
Schoenfeld calls problems “starting points for serious explorations, rather than tasks to be completed”

As opposed to most students’ approach—find the answer and move to the next problem.

Explore, Develop, Extend, Wonder

Principles and Standards

“A problem-centered approach to teaching mathematics uses interesting and well-selected problems to launch mathematical lessons and engage students. In this way, new ideas, techniques, and mathematical relationships emerge and become the focus of discussion”

Go to a DESCRIPTION of these activities found at the bottom of the page.
What's New

2005 Mathematics Game
Dr. Math® Presents More Geometry
Park City Mathematics Institute 2005

Forum Features

Ask Dr. Math
Discussion Groups
Internet Newsletter
MathTools
Problems of the Week
Teacher2Teacher
Teacher Exchange
Workshops

Math Resources by Subject
K-12, College, & Advanced Math

Math Education
Innovations and Concerns

Key Issues in Math

[Suggestion Box] [Math Library] [Help] [Quick Reference] [Search Our Site]

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Problems of the Week

The Math Forum’s Problems of the Week service (the PoWs), is one of the most successful examples of challenging mathematical learning on the web, having mentored approximately 120,000 students. Students access non-routine problems online. They submit solutions and explanations, then receive feedback from volunteer math professionals/enthusiasts with encouragement to reflect more on their solutions, refine, and re-submit.
The Math Forum's Problems of the Week (PoWs) are designed to provide creative, non-routine challenges for students in grades three through twelve. Problem-solving and mathematical communication are key elements of every problem. **Program Update, August 16, 2004.**

Memberships and Subscriptions

Info: [Mentoring](#) || [Teacher Accounts](#) || [Pricing Guide](#) || [Purchase Online](#)
Free browsing, full access to members: [Active Problem Library](#) || [Lesson Support Pages](#)
Answer Check

After submitting a solution, students are encouraged to view the Answer Check. If their answer doesn’t match ours, they might:

- See that they’ve made a calculation error.
- Notice that they’ve misinterpreted the problem.
- Realize they’ve completed only part of the problem or not quite answered the question asked.
- Tell us where they think they are stuck.
Mentoring Student Submissions

- The Math Forum provides limited free mentoring with the help of volunteers and groups of education students.
- Mentoring involves careful reading of submissions, scoring, and replying to students about their solutions.
- Priority Mentoring is available by subscription.
Evaluation

Evaluation has shown that both weak and strong elementary, middle, and high school students who worked with PoWs over a 10-month period were found to make more connections to, generate more effective strategies for, and work more independently with the problems than they had before.
PoW Scoring Rubric

Reflection: check the answer, reflect on its reasonableness, summarize the process, and connect it to prior knowledge and experience.
Congruent Rectangles

These seven congruent rectangles form a larger rectangle. If the area of the larger rectangle is 756 units$^2$, what is its perimeter?
These seven congruent rectangles form a larger rectangle. If the area of the larger rectangle is 756 units$^2$, what is its perimeter?

**What does the answer show that James understands?**

James:

The perimeter is 166 units. I knew that area was length times width. So I knew that 27 times 28 gave me 756. So I added 28 plus 28 plus 27 plus 27 to get my answer of 166.
PoW Scoring Rubric

Reflection: check the answer, reflect on its reasonableness, summarize the process, and connect it to prior knowledge and experience.
Congruent Rectangles

These seven congruent rectangles form a larger rectangle. If the area of the larger rectangle is 756 units², what is its perimeter?

What I would tell James.

James:

The perimeter is 166 units. I knew that area was length times width. So I knew that 27 times 28 gave me 756. James, it is good that you understood that you needed to find two numbers whose product was 756, but are there other numbers that also give 756? Is there other information given in the problem that you need to consider to decide which one to use?
Bob:

If 756 units$^2$ is the area, and area is calculated by multiplying width times length, then the width and height must both be 756 units. Then if perimeter is calculated by $2L + 2W$ the perimeter must be 3024 units.
Congruent Rectangles

Lisa: What does Lisa understand?

Let the dimensions of the small rectangles be \( a, b \), then the area of the large rectangle is:

\[ 4b(a+b)=756 \]

From the graph, we can see that \( 4b=3a \), so \( b=\frac{3a}{4} \), then we have

\[ 3a\left(a + \frac{3a}{4}\right)=756 \]
\[ \frac{7}{4}a^2=252 \]
\[ a^2=144 \]
\[ a=12 \ (\text{-}12 \text{ rejected}) \]
\[ b=\frac{3}{4}a=\left(\frac{3}{4}\right)\times 12=9 \]

The perimeter of the larger rectangle is \( 4b+a+b=a+5b=12+5\times 9=12+45=57 \)
Congruent Rectangles

These seven congruent rectangles form a larger rectangle. If the area of the larger rectangle is 756 units$^2$, what is its perimeter?

\[ 3l = 4w \]
\[ l = \frac{4w}{3} \]

\[
\begin{align*}
(l+w)(4w) &= 756 \\
(4w/3+w)(4w) &= 756 \\
28w^2/3 &= 756 \\
w^2 &= 81 \\
w &= 9, \quad l = 12 \\
P &= 10w + 2l = 114
\end{align*}
\]
These seven congruent rectangles form a larger rectangle. If the area of the larger rectangle is 756 units$^2$, what is its perimeter?

Carson:

$$\frac{756}{7} = 108$$

$$lw = 108$$

Look for factors of 108 in the ratio of 4:3. Trial and error gave 12 and 9.
Note to Cynthia: not yet
Encouragement

This problem may seem different from the types of problems that you're used to, but don't assume that that means you can't do it. Think about how the shape of the glass changes the rate at which the water rises. If you're stuck, no problem. Tell us what you are thinking and make sure to leave a comment about the help you want.
algebra
problem of the week

Graph 1
Graph 2
Graph 3
Graph 4

Note to Cynthia: not yet
algebra
problem of the week

Graph 1
Phone a friend

Graph 2

Graph 3
Poll the audience

Graph 4
Ask Dr. Math®

Have a math question?

1. Browse the Archive:
   - FAQ
   - Formulas
   - Selected Answers
   - Elementary School
   - Middle School
   - High School
   - College & Beyond

2. Search the Archive:

3. Can't find the answer?
   Write to Dr. Math

Take Dr. Math with you!

Get the books:

- Dr. Math Gets You Ready for Algebra
- Dr. Math Explains Algebra
- Dr. Math Introduces Geometry
- Dr. Math Presents More Geometry
Ask Dr. Math
Elementary Archive

Browse questions and answers:

- Arithmetic
  - Addition
  - Subtraction
  - Multiplication
  - Division
  - Fractions/Decimals
- Definitions
- Geometry
  - Two-Dimensional
    - Circles
    - Triangles/Polygons
  - 3D and Higher
    - Polyhedra
- Golden Ratio/Fibonacci
- History/Biography
- Measurement
  - Calendars/Dates/Time
  - Temperature
  - Terms/Units of Measurement
- Number Sense/About Numbers
  - Infinity
  - Large Numbers
  - Place Value
  - Prime Numbers
  - Square Roots
- Projects
- Puzzles
- Word Problems

See also our selected answers to common questions.
Ask Dr. Math

Elementary Archive

Browse Elementary Word Problems

Stars indicate particularly interesting answers or good places to begin browsing.

How can you have 21 coins of nickels, dimes, pennies that equal one dollar? No quarters. My son and I need an explanation.

8 dimes, 5 quarters, 4 half dollars [1/13/1995] ★
Jacob has 8 dimes, Laura has 5 quarters, and Mellisa has 4 half dollars. How much money is this all together?

Blending Seed [09/27/1998] ★
How much of $0.65/lb seed does a dealer need to combine with 200 lbs of $0.45/lb seed to make a $0.55/lb blend?
Ask Dr. Math: FAQ

Roman Numerals

Table of Contents

Basics
How do I read and write Roman numerals?
What are the rules for subtracting letters? Can I write MIM? What about IIII?
How do I write large numbers?
Web resources: introductions, charts, calculators, and converters

Calculations
Did the Romans use fractions?
How can I use Roman numerals to do arithmetic problems?

Uses Past and Present
How did the Romans use math?
How are Roman numerals used today?
Resources for other numeral systems
A community library of technology tools, lessons, activities, and support materials for teaching and learning mathematics.

New to Math Tools? Welcome!

There's a lot of stuff in here. Join our community! You can get a nice tour in the Getting Started section.

What's Hot?

[January 26] Upper middle schoolers that haven't yet mastered multiplication facts: Bethy writes, "We have a number of students who, at 7th and 8th grade, still don't know 75% or more of the multiplication facts. What does the research indicate? Does anyone know of an effective method for equipping these students with a procedure so that they can move ahead?"

[January 24] Fractions, concept and calculations: Cynthia writes, "How do you teach conceptual understanding of fractions? What tools have you
### Resource Name | Topic (Course) | Technology | Type | $? | Rating
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Attribute Trains | Patterns and sequences (Kindergarten) + | Java Applet | Tool |  | 🌟🌟🌟🌟🌟
Expanding Our Pattern Block Fr... | Writing fractions (Math 3)+ | Java Applet | Lesson Plan |  | 🌟🌟🌟🌟🌟
Investigating Tessellations Us... | Symmetry (Math 3)+ | Java Applet | Lesson Plan |  | 🌟🌟🌟🌟🌟
Line Symmetry | Symmetry (Kindergarten)+ | Java Applet | Lesson Plan |  | 🌟🌟🌟🌟🌟
Making Patterns | Patterns and sequences (Pre-Kindergarten)+ | Java Applet | Tool |  | 🌟🌟🌟🌟🌟
No Matter What Shape Your Fra... | Fractions (Math 4)+ | Java Applet | Activity |  | 🌟🌟🌟🌟🌟
PatBlocks | Shapes (Pre-Kindergarten)+ | Java Applet | Tool |  | 🌟🌟🌟🌟🌟
Pattern Blocks | General (Math 3)+ | Java Applet | Tool |  | 🌟🌟🌟🌟🌟
Pattern Blocks | General (Math 6)+ | Java Applet | Tool |  | 🌟🌟🌟🌟🌟
Pattern Blocks | General (Pre-Kindergarten)+ | Java Applet | Tool |  | 🌟🌟🌟🌟🌟
Pattern Blocks: ABAB | General (Pre-Kindergarten)+ | Java Applet | Activity |  | 🌟🌟🌟🌟🌟
Pattern Blocks: Describing a P... | General (Pre-Kindergarten)+ | Java Applet | Activity |  | 🌟🌟🌟🌟🌟
Pattern Blocks: Finding Similar... | General (Math 3)+ | Java Applet | Activity |  | 🌟🌟🌟🌟🌟
Pattern Blocks: Finding Similar... | Tessellations (Geometry) | Java Applet | Activity |  | 🌟🌟🌟🌟🌟
Searching For: fraction (148 results)

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<tr>
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