



Algebraic Thinking for All Students



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Handouts available electronically at
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Why Algebraic Reasoning or Thinking?



What is Algebraic Thinking?

“Algebraic thinking or algebraic reasoning involves forming generalizations from experiences with number and computation, formalizing these ideas with the use of a meaningful symbol system, and exploring the concepts pattern and function.”

(Van De Walle, 2010, p. 254)



Algebraic Reasoning includes:

- ❖ Pictorial, graphic and verbal descriptions
- ❖ Numeric representations



Where is number in algebraic reasoning?



Algebraic Reasoning

- ❖ Generalization from arithmetic
- ❖ Meaningful use of symbols
- ❖ Study of patterns and functions



Generalization from Arithmetic



Developing Arithmetic in the Elementary Grades

- ❖ The separation of arithmetic and algebra deprives students of powerful ways of thinking about mathematics.
- ❖ Fundamental properties that children use in calculating are the basis for most of symbolic manipulation in algebra.



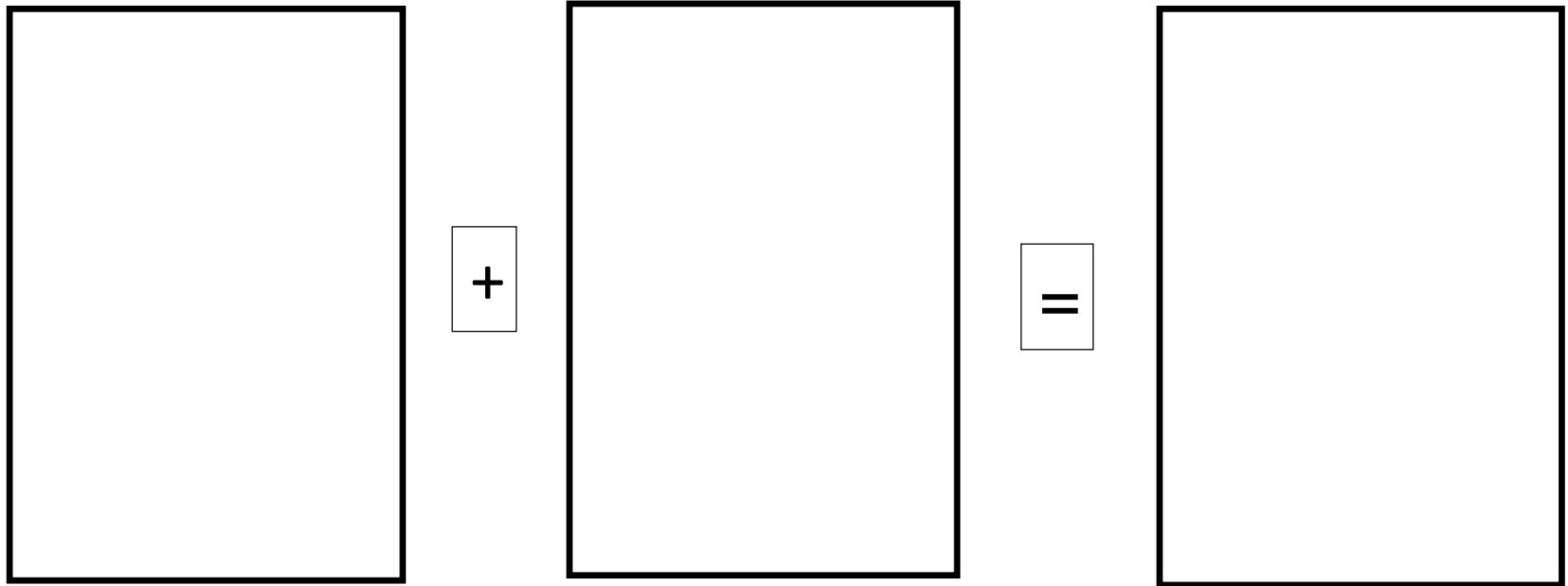
Using Playing Cards

Let's play the game 'Salute'

- ❖ Three players on each team
- ❖ Deck of cards
- ❖ Paper to record (optional)



Using Playing Cards to form Equations





Using Playing Cards to form Equations

Four problems involving playing cards

$$\square + \boxed{7} = \boxed{10} \quad (x + 7 = 10)$$

$$\square + \square = \boxed{8} + \boxed{6} \quad (2x = 8 + 6)$$

The first two cards are the same.

$$\square + \square + \square = \square + \square \quad (3x = 2y)$$

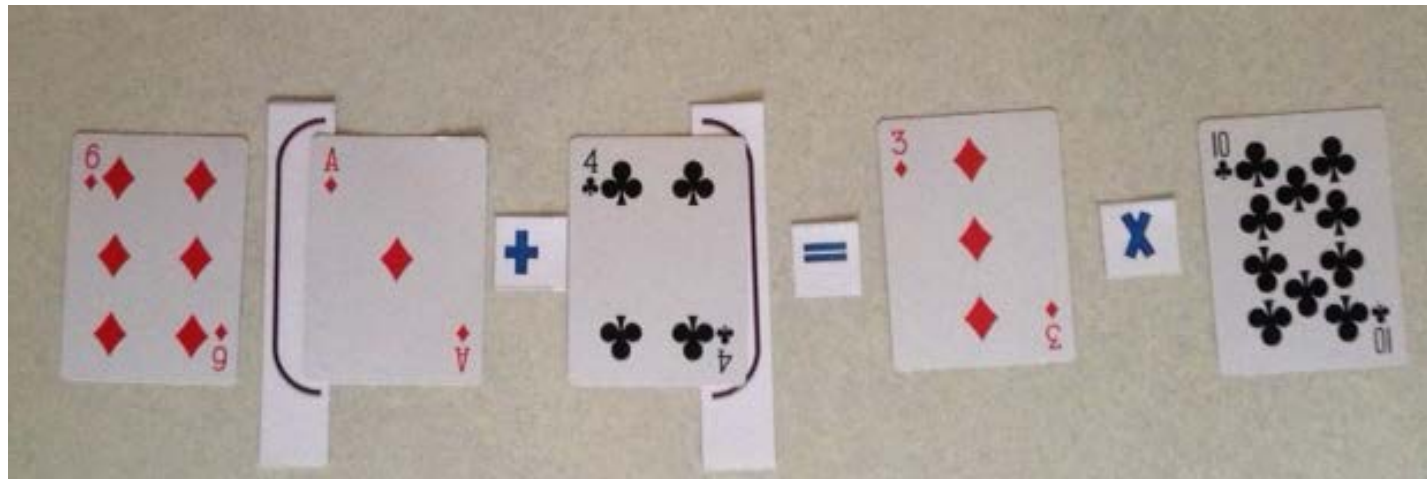
The first three cards are the same, and the last two cards are the same.

$$\square + \square + \square + \square = \square + \square + \square + \square$$

All eight cards must be different.



Using order of operations to evaluate expressions and solve equations




















Meaningful Use of Symbols



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The students were introduced to a system of equations.
HOW MUCH IS EACH SYMBOL WORTH?
















			32
			29
			25
			24
33	40	37	
			



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HOW MUCH DOES EACH FISH COST?
















			SUM	
				\$6
				\$12
				\$19
				\$16
SUM	\$13	\$19	\$21	
				
	Goldfish	Beta	Clown Fish	



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HOW MUCH DOES EACH WHALE WEIGH IN TONS?

			SUM
			330
			170
			200
			185
SUM	215	375	295
			
Right Whale	Gray Whale	Blue Whale	



Questions to ask students

- ❖ Can you tell me what you were thinking?
- ❖ Did you solve this in a different way?
- ❖ How do you know this is true?
- ❖ Does this always work?



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Figure This!
Math Challenges for Families

Which is worth more, a
SMILE or a **FROWN?**



Figure This! The costs of combinations of frowns, smiles, and neutral faces are shown. How much is a smile worth?

Hint: Find a way to combine two of the rows or columns that have something in common.

Reasoning about unknowns is essential in studying equations. Economists, nurses, chemists, and engineers all use equations in their work.



Systems of Equations

Objectives of the investigation

Students will:

- ❖ Develop their ability to reason with and represent with variables;
- ❖ Move away from random guess-and-check to a more logical approach for finding values for variables in a system of equations; and
- ❖ Understand various approaches to solving the same problem.



Make up your own chart

—
—
—

— — —



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Study of Patterns and Functions



Two of Everything
By Lily Toy Hong





Two Of Everything

- ❖ Read the book.
- ❖ Act out the story using a magical pot.
- ❖ Develop a table of values using Input and Output.
- ❖ Utilize pattern found from the table to generalize a rule verbally and using symbols.



What would you choose ?

- ❖ Choice A: 100 coins each day for 10 days
- ❖ Choice B: 5 coins and a magical pot that doubled the coins each day for 10 days

Justify your reasoning



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Study of Patterns and Functions



The Birthday Party Problem

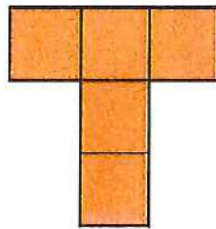
Scenario:

Tom is having a birthday party. For his birthday, he has decided to arrange square tables in the shape of a T, for Tom. Tom is trying to figure out how many tables he will need for different sizes of T's. The different sizes of T's are referred to as arrangements.

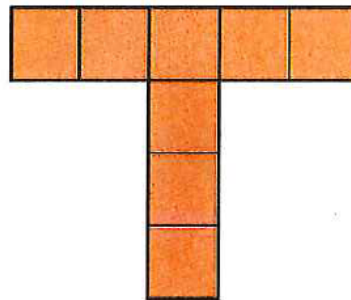


Birthday Party Patterns

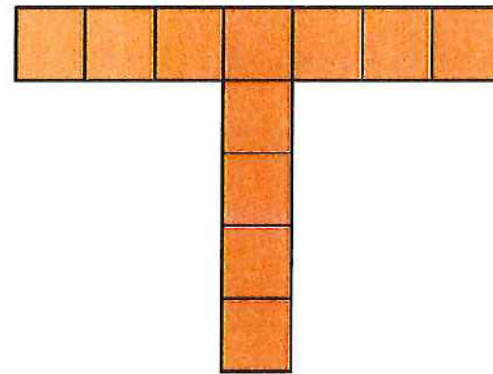
Tom wants to arrange the tables for his birthday party in the shape of a T, for Tom. He is trying to figure out how many tables he will need for different sizes of T's.



Arrangement 1



Arrangement 2



Arrangement 3

Use color tiles to build the arrangements 1-5 and look for patterns.



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Birthday Party Patterns

Sam wants to arrange the tables for his birthday party in the shape of a T, for Sam. He is trying to figure out how many tables he will need for different sizes of T's.



1. Question

2. Without drawing or building

3. Use colored tiles to build arrangements, show your work





Birthday Party Problem

Arrangement	Number of Tables
1	5
2	
3	
4	



Birthday Party Problem

Arrangement	Number of Tables	Number of Tables	
1	5	5	
2	8	$5 + 3$	
3	11		
4	14		



Birthday Party Problem

Arrangement	Number of Tables	Number of Tables	Number of Tables
1	5	5	$5 + 0 \times 3$
2	8	$5 + 3$	$5 + 1 \times 3$
3	11	$5 + 3 + 3$	$5 + 2 \times 3$
4	14	$5 + 3 + 3 + 3$	$5 + 3 \times 3$

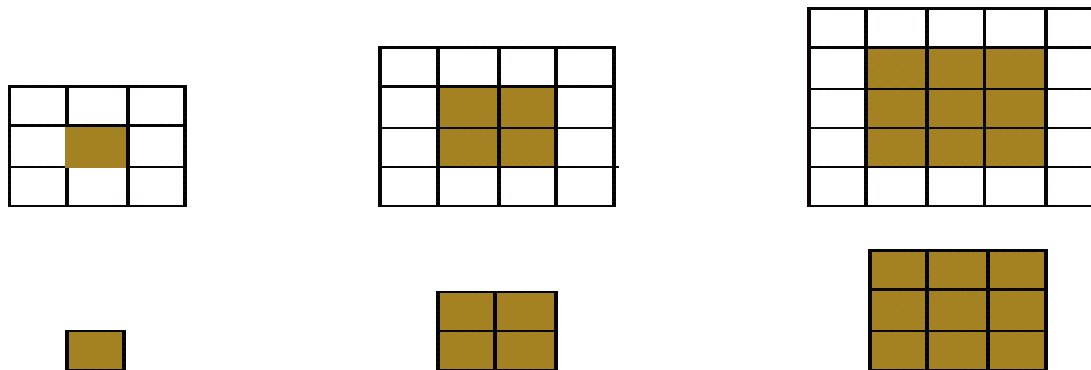


Finding Patterns and Functions



The Tiling a Patio problem

Alfredo Gomez is designing square patios. Each patio has a square garden area in the center. Alfredo uses brown tiles to represent the soil of the garden. Around each garden, he designs a border of white tiles. The pictures show the three smallest square patios that he can design with brown tiles for the garden and white tiles for the border.





Tiling a Patio

Patio Number	Number of Brown Tiles	Number of White Tiles	Total Number of Brown and White Tiles
1	1	8	9
2	4	12	16
3			

**It is now time for GETS
(Graph, Equation, Table, Solution)**



Crossing the River Problem

Students will:

- develop their ability to reason with and represent with variables
- move away from random guess-and-check to a more logical approach for finding values for variables in a system of equations and
- understand various approaches to solving the same problem.



Crossing the River Problem

Scenario

Eight adults and two children need to cross a river. A small boat is available that can hold one adult, or one or two children. Everyone can row the boat. How many one-way trips does it take for them all to cross the river?

Lets act the story out with:

- ❖ one adult and two children
- ❖ two adults and two children



Crossing the River Problem

Scenario

Eight adults and two children need to cross a river. A small boat is available that can hold one adult, or one or two children. Everyone can row the boat.

How many one-way trips does it take for them all to cross the river?



Crossing the River Problem

Extension

Can you describe how to work it out for two children and any number of adults?



Internet Resources

- ❖ Pan Balance Shapes
<http://illuminations.nctm.org/Activity.aspx?id=3531>
- ❖ Function Machine
<http://www.shodor.org/interactivate/activities/FunctionMachine/>
- ❖ Function Machine Math Playground
<http://www.mathplayground.com/functionmachine.html>
- ❖ Stop That Creature!
<http://pbskids.org/cyberchase/media/games/functions/>



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- **Visual Algebra Puzzles**



Create your own algebra puzzles then try to solve them! This easy to use, educational tool was designed to work together with Shuttle Mission Math, an algebraic reasoning game in the app store. Puzzles can be solved with at least one of the following visual strategies: Scale Up, Scale Down (multiply or divide)

<https://itunes.apple.com/us/app/visual-algebra-puzzles/id662990649?mt=8>

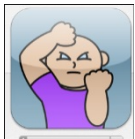
- **Shuttle Mission Math**



Shuttle Mission Math is a mathematical puzzle game that makes algebraic thinking both visual and interactive. The goal is to find the weight of each space creature and assemble a team for the next shuttle mission.

<https://itunes.apple.com/us/app/shuttle-mission-math/id498617241?mt=8>

- **Algebra Champ**



Game like environment for solving linear equations

<https://itunes.apple.com/us/app/algebra-champ/id398873050?mt=8>



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