

Strengthening Mathematics Teachers' Pedagogical Content Knowledge through Collaborative Investigations in Combinatorics

Rice University

Mathematics Leadership Institute (MLI)

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Lake Tahoe, NV

MLI Goals

- Develop a cadre of lead teachers in mathematics.
- **Provide mathematics content support**
- Provide pedagogical support.
- Develop highly-qualified mathematics teachers.
- Ensure that ***all*** high school students have access to challenging mathematics courses.
- Impact the instructional practices of mathematics faculty, post-docs, and graduate students.

Summer 2 Mathematics Strand

Why?

- Learn new mathematics
- Change mindsets about what math is not
- Experience an alternative learning method and build confidence

How?

- PCMI approach

Pre- and Post-Test Items

Suppose we have stamps of every cent denomination, how many ways can you arrange stamps in a line (order matters) on an envelope to produce the following totals of postage? (Assume the stamps all have the same orientation.)

- (a) 5 cents
- (b) 6 cents
- (c) 9 cents

Suppose we have stamps of every cent denomination **except for 5 cents**, how many ways can you arrange stamps in a line (order matters) on an envelope to produce the following totals of postage? (Assume the stamps all have the same orientation.)

- (a) 5 cents
- (b) 6 cents
- (c) 9 cents

Pre- and Post-Test Items

Expand the following polynomial $(a + b)^9$.

What is the sum of the coefficients of the expression $(1 + x)^{10}$ when it is fully expanded?

How many "words" (they don't have to be real words in a language just a sequence of letters) can one make by rearranging the letters in *NATASHA*?

Pre-Test Performance

n = 30

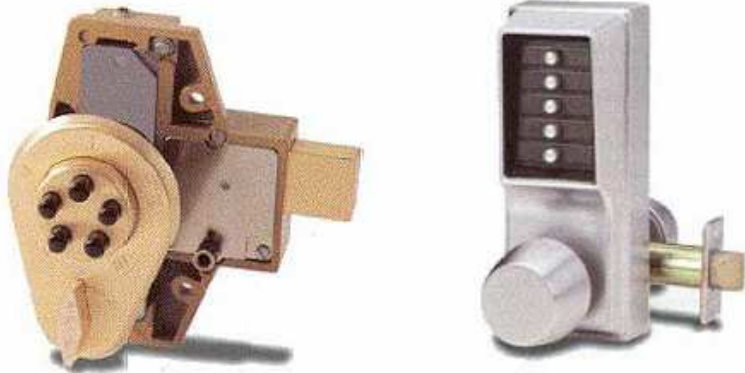
Maximum possible score 40

Pre-test: Mean= **10.4** Median = **11**

Teachers' problem-solving knowledge in combinatorics needed serious attention!

The Mathematics: A Pushbutton Lock

Here's a problem that will weave its way in and out of many of the topics we discuss this summer. We'll start working on it right now, but don't feel that you have to get the "right answer" right away. We'll keep coming back to it, even after you might be satisfied that you know what's going on. There's a lot to discover in this problem.



At least one company advertises that there are thousands of combinations, and the question is, "Is the company telling the truth?"

PROBLEM

How many combinations are there on a 5-button lock?

The Mathematics: A Pushbutton Lock

These 5-button devices are purely mechanical (no electronics). You can set the combination using the following rules:

1. A combination is a sequence of 0 or more pushes, each push involving at least one button.
2. Each button may be used at most once (once you press it, it stays in).
3. Each push may include any of the buttons that haven't been pushed yet, up to and including all remaining buttons.
4. The combination does *not* need to include all buttons.
5. When two or more buttons are pushed at the same time, order doesn't matter.

The Mathematics: A Pushbutton Lock

Here are some possible combinations:

$\{\{1, 3\}, \{4\}\}$

$\{\{1, 2, 4\}, \{3, 5\}\}$

$\{\{3\}, \{1, 2\}\}$

$\{\{1, 2\}, \{3\}\}$

$\{\{1, 2, 3, 4, 5\}\}$

$\{\{2\}, \{1\}, \{3\}\}$

$\{\{1, 2\}, \{4\}, \{3, 5\}\}$

$\{2\}$

$\{\}$

Notation:

$\{\{1, 2\}, \{3\}\}$ means

“press 1 and 2

together, then press 3.”

There's one possible no-push combination: the door's just already unlocked. Not a great combination, but it counts.

Impact: Pre- and Post-Tests

n = 30

Maximum possible score 40

Pre-test: Mean = **10.4** Median = **11**

Post-test: Mean = **33.9** Median = **36.5**

$t (df = 31) = -20.79, p < .0001$

Teachers' problem-solving knowledge increased significantly as a result of participating in the summer leadership institute.

Impact: Lead Teacher Survey

Over 90 percent of lead teachers agreed that as a result of participating in the 2006 MSLI:

- Their knowledge and understanding of combinatorics increased.
- Their ability to solve mathematic problems increased.

Impact: Reflection Journals

- *The combinatorics problems are finally starting to make sense. From day one, as a group we generated answers and answers only. I had an answer but could not explain my thought process. They are more challenging every day and it challenges my understanding to not just get an answer but explain my process and thought.*
- *I feel lost occasionally with the discovery/learn-on-your-own method. I wish there could be a balance between presentation (direct instruction) and discovery.*

Impact: Reflection Journals

- *I like solving the problems but would also enjoy sometime to be taught! [Still waiting for the “pull it back together” moment].*
- *I am enjoying working with our group a lot. It was wonderful that we would be able to be together and share our ideas and ways of thinking.*

Impact: Reflection Journals

- *Morning session – more challenging problems. I worked with a new group today. It is wonderful to see how different people work on a problem.*
- *I wish I could get across to my students the satisfaction of the challenge of an advanced math problem. I want them to enjoy the struggle the way I am.*

Impact: Reflection Journals

- *Landon [the MATH graduate student] is so calm and assuring and uses wait time well – we were all saying the same thing in English – Landon wanted us to jump to math language, function notation which we haven't used in a long time – notation we normally do not use!! We must speak the same language.*
- *I've really enjoyed the problem solving. It's much different from doing problems on your own. It's much more interesting when we learn within a community with support.*

MLI Information and Contacts

MLI webpage <http://nsfmli.rice.edu/index.htm>

Antwanette Hill, Ed.D., External Evaluator
antwanette.hill@cfisd.net

Ann McCoy, Ph.D., External Evaluator
ann.mccoy@att.net

Jackie Sack, Ed.D., MLI Co-Manager
jsack@rice.edu