

Promoting Algebraic Thinking through the Lens of Number



Rice University School Mathematics Project Houston, Texas

CAMT 2015 Houston, Texas



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Why Algebraic Reasoning?



What is Algebraic Reasoning?

"Algebraic thinking or algebraic reasoning involves forming generalizations from experiences with number and computation, formalizing these ideas with the use of a meaningful symbol system, and exploring the concepts pattern and function." (Van De Walle, 2010, p. 254)



Where is number in algebraic reasoning?



Algebraic Reasoning

- ***** Generalization from arithmetic
- Meaningful use of symbols
- Study of patterns and functions



Generalization from Arithmetic



Developing Arithmetic in the Elementary Grades

- The separation of arithmetic and algebra deprives students of powerful ways of thinking about mathematics.
- Fundamental properties that children use in calculating are the basis for most of symbolic manipulation in algebra.



Using Playing Cards

Let's play the game 'Salute'

- Three players on each team
- Deck of cards
- Paper to record (optional)



Using Playing Cards to form Equations





Using Playing Cards to form Equations

Figure 1





Using order of operations to evaluate expressions and solve equations





Meaningful Use of Symbols



Figure 2

The students were introduced to a system of equations. HOW MUCH IS EACH SYMBOL WORTH?





Goldfish

Figure 3

(a) The second activity used symbols familiar to the students.

SUM \$6 \$12 \$19 \$16 \$13 \$19 \$21 SUM

Beta

Clown Fish

HOW MUCH DOES EACH FISH COST?



Figure 4

(a) Students had more difficulty when the activity dealt with larger sums.

SUM 330 170 200 185 SUM 375 215 295 **Right Whale** Gray Whale Blue Whale

HOW MUCH DOES EACH WHALE WEIGH IN TONS?



Questions to ask students

- Can you tell me what you were thinking?
- Did you solve this in a different way?
- ***** How do you know this is true?
- Does this always work?



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Challenges for Families

Which is worth more, a SI E or a **FROWN**?

Figure This! The costs of combinations of frowns, smiles, and neutral faces are shown. How much is a smile worth?

Hint: Find a way to combine two of the rows or columns that have something in common.

Reasoning about unknowns is essential in studying equations. Economists, nurses, chemists, and engineers all use equations in their work.

Sum



Systems of Equations

Objectives of the investigation

Students will:

- Develop their ability to reason with and represent with variables;
- Move away from random guess-and-check to a more logical approach for finding values for variables in a system of equations; and
- Understand various approaches to solving the same problem.



Make up your own chart



Study of Patterns and Functions



<u>Two of Everything</u> By Lily Toy Hong





Two Of Everything

- Read the book.
- ✤ Act out the story using a magical pot.
- Develop a table of values using Input and Output.
- Utilize pattern found from the table to generalize a rule verbally and using symbols.



Activity Sheet 1

What would you choose ?

Choice A: 100 coins each day for 10 days

Choice B: 5 coins and a magical pot that doubled the coins each day for 10 days

Justify your reasoning



Activity Sheet 2





Study of Patterns and Functions



The Dinner Table Problem

Scenario:

Susan is preparing for a dinner party. She has seven tables that will be placed with one side close to one side of another table to make a long row. How many people can she invite?



Dinner Table Problem

| Dinner Tables | Show How | Number of People |
|----------------------|----------|------------------|
| 1 | | 4 |
| 2 | | 6 |
| 3 | | |



Finding Patterns and Functions



The Tiling a Patio problem

Alfredo Gomez is designing square patios. Each patio has a square garden area in the center. Alfredo uses brown tiles to represent the soil of the garden. Around each garden, he designs a border of white tiles. The pictures show the three smallest square patios that he can design with brown tiles for the garden and white tiles for the border.



Navigating Through Algebra grades 3-5



Tiling a Patio

| Patio Number | Number of Brown Tiles | Number of White Tiles | Total Number of Brown and White Tiles |
|--------------|--------------------------|--------------------------|---|
| 1 | 1 | 8 | 9 |
| 2 | 4 | 12 | 16 |
| 3 | | | |

It is now time for GETS (Graph, Equation, Table, Solution)



Students will:

- develop their ability to reason with and represent with variables
- move away from random guess-and-check to a more logical approach for finding values for variables in a system of equations and
- understand various approaches to solving the same problem.



Scenario

Eight adults and two children need to cross a river. A small boat is available that can hold one adult, or one or two children. Everyone can row the boat. How many one-way trips does it take for them all to cross the river?

Lets act the story out with:

- one adult and two children
- two adults and two children



Scenario

Eight adults and two children need to cross a river. A small boat is available that can hold one adult, or one or two children. Everyone can row the boat.

How many one-way trips does it take for them all to cross the river?



Extension

Can you describe how to work it out for two children and any number of adults?



Internet Resources

Scales and balance <u>http://nlvm.usu.edu/en/nav/frames_asid_324_g_3_t_2.html</u>

Pan Balance Shapes

http://illuminations.nctm.org/Activity.aspx?id=3531

Function Machine

http://www.shodor.org/interactivate/activities/FunctionMachine/

Function Machine Math Playground <u>http://www.mathplayground.com/functionmachine.html</u>

Stop That Creature!

http://pbskids.org/cyberchase/media/games/functions/



• Visual Algebra Puzzles



Create your own algebra puzzles then try to solve them! This easy to use, educational tool was designed to work together with Shuttle Mission Math, an algebraic reasoning game in the app store. Puzzles can be solved with at least one of the following visual strategies: Scale Up, Scale Down (multiply or divide)

https://itunes.apple.com/us/app/visual-algebra-puzzles/id662990649?mt=8

Shuttle Mission Math



Shuttle Mission Math is a mathematical puzzle game that makes algebraic thinking both visual and interactive. The goal is to find the weight of each space creature and assemble a team for the next shuttle mission.

https://itunes.apple.com/us/app/shuttle-mission-math/id498617241?mt=8

Algebra Champ



Game like environment for solving linear equations

https://itunes.apple.com/us/app/algebra-champ/id398873050?mt=8







This presentation is based in part on a project partially funded by the Teacher Quality Grants program at the Texas Higher Education Coordinating Board (grant #496). The Teacher Quality Grants Program is supported through federal funds under NCLB Title II, Part A.



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