Fun with Fractals and Functions

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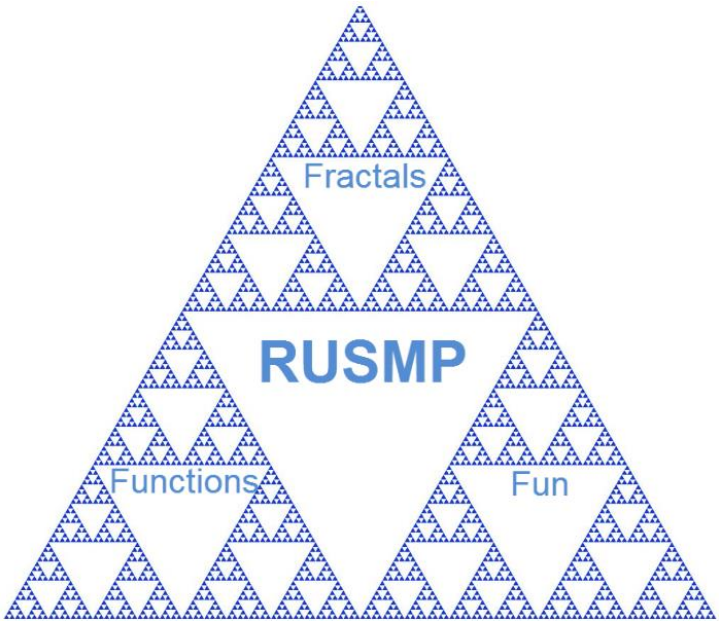

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The Sierpinski Triangle & Functions

The **Sierpinski triangle** is a fractal named after the Polish mathematician Waclaw Sierpiński who described it in 1915. Fractals are self-similar patterns that repeat at different scales.

Let's draw the first three iterations of the Sierpinski's Triangle!

Iteration 1: Draw an equilateral triangle with side length of 8 units on triangular grid paper. Use the bottom line of the grid paper to draw the base of this triangle. Mark the midpoints of the three sides. Then connect the three midpoints and shade in the triangle that is pointing downward.

Iteration 2: Repeat the first iteration with a new triangle. Now mark the midpoints of the three sides of each of the three unshaded triangles. Connect the midpoints and shade the three triangles that are pointing downward.

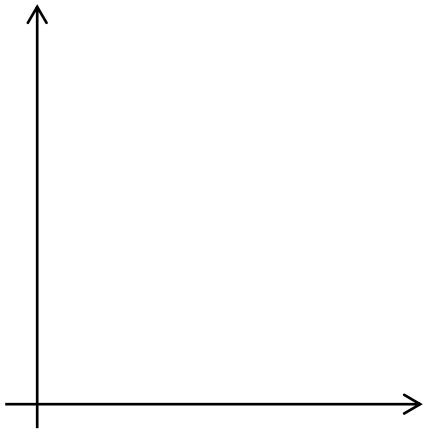
Iteration 3: Repeat the first and second iterations with a new triangle. Now mark the midpoints of the three sides of each of the nine unshaded triangles. Connect the midpoints and shade/color the nine triangles that are pointing downward. Be creative when you shade in your triangles! Cut out the 3 triangles.

*Look at the iterations. Write down patterns you notice. Share with your neighbor.

1. (a) How many unshaded triangles are there at each iteration? Fill in the table of values below for the first 5 iterations and describe the pattern in a sentence.

   (b) Write the function for the table of values, and then sketch a graph of the function. What type of function is this?

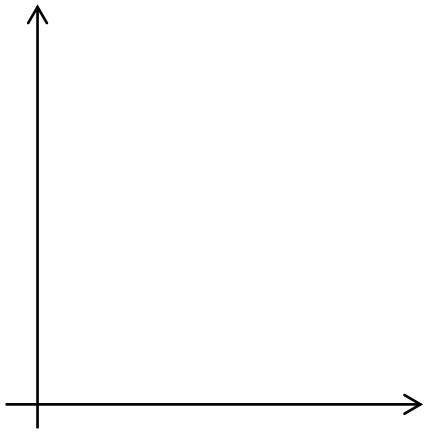
Iteration	**# of unshaded triangles**
0	1
1	3
2	9
3	
4	
5	
n	



2. (a)What fraction of the triangle is unshaded at each iteration? Fill in the table of values for the first 5 iterations and describe the pattern in a sentence.

(b) Write the function for the table of values, and then sketch a graph of the function. What type of function is this?

Iteration	**Fraction of triangle that is unshaded**
0	1
1	$\frac{3}{4}$
2	
3	
4	
5	
n	



(c) The Sierpinski's triangle is the area of the triangle that is left after the shaded triangles are removed, i.e., the unshaded part of the triangle. What can you say about the area of the triangle as the number of iterations approach infinity? Why? Explain below.

3. Let's now look at the boundary of the Sierpinski's triangle. Let's assume that the side length of the triangle is 1. Thus, the boundary is 3 units at iteration 0. For iteration 1, in addition to the outer edges, you will need to add the inside edges as well. Thus the boundary is $3 + \frac{3}{2} = \frac{9}{2}$. For iteration 2, the boundary is $3 + \frac{3}{2} + \frac{9}{4} = \frac{27}{4}$.

   (a) Fill in the table of values for the first 5 iterations and describe the pattern in a sentence.

   (b) Write the function for the table of values.

Iteration	**Boundary of the triangle**
0	3
1	$\frac{9}{2}$
2	$\frac{27}{4}$
3	
4	
5	
n	

   (c) What can you say about the boundary as the number of iterations approach infinity?

4. (a) How many shaded triangles are there at each iteration? Fill in the table of values for the first 5 iterations and describe the pattern in a sentence.

   (b) Write the function for the table of values.

Iteration	**Process**	**# of shaded triangles**
0	0	0
1	1	1
2	1+3	4
3	1+3+9	
4		
5		
n		

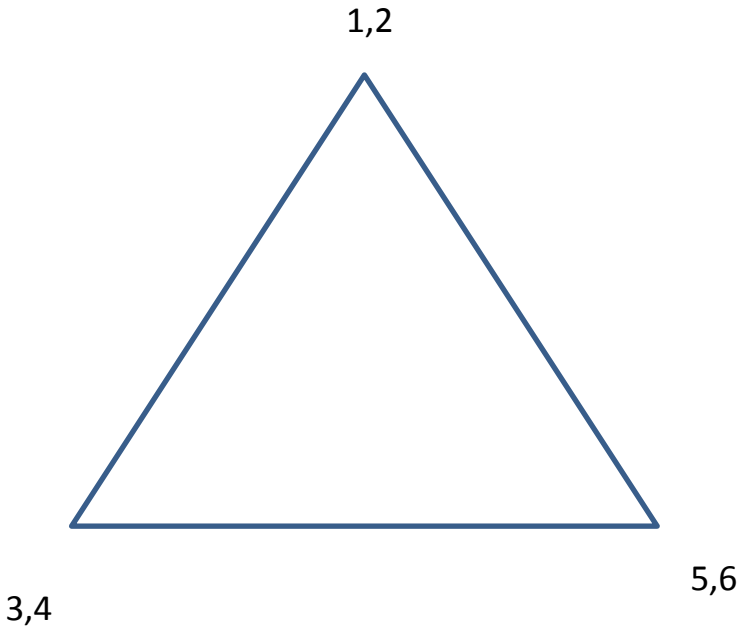
*Find another interesting pattern from Sierpinski's Triangle and share with your neighbor.

*How is the quote below related to the Sierpenski's triangle?

To see a world in a grain of sand,
And a heaven in a wildflower:
Hold infinity in the palm of your hand,
And eternity in an hour.
-- William Blake

The Chaos Game

- Pick a point inside the triangle to begin the game. This point is called the seed.
- Each vertex of the triangle has been labeled with two numbers (see below).
- One person rolls the die and the other person plots the next point half the distance from the seed to the vertex that corresponds to the result of the rolled die.
- Roll the die again, and then plot the next point half the distance from the last plotted point and the vertex that corresponds to the result of the rolled die.
- Plot 5-10 points on the triangle.



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Free Resources for Exploring Fractals:

- One-page flyer about fractals: http://fractalfoundation.org/fractivities/WhatIsaFractal-1pager.pdf
- Instructions to download free XaoS software (fractal zoomer): http://fractalfoundation.org/resources/fractal-software/xaos-instructions/
- Chaos Game, TI-Nspire file: http://www.johnhanna.us/TI-nspire.htm
- Chaos Game, GeoGebra: http://www.geogebratube.org/material/show/id/8896
- Chaotica, iOS app: https://itunes.apple.com/us/app/chaotica/id380892615?mt=8&ign-mpt=uo%3D4
- Chaos game, Shodor Interactivate: http://www.shodor.org/interactivate/activities/TheChaosGame/
- Chaos game, how to program in Python: http://www.billthelizard.com/2011/02/chaos-game.html