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Exploring Finite Differences through Multiple Representations of Functions

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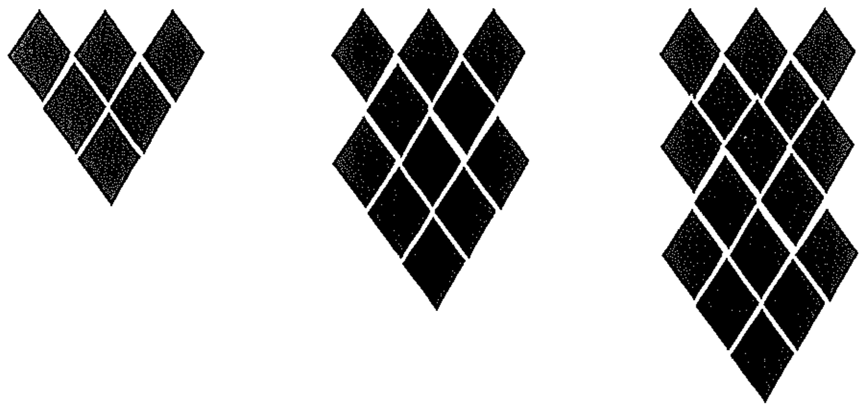
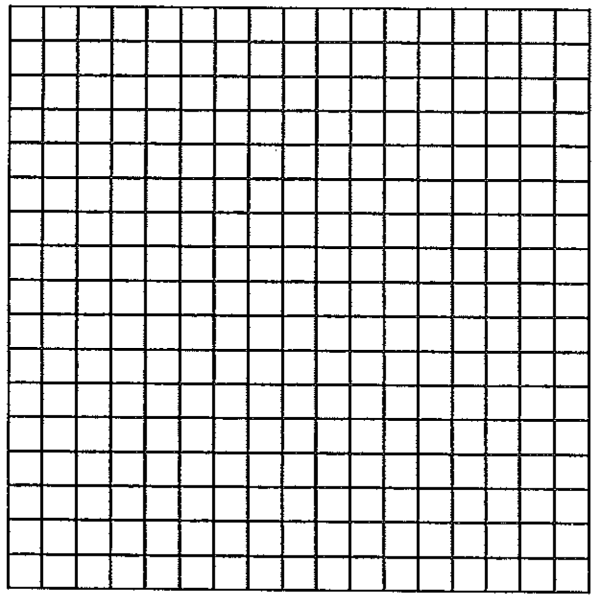
1. Study this pattern. Count the number of tiles in each step. Identify how the pattern stays the same and how it grows. Then complete problems 2 through 7.



2. Complete the table.

Term Number	Process Column	Numerical Value of Term
1		
2		
3		
4		
5		
...		
n		

3. Graph the function



4. Find the number of tiles for:

$n = 10$ ______________

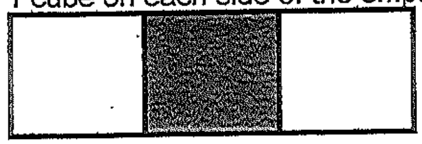
$n = 25$ ______________

$n = 57$ ______________

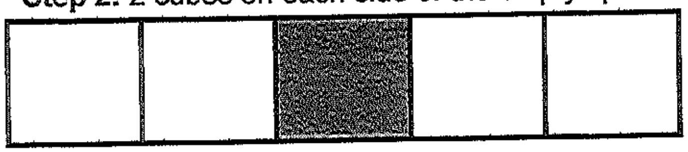
5. Find the rule for the pattern. ______________

SHUTTLE GAME

Step 1: 1 cube on each side of the empty space

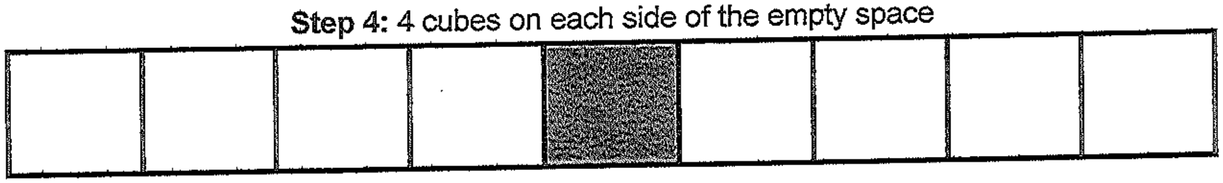


Step 2: 2 cubes on each side of the empty space

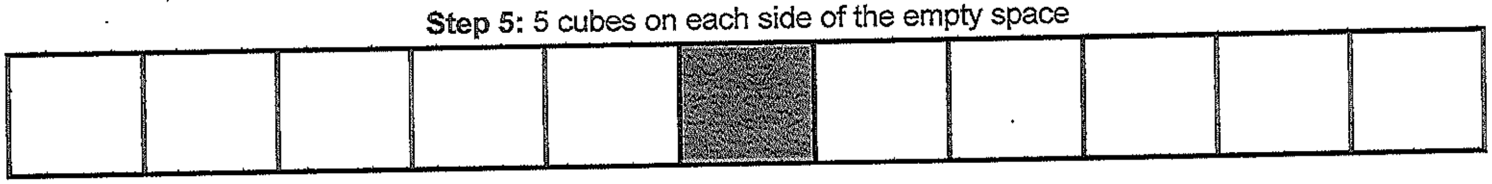


Step 3: 3 cubes on each side of the empty space

Step 4: 4 cubes on each side of the empty space



Step 5: 5 cubes on each side of the empty space



Start with green cubes on one side and yellow cubes on the other side. The goal is to reverse the colors in the fewest number of moves following these rules:

(1) A cube may be moved forward one space or may jump forward over one cube of the opposite color.

(2) A cube may not be moved backwards.

The Frog Puzzle (http://www.hellam.net/maths2000/frogs.html)

Cubes	Moves
1	
2	
3	
4	
5	
6	
7	
10	
50	
n	

Pizza Pieces

Find a function that relates the greatest number of pizza pieces, n, that a pizza cutter can obtain from c cuts on a circular pizza. The pieces do not have to be equal.

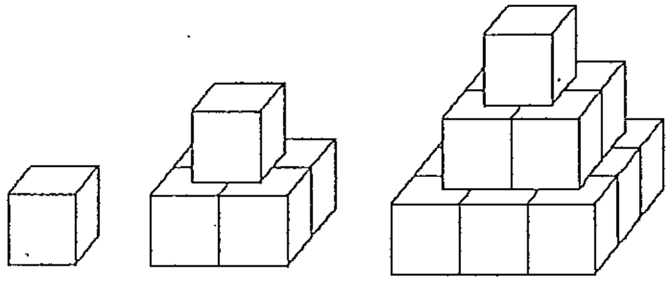
Procedure:

1. Draw circular pizzas indicating the maximum number of pieces obtained with one cut, two cuts, three cuts, etc.
2. Organize your data in a chart like the one below.

Number of cuts (c)	Number of pieces (n)
0	
1	
2	
3	
4	
5	

3. Sketch a graph of your data.
4. Find a function rule that expresses the relationship between the number of pieces (n) and the number of cuts (c).
5. What is the maximum number of pizza pieces obtained with five cuts? With ten cuts?
6. Is there a greatest number of pieces?

Pyramid Functions



You can use blocks to build pyramids such as those shown above. Complete the table showing the number of layers in each pyramid and the number of blocks needed to build it. All pyramids are solid with no empty space inside.

a.

Layers (n)	1	2	3	4	5	8	50
Blocks (b)							

b. Use finite differences to find the degree of the relationship.

c. Write an equation for this relationship.

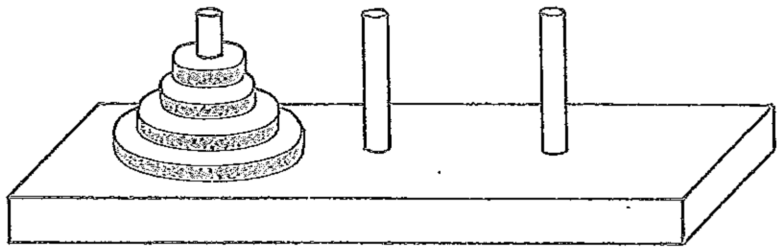
d. Use your model to predict the number of blocks needed to build a pyramid 8 layers high.

e. Graph and trace the curve to find the number of layers in a pyramid built with 650 blocks.

Advanced Algebra Through Data Exploration, p. 480 (Key Curriculum)

TOWER OF HANOI

The object of this ancient puzzle is to transfer the tower of discs to either of the two vacant pegs in the fewest possible moves. You may only move one disc at a time. You may not place a disc on one that is smaller.



In the table to the right, n represents the number of discs in the tower. M represents the fewest number of moves it takes to transfer those discs to the vacant pegs.

What is the fewest number of moves with four discs?

Complete the table at the right through seven discs.

Find a pattern which would give you the solution for 64 discs.

How did you find the pattern?

What is the formula for the fewest number of moves needed to transfer n number of discs?

Find a pattern for the number of moves each disc makes. Consider the smallest disc number one.

n	M
1	_____
2	_____
3	_____
4	_____
5	_____
6	_____
7	_____
⋮	
64	_____
⋮	
n	_____

Finite Differences: A Pattern-Discovery Approach to Problem-Solving
D. Seymour & M. Shedd; Dale Seymour Publications (1973)

REFERENCE CHART

First Degree

$y = ax + b$

x	y	
0	b	
		a
1	$a+b$	
		a
2	$2a+b$	
		a
3	$3a+b$	
		a
4	$4a+b$	
		a
5	$5a+b$	

Second Degree

$y = ax^2 + bx + c$

x	y		
0	c		
		$a+b$	
1	$a+b+c$		$2a$
		$3a+b$	
2	$4a+2b+c$		$2a$
		$5a+b$	
3	$9a+3b+c$		$2a$
		$7a+b$	
4	$16a+4b+c$		$2a$
		$9a+b$	
5	$25a+5b+c$		

Third Degree

$y = ax^3 + bx^2 + cx + d$

x	y			
0	d			
		$a+b+c$		
1	$a+b+c+d$		$6a+2b$	
		$7a+3b+c$		$6a$
2	$8a+4b+2c+d$		$12a+2b$	
		$19a+5b+c$		$6a$
3	$27a+9b+3c+d$		$18a+2b$	
		$37a+7b+c$		$6a$
4	$64a+16b+4c+d$		$24a+2b$	
		$61a+9b+c$		
5	$125a+25b+5c+d$			

Fourth Degree

$y = ax^4 + bx^3 + cx^2 + dx + e$

x	y				
0	e				
		$a+b+c+d$			
1	$a+b+c+d+e$		$14a+6b+2c$		
		$15a+7b+3c+d$		$36a+6b$	
2	$16a+8b+4c+2d+e$		$50a+12b+2c$		$24a$
		$65a+19b+5c+d$		$60a+6b$	
3	$81a+27b+9c+3d+e$		$110a+18b+2c$		$24a$
		$175a+37b+7c+d$		$84a+6b$	
4	$256a+64b+16c+4d+e$		$194a+24b+2c$		$24a$
		$369a+61b+9c+d$		$108a+6b$	
5	$625a+125b+25c+5d+e$		$302a+30b+2c$		
		$671a+91b+11c+d$			
6	$1296a+216b+36c+6d+e$				

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