Investigating Functional Inverses from the Concrete to the Symbolic

Materials: Patty Paper, rulers, colored pencils, Miras, grid paper

Investigation I:

1. Fold a sheet of Patty Paper twice to form the axes of a Cartesian plane. Label the x-axis and the y-axis.

2. Carefully fold your sheet of Patty Paper to form the line $y = x$. Label this line.

3. What are the characteristics of all the points that lie on the line $y = x$?

4. Sketch the function $y = 2x + 1$ on your Cartesian plane.

5. Fold the sheet of Patty Paper along the line $y = x$ to find the reflection of $y = 2x + 1$ across the line $y = x$. You may also use a Mira to find the reflection. Sketch this image a different color from your sketch of $y = 2x + 1$.

6. Unfold the sheet of Patty Paper. Describe the relationship between the original graph and its reflection across the line $y = x$.

7. Make two tables of x- and y-values, one for $y = 2x + 1$ and one for its reflection across the line $y = x$. Describe the relationship between the two tables.

8. What are the x- and y-intercepts of $y = 2x + 1$ and its reflection across the line $y = x$? Label them on your Cartesian plane. Describe the relationships that you see.

9. Find the equation of this image of $y = 2x + 1$?

10. Is this image of $y = 2x + 1$ a function? Explain.

Investigation II:

1. Fold a sheet of Patty Paper twice to form the axes of a Cartesian plane. Label the x-axis and the y-axis.

2. Carefully fold your sheet of Patty Paper to form the line $y = x$. Label this line.

3. Sketch the function $y = x^2$ on your Cartesian plane.

4. Fold the sheet of Patty Paper along the line $y = x$ to find the reflection of $y = x^2$ across the line $y = x$. You may also use a Mira to find the reflection. Sketch this image a different color from your sketch of $y = x^2$.

5. Unfold the sheet of Patty Paper. Describe the relationship between the original graph and its reflection across the line $y = x$.

6. Make two tables of x- and y-values, one for $y = x^2$ and one for its reflection across the line $y = x$. Describe the relationship between the two tables.

7. What are the x- and y-intercepts of $y = x^2$ and its reflection across the line $y = x$? Label them on your Cartesian plane. Describe the relationships that you see.

8. Find the equation of this image of $y = x^2$?

9. Is this image of $y = x^2$ a function? Explain.

Investigation III:

1. Fold a sheet of Patty Paper twice to form the axes of a Cartesian plane. Label the x-axis and the y-axis.

2. Carefully fold your sheet of Patty Paper to form the line $y = x$. Label this line.

3. Sketch the function $y = x^3$ on your Cartesian plane.

4. Fold the sheet of Patty Paper along the line $y = x$ to find the reflection of $y = x^3$ across the line $y = x$. You may also use a Mira to find the reflection.Sketch this image in a different color from your sketch of $y = x^3$.

5. Unfold the sheet of Patty Paper. Describe the relationship between the original graph and its reflection across the line $y = x$.

6. Make two tables of x- and y-values, one for $y = x^3$ and one for its reflection across the line $y = x$. Describe the relationship between the two tables.

7. What are the x- and y-intercepts of $y = x^3$ and its reflection across the line $y = x$? Label them on your Cartesian plane. Describe the relationships that you see.

8. Find the equation of this image of $y = x^3$?

9. Is this image of $y = x^3$ a function? Explain.

Summary:

1. In the three investigations, which of the functions had images that were functions?

2. What would you have to do to a function whose image was not a function to make its image a function?

3. Describe the images of points on the original function that intersected the line $y = x$?

4. Repeat Investigation I for $y = 4$? Explain what you observe.

5. Explain the relationship between a function and its inverse. Your explanation should include descriptions of the relationships in tables, graphs, and equations.

6. Is the inverse of a function always a function? Explain.

7. Is the inverse of a function always a relation? Explain.

8. Name two functions that are their own inverses.