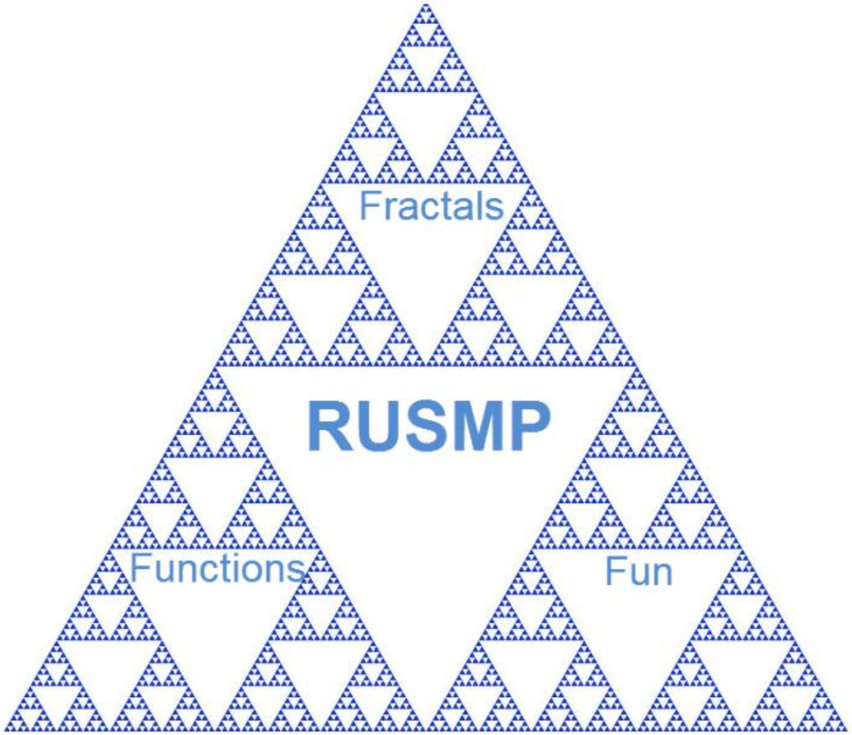
Fun with Fractals and Functions



2015 Conference for the Advancement of Mathematics Teaching

June 25, 2015

Houston, Texas

RICE UNIVERSITY
SCHOOL MATHEMATICS PROJECT

Alice Fisher
afisher@rice.edu
Director of Technology Applications & Integration

What is a fractal?

Fractal comes from the Latin word "fractus" for *broken* or *fractured* and was coined by the mathematician Benoit Mandelbrot in 1975.

Mandelbrot showed how visual complexity can be created from simple rules.

What is a fractal?

- Fractals are formed by evaluating a function over and over again for which the output becomes the input in the next iteration. This process is called **recursion.**

Example: $\mathbf{z_{n+1} = (z_n)^2 + C}$

With $z_0 = C$ where C is a point in the complex plane. The Mandelbrot set is the set of values of C for which the orbit under iteration remains **bounded**.

What is a fractal?

$$z_{n+1} = (z_n)^2 + C$$

When C = 1:

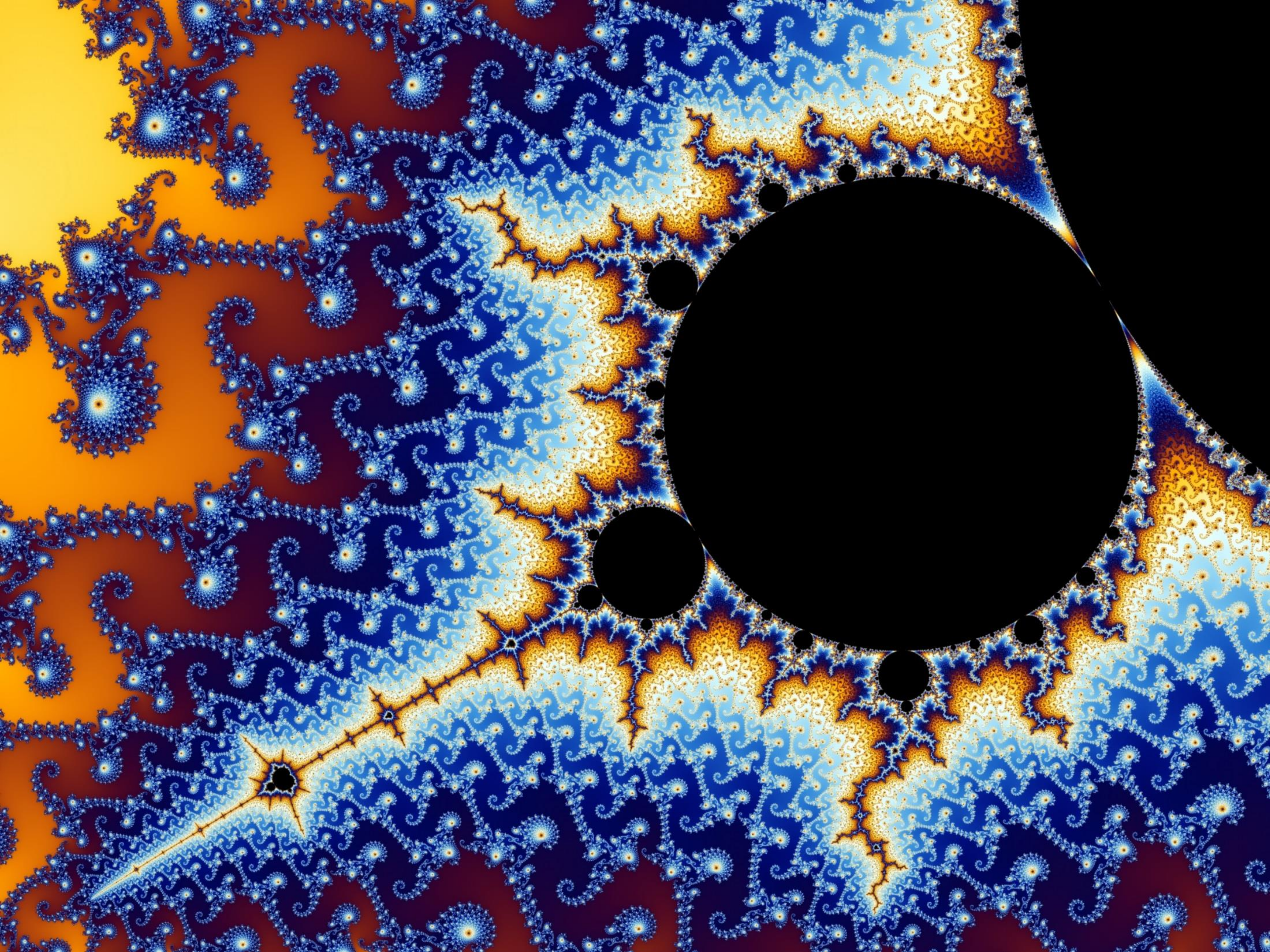
$z_0 = 1,\ z_1 = 1^2 + 1 = 2,\ \ z_2 = 2^2 + 1 = 5,\ \ z_3 = 5^2 + 1 = 26, \ldots$

Since this sequence tends to infinity, C = 1 is not in the Mandelbrot set.

When C = -1:

$z_0 = -1,\ \ z_1 = (-1)^2 + (-1) = 0,\ \ z_2 = 0^2 + (-1) = -1,$
$z_3 = (-1)^2 + (-1) = 0, \ldots$

Since this sequence is bounded, C = -1 is in the Mandelbrot set.



What is a fractal?

- When viewed closely, fractals are **self-similar**. Thus, if we zoom in on a fractal, we essentially see the same shape, but at a different scale.
- Thus, “smaller and smaller copies of a pattern are successively nested inside each other, so that the same intricate shapes appear no matter how much you zoom in to the whole.”(Stephen Wolfram, 2012)

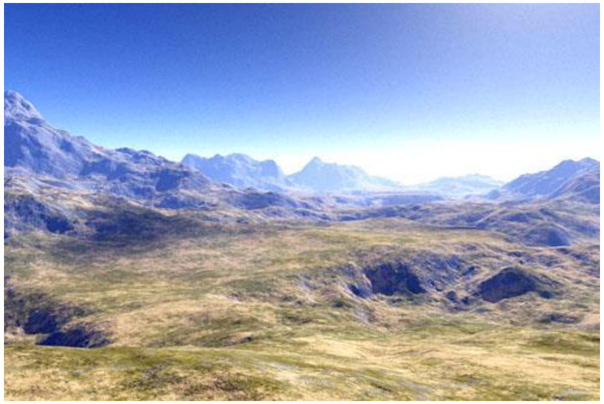
Xaos Software Demo

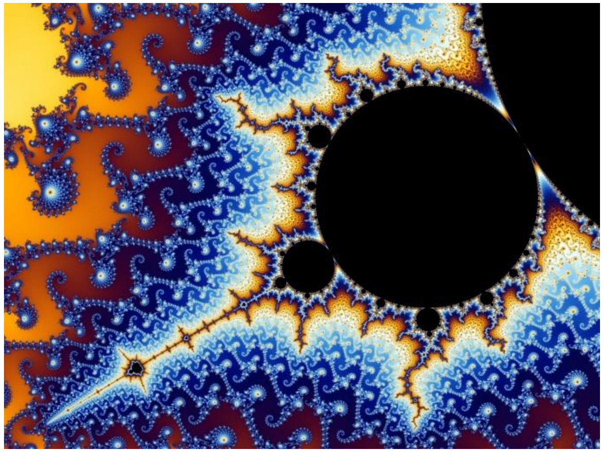
Let’s zoom in!

Where do we find fractals?

We find fractals in nature! Examples include **branching patterns** such as trees, river networks, blood vessels, mountains (below left) and **spiral patterns** such as seashells, hurricanes and galaxies (below right).

Fractals Created by Software



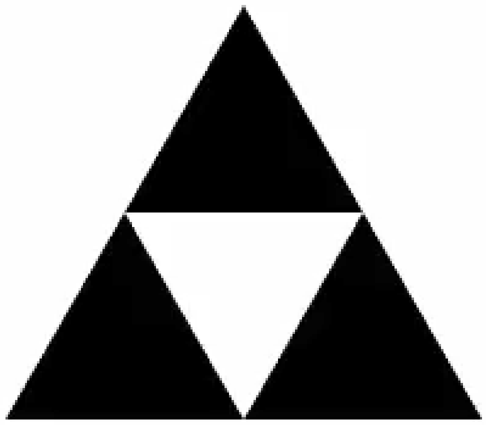


Why study fractals in the mathematics classroom?

- Fractals are relevant! They are found almost everywhere in nature.
- There is a lot of mathematics underlying fractals that is accessible to secondary school students.
- Fractals combine mathematics and art, and their beauty and diversity may engage students who are not normally “hooked” in a mathematics classroom.

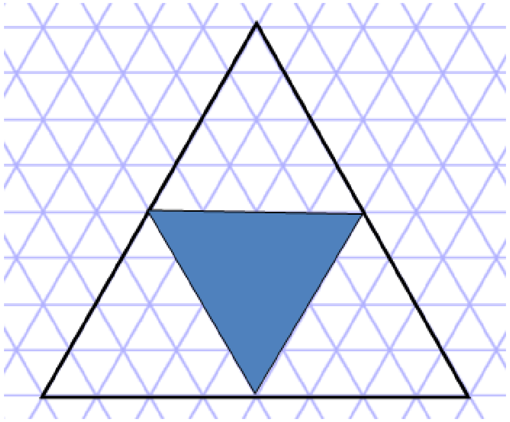
The Sierpinski Triangle

The **Sierpinski triangle** is a fractal named after the Polish mathematician, Wacław Sierpiński, who described it in 1915.

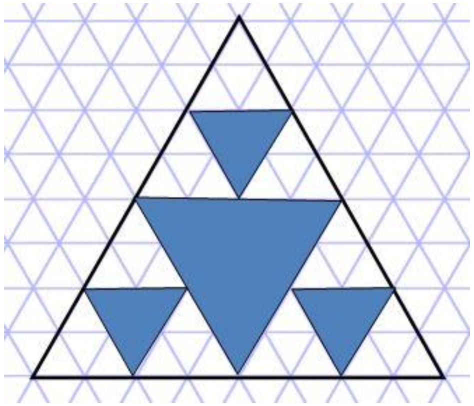


Let's create the first three iterations of the Sierpinski triangle:

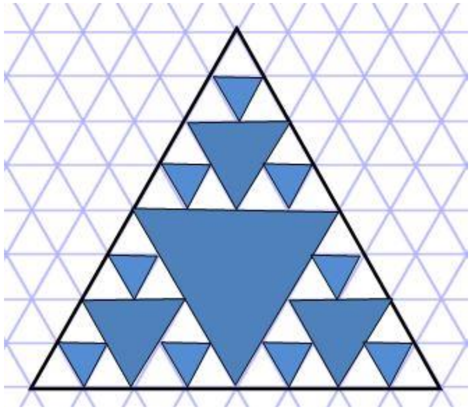
Iteration 1: Draw an equilateral triangle with side length of 8 units on triangular grid paper. Mark the midpoints of the three sides. Then connect the three midpoints and shade in the triangle that is pointing downward.



Iteration 2: Repeat the first iteration with a new triangle. Now mark the midpoints of the three sides of each of the three unshaded triangles. Connect the midpoints and shade the three triangles that are pointing downward.



Iteration 3: Repeat the first and second iterations with a new triangle. Now mark the midpoints of the three sides of each of the nine unshaded triangles. Connect the midpoints and shade/color the nine triangles that are pointing downward. Be creative when you shade in your triangles…use colors! Cut out the three triangles.



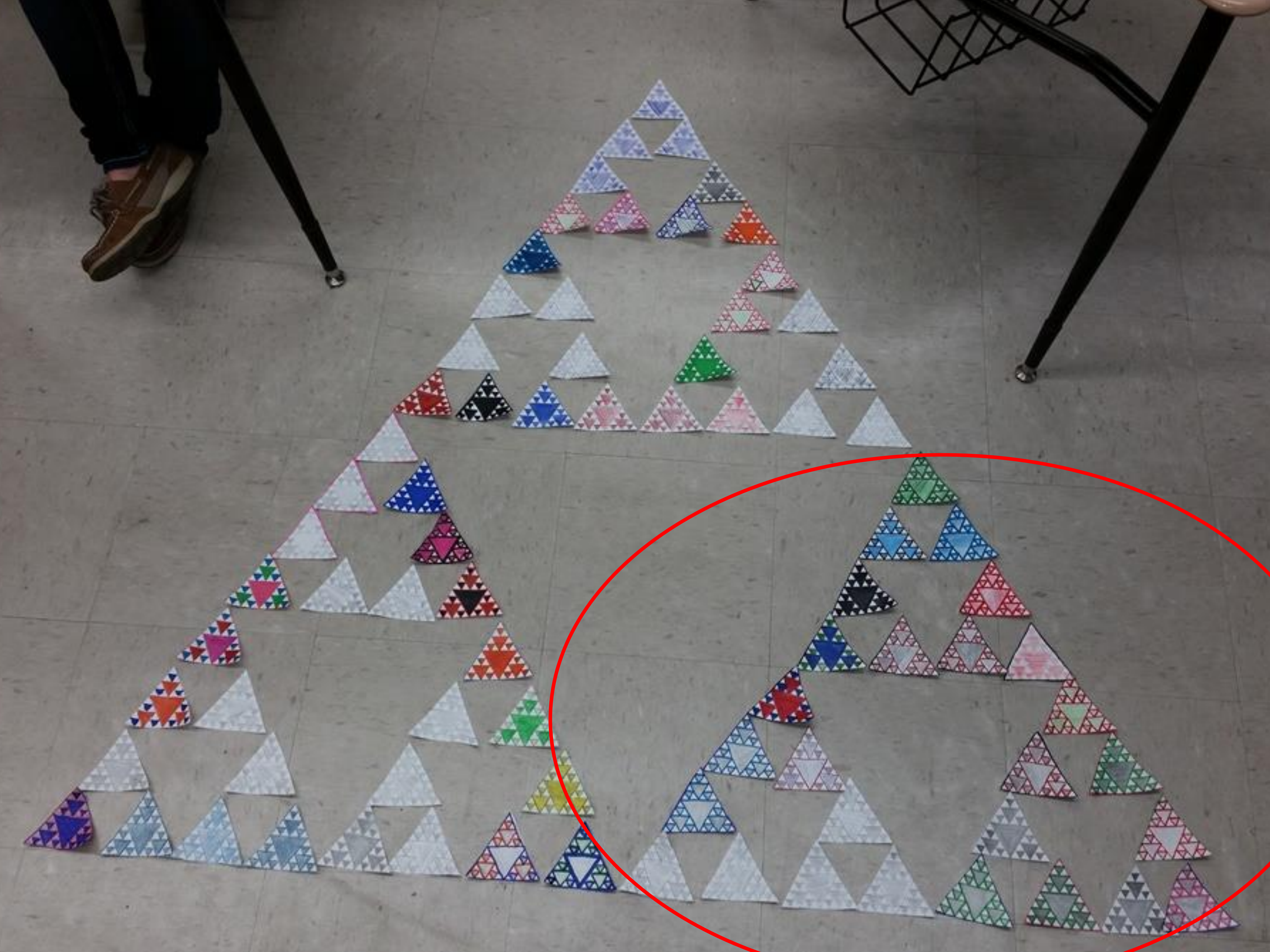
What do you notice?

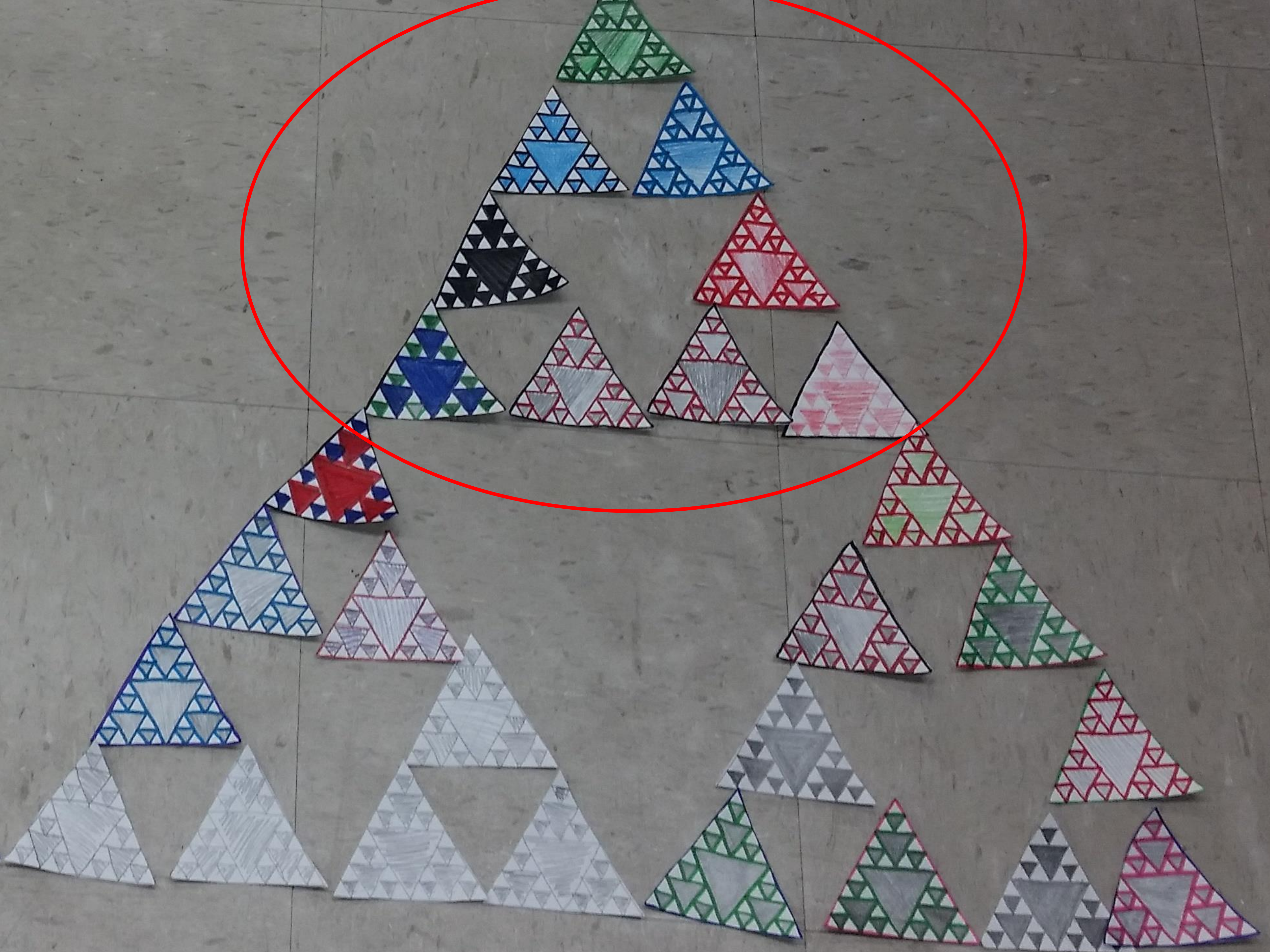
What do you wonder?

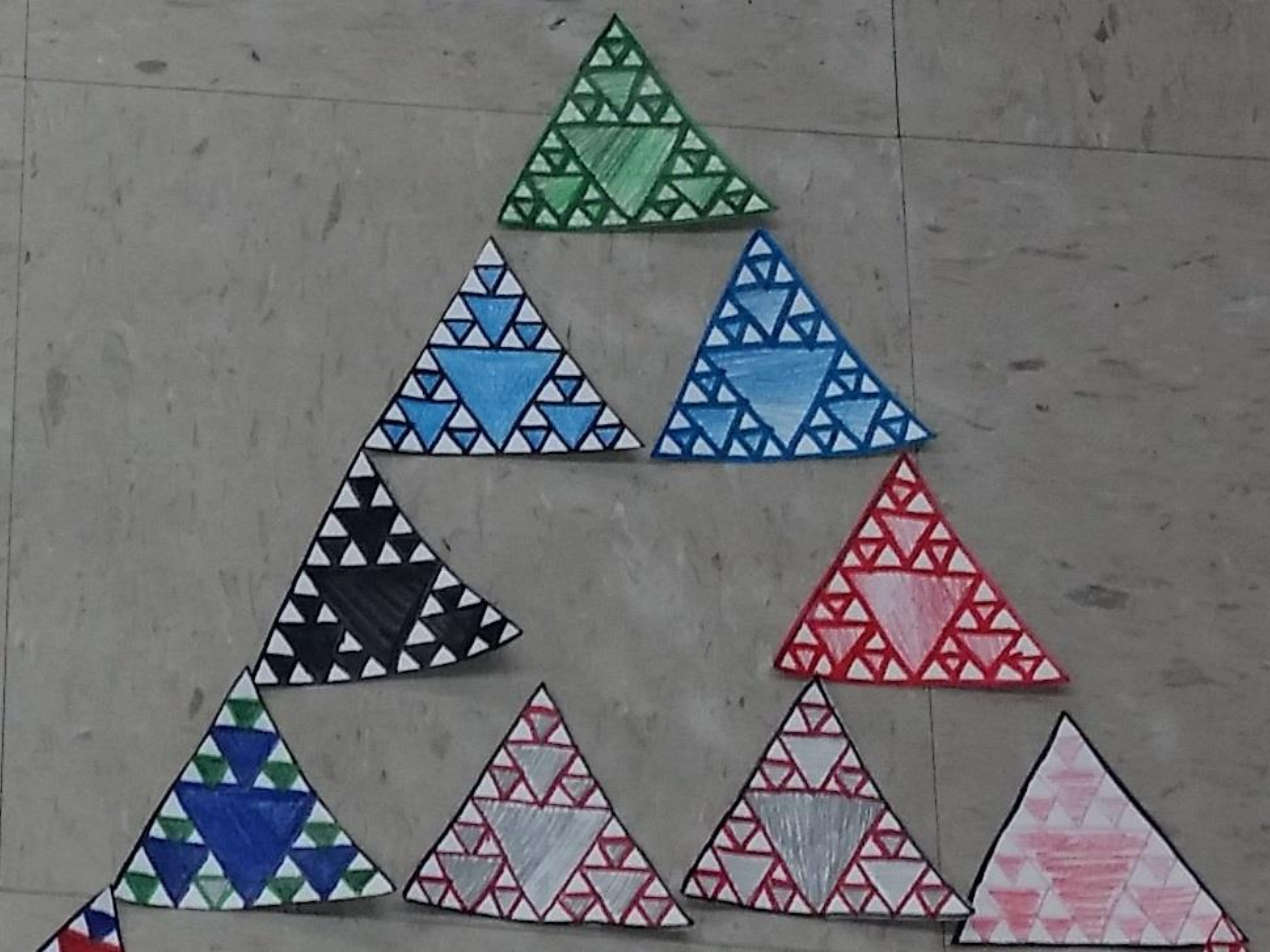
Share with your neighbor.

We will examine **3 functions** derived from looking at **successive iterations** of the Sierpinski triangle:

1. The number of unshaded triangles within the Sierpinski triangle
2. The ratio of the area of the Sierpinski triangle that is unshaded to the area of the entire Sierpinski triangle
3. The number of shaded triangles within the Sierpinski triangle

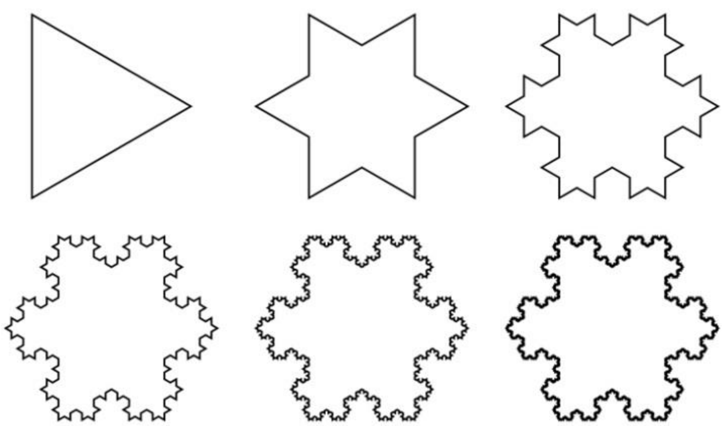




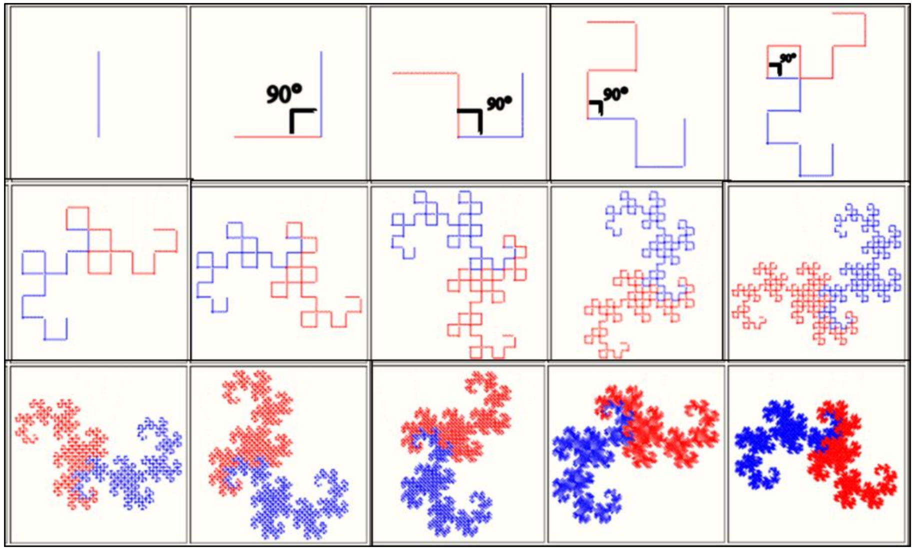


Fractals that lend themselves to study in a secondary mathematics classroom:

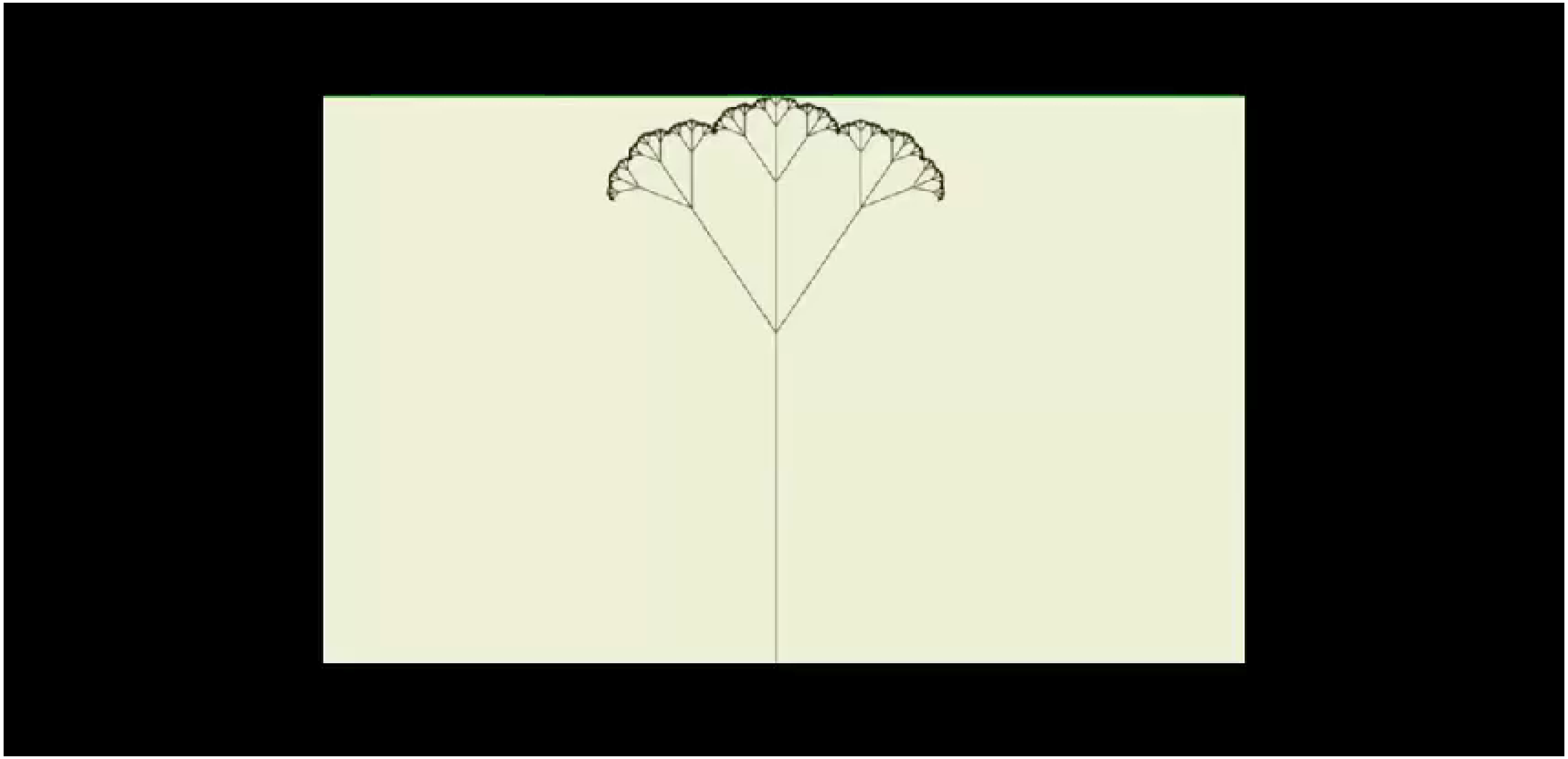
Koch snowflake



Jurassic Park fractal



Finding the Sierpinski triangle in surprising places!
(iOS app: Geom-e-Tree)



Rice University School Mathematics Project
Houston, Texas

Website: www.rusmp.rice.edu

Alice Fisher
afisher@rice.edu
Director of Technology Applications & Integration