

Alief Independent School District



**Rice University  
School Mathematics Project**

**RUSMP**

Southwest Educational Development Laboratory



**Mathematics Assessment Project**

2000-2001

**Mathematics Assessment Project**

This project was sponsored by the Southwest Consortium for the Improvement for Mathematics and Science Teaching (SCIMAST).

# Mathematics Assessments

Under the Direction of:

Dr. Anne Papakonstantinou  
Director, RUSMP  
Rice University

Richard Parr  
Director of Educational Technology and  
Secondary Education, RUSMP  
Rice University

Dr. Marsha Lilly  
Secondary Mathematics Coordinator  
Alief ISD

Holly MacLean  
Elementary Mathematics Coordinator  
Alief ISD

# Alief ISD - RUSMP - SEDL

## Mathematics Assessment Project

### Author

### Campus

|                 |                          |
|-----------------|--------------------------|
| Amy Brown       | Elsik High School        |
| Mary Ann Frazee | Hastings High School     |
| Caryn Gurt      | Youngblood Intermediate  |
| Terri Johnson   | Holub Middle School      |
| Monique Mays    | Elsik Ninth Grade Center |
| EuRhonda Morgan | Hastings High School     |
| Jennifer Perez  | Cummings Elementary      |
| Cindy Scott     | Elsik High School        |
| Heather Webb    | Hastings High School     |

### Compiled by

Rachel Pinkston  
Rozanne Rubin

Hastings High School  
Elsik High School

# Introduction

Teachers from Alief Independent School District, with generous funding from Shell Westhollow Technology Center, attended the Rice University School Mathematics Project (RUSMP) in June of 2000. In the fall of 2000, participants were invited to take part in a semester-long graduate seminar on Alternative Assessment. Southwest Educational Development Laboratory provided generous financial backing for this follow up assessment project.

One goal of this seminar was to provide these teachers with an opportunity for extended learning by writing and refining assessments with corresponding rubrics. Under the supervision of Dr. Anne Papakonstantinou, Mr. Richard Parr, Dr. Marsha Lilly, and Ms. Holly MacLean, the group was provided further instruction in designing rubrics and developing more open-ended assessments that address *concepts* rather than isolated skills. The group met on multiple occasions throughout the fall of 2000 to create and refine this document.

Another goal was to have participants become more reflective practitioners by providing them continued support as they implemented the methodologies and activities that were presented to them in the summer course. Participants also shared with colleagues on their respective campuses which led to further professional growth within the Alief Independent School District. Curriculum permitting, participants were encouraged to field test their assessments during the time frame of the seminar.

The seminar culminated in a presentation to fellow participants, their building administrators, and representatives from Shell Westhollow Technology Center and Southwest Educational Development Laboratory. For participation in this seminar, each person was eligible to receive two hours of graduate credit in education from Rice University.

This project was sponsored by the Southwest Consortium for the Improvement for Mathematics and Science Teaching (SCIMAST).

# Table of Contents

|  |            |
|--|------------|
| <b>Assessments for Grades 5 and 6 Mathematics .....</b>      | <b>6</b>   |
| Number and Operations .....                                  | 7          |
| Patterns, Functions, and Algebra .....                       | 12         |
| Statistics.....  | 17         |
| Probability .....  | 22         |
| Measurement .....  | 27         |
| <br>   |            |
| <b>Assessments for Algebra I .....</b>                       | <b>33</b>  |
| Interpreting Graphs .....                                    | 34         |
| Solving Linear Equations.....                                | 43         |
| Graphing Linear Functions .....                              | 51         |
| Systems of Equations .....                                   | 89         |
| <br>   |            |
| <b>Assessments for High School Mathematics .....</b>         | <b>94</b>  |
| Systems of Linear Equations.....                             | 95         |
| Volume and Surface Area .....                                | 101        |
| Quadratic Equations .....                                    | 105        |
| Parametric Equations.....                                    | 113        |
| Indirect Measurement Using Trigonometric Ratios .....        | 125        |
| <br>   |            |
| <b>Annotated Bibliography of Assessment Resources.....</b>   | <b>132</b> |
| <br>   |            |
| <b>Websites with Mathematics Assessment Information.....</b> | <b>137</b> |

Alief ISD - RUSMP - SEDL

## Mathematics Assessment Project

### Assessments for Grades 5 and 6 Mathematics

Submitted by

Caryn Gurt

Jennifer Perez

Youngblood Intermediate

Cummings Elementary

# **Number and Operations**

## Number and Operations Grades 5-6 Example Assessment

Bill and Jane are planning a barbecue. They want to serve  $\frac{1}{2}$  pound of ribs to each person. They also want to have an extra  $\frac{1}{3}$  pound of ribs per person in case some people are still hungry. Jane has already bought  $7\frac{3}{4}$  pounds of ribs. Five guests are coming to the barbecue.

- Do Bill and Jane have enough ribs for each person to have  $\frac{1}{2}$  pound and still have an extra  $\frac{1}{3}$  pound per person left?
- Will they need to buy more ribs before the barbecue? If so, how much should they buy?

Show your work using pictures, diagrams, or words. Explain how you solved this problem.



## Teacher Notes

**Course/Grade Level:** 5<sup>th</sup> and 6<sup>th</sup> Grade Math (Some 4<sup>th</sup> Grade G/T students may be able to do this.)

**Objectives:** Compare 2 fractional quantities using a variety of methods, use addition to solve problems involving fractions

**Materials:** Copy of assessment (one per student or one transparency), writing paper, pencils, fraction manipulatives (optional), calculator (optional)

**Procedure:** Give each student a copy of the assessment or show a copy on the overhead. They may complete the assessment on the assessment sheet or on a separate sheet of paper.

## Answer Key

- Do Bill and Jane have enough ribs for each person to have  $\frac{1}{2}$  pound and still have an extra  $\frac{1}{3}$  pound per person left?

Student answers and work will vary depending on the number of people they counted. If they counted just the five people who were invited, they will need  $4\frac{1}{6}$  pounds of ribs. If the students included Bill and Jane as well, then they will need  $5\frac{5}{6}$  pounds of ribs. Either way,  $7\frac{3}{4}$  pounds will be enough.

- Will they need to buy more ribs before the barbecue? If so, how much should they buy?

They will not need to buy any more ribs before the barbecue.

## Number and Operations: 5<sup>th</sup> and 6<sup>th</sup> Grade Rubric

- 4 Complete understanding of the concept, idea, and processes is shown by: using math words and symbols, showing all the work including labeled diagrams and sketches, answering the questions correctly using a solution sentence. No serious flaws in reasoning.
- 3 Almost complete understanding of the concept, ideas, and processes is shown. Some math words and symbols are used. Some basic diagrams and/or sketches were shown. Answers were generally correct, but may have had a few minor errors. No serious flaws in reasoning.
- 2 Some of the concept was understood. Some math words and symbols were used. Unclear or confusing diagram or sketch. The answers contained some serious errors. Flaws in reasoning.
- 1 Only the basic concept was understood. Math words and symbols were misused or not used at all. No diagram or sketch. The answers contain major errors. Serious errors in reasoning.
- 0 Shows no understanding of the concept or didn't attempt to answer the questions. No math words or symbols were used. No diagram or sketch. No attempt was made to find the solution or a reasonable answer. The answer was a guess.

# **Patterns, Functions, and Algebra**

**Patterns, Functions, and Algebra**  
**Grades 5-6**  
**Example Assessment**

Bob's Burger Barn sells 57 Veggie Burger Specials each day.

| Day | Number of Veggie Burger Specials |
|-----|----------------------------------|
| 1   |                                  |
| 2   |                                  |
| 3   |                                  |
| 4   |                                  |
| 5   |                                  |
| 18  |                                  |
| 100 |                                  |
| n   |                                  |

- a. Complete the table.
- b. Write a rule that describes how to find the number of Veggie Burger Specials sold on the  $n$ th day.
- c. On what day did Bob's Burger Barn sell 798 Veggie Burger Specials? Explain your reasoning.

## Teacher Notes

**Course/Grade Level:** 5<sup>th</sup> and 6<sup>th</sup> Grade Math (Some 4<sup>th</sup> Grade G/T students may be able to do this.)

**Objectives:** Use lists, tables, charts, and diagrams to find patterns; generate formulas to represent relationships from a table of data

**Materials:** One copy of the assessment per student, pencils, calculator (optional)

**Procedures:** Give each student a copy of the assessment. They may complete the assessment on the assessment sheet or on a separate sheet of paper. If they use a separate sheet of paper, they will need to draw and complete the entire table.

## Answer Key

a.

| Day | Number of Veggie Burger Specials |
|-----|----------------------------------|
| 1   | 57                               |
| 2   | 114                              |
| 3   | 171                              |
| 4   | 228                              |
| 5   | 285                              |
| 18  | 1026                             |
| 100 | 5700                             |
| n   | 57n                              |

b.  $57n$

c. Day 14

Explain your reasoning: answers will vary

## **Patterns, Relations, and Algebraic Thinking: 5<sup>th</sup> and 6<sup>th</sup> Grade Rubric**

- 4 Complete understanding of the concept, idea, and processes is shown by: using math words and symbols, showing all the work including labeled diagrams and sketches, answering the questions correctly using a solution sentence. No serious flaws in reasoning.
- 3 Almost complete understanding of the concept, ideas, and processes is shown. Some math words and symbols are used. Some basic diagrams and/or sketches were shown. Answers were generally correct, but may have had a few minor errors. No serious flaws in reasoning.
- 2 Some of the concept was understood. Some math words and symbols were used. Unclear or confusing diagram or sketch. The answers contained some serious errors. Flaws in reasoning.
- 1 Only the basic concept was understood. Math words and symbols were misused or not used at all. No diagram or sketch. The answers contain major errors. Serious errors in reasoning.
- 0 Shows no understanding of the concept or didn't attempt to answer the questions. No math words or symbols were used. No diagram or sketch. No attempt was made to find the solution or a reasonable answer. The answer was a guess.



# Statistics

**Statistics**  
**Grades 5-6**  
**Example Assessment**

Use one fun size bag of Plain M&M's to complete the following:

- a. Draw a frequency chart to show the different colors of M&M's in your bag.
- b. Construct a bar graph using appropriate scale and labeling to represent the frequency chart made in *a*.
- c. Explain the shape of the data in your bar graph.
- d. Find the median, mode, range, and mean of the data collected in your FREQUENCY CHART. Which measure best describes your bag of M&M's? Why?

## Teacher Notes

**Course/Grade Level:** 5<sup>th</sup> and 6<sup>th</sup> Grade Math

**Objectives:** Reading and interpreting graphs, displaying data, and describing data.

**Materials:** 1 Fun Size bag of M&M's, pencil, paper, colored pencils-optional. (You may choose to use a pre-determined amount of colored tiles or chips instead of the M&M's)

**Procedures:** Give each student 1 Fun Size bag of M&M's (colored tiles or chips) to complete the assessment. They may complete the assessment on the assessment sheet or on a separate sheet of paper.

## Answer Key

Use one fun size bag of Plain M&M's to complete the following:

- a. Draw a frequency chart to show the different colors of M&M's in your bag.
- b. Construct a bar graph using appropriate scale and labeling to represent the frequency chart made in *a*.
- c. Explain the shape of the data in your bar graph.
- d. Find the median, mode, range, and mean of the data collected in your FREQUENCY CHART. Which measure best describes your bag of M&M's? Why?

Answers:

- A.) Answers will vary.
- B.) Answers will vary.
- C.) Answers will vary.
- D.) Answers will vary.

\*Note: If you should choose to use colored tiles or chips, you can make the numbers come out the same for every student; therefore, a key could be made with exact answers.

## Statistics: 5<sup>th</sup> and 6<sup>th</sup> Grade Rubric

- 4 Complete understanding of the concept, idea, and processes is shown by: using math words and symbols, showing all the work including labeled diagrams and sketches, answering the questions correctly using a solution sentence. No serious flaws in reasoning.
- 3 Almost complete understanding of the concept, ideas, and processes is shown. Some math words and symbols are used. Some basic diagrams and/or sketches were shown. Answers were generally correct, but may have had a few minor errors. No serious flaws in reasoning.
- 2 Some of the concept was understood. Some math words and symbols were used. Unclear or confusing diagram or sketch. The answers contained some serious errors. Flaws in reasoning.
- 1 Only the basic concept was understood. Math words and symbols were misused or not used at all. No diagram or sketch. The answers contain major errors. Serious errors in reasoning.
- 0 Shows no understanding of the concept or didn't attempt to answer the questions. No math words or symbols were used. No diagram or sketch. No attempt was made to find the solution or a reasonable answer. The answer was a guess.

# Probability

**Probability**  
**Grades 5-6**  
**Example Assessment**

Jacob had one standard die and one penny in his hand. He threw both the coin and the die into the air and watched them both hit the ground. Assuming that both landed flat and did not balance on an edge or corner, complete the following:

- a. Draw a tree diagram to show all of the possible ways his penny and die could have landed.
- b. Find the probability of Jacob's toss landing both on tails and an odd number.
- c. Suppose this were a game between 2 players. Player 1 wins if the penny lands on tails and the die toss is a 5 or 6. Player 2 wins if the penny lands on tails and the die toss is any of the other numbers. Would this game be fair? Explain.

## Teacher Notes

**Course/Grade Level:** 5<sup>th</sup> and 6<sup>th</sup> Grade Math

**Objectives:** Construct tree diagrams, find the probability of compound events, and tell if a game is fair.

**Materials:** Pencil, paper.

**Procedures:** Students are given a copy of the assessment. They may write on the assessment itself or write on their own paper.



## Answer Key

Jacob had one standard die and one penny in his hand. He threw both the coin and the die into the air and watched them both hit the ground. Assuming that both landed flat and not balancing on an edge or corner, complete the following:

- a. Draw a tree diagram to show all of the possible ways his penny and die could have landed.
- b. Find the probability of Jacob's toss landing both on tails and an odd number.
- c. Suppose this were a game between 2 players. Player 1 wins if the penny lands on tails and the die toss is a 5 or 6. Player 2 wins if the penny lands on tails and the die toss is any of the other numbers. Would this game be fair? Explain.

Answers:

- A.) Check student's tree diagram.
- B.)  $\frac{1}{4}$
- C.) No. Answers will vary. Possible answer: The game is not fair because Player 1 will have only  $\frac{1}{6}$  of a chance of winning where Player 2 has  $\frac{1}{3}$ . Player 2 has a better chance of winning. Both players would have to have an equal chance of winning for the game to be fair.

## Probability: 5<sup>th</sup> and 6<sup>th</sup> Grade Rubric

- 4 Complete understanding of the concept, idea, and processes is shown by: using math words and symbols, showing all the work including labeled diagrams and sketches, answering the questions correctly using a solution sentence. No serious flaws in reasoning.
- 3 Almost complete understanding of the concept, ideas, and processes is shown. Some math words and symbols are used. Some basic diagrams and/or sketches were shown. Answers were generally correct, but may have had a few minor errors. No serious flaws in reasoning.
- 2 Some of the concept was understood. Some math words and symbols were used. Unclear or confusing diagram or sketch. The answers contained some serious errors. Flaws in reasoning.
- 1 Only the basic concept was understood. Math words and symbols were misused or not used at all. No diagram or sketch. The answers contain major errors. Serious errors in reasoning.
- 0 Shows no understanding of the concept or didn't attempt to answer the questions. No math words or symbols were used. No diagram or sketch. No attempt was made to find the solution or a reasonable answer. The answer was a guess.

# Measurement

**Measurement**  
**Grades 5-6**  
**Example Assessment**

Kim has a new rectangular rug for her living room. The rug has an area of 64 square feet and a perimeter of 40 feet.

- a. Sketch and label with proper units the dimensions of Kim's rug.
- b. What is the perimeter in inches? Yards?
- c. Kim decided a round living room table would go just perfectly. The table she found has a diameter of 34 cm. What would be the circumference of the table? Area?

## Teacher Notes

**Course/Grade Level:** 5<sup>th</sup> and 6<sup>th</sup> Grade Math

**Objectives:** Finding the perimeter of a geometric figure, convert units within metric and customary systems, finding area of squares, rectangles, parallelograms, and triangles, finding circumference and area of circles.

**Materials:** Pencil, paper, calculator-optional.

**Procedures:** Students are given a copy of the assessment. They may write on the assessment itself or use a separate sheet of paper.  
\*You may have the students use 3.14 for *pi* unless you specifically have been practicing using *pi* on the calculator in a keystroke sequence.

## Answer Key

Kim has a new rectangular rug for her living room. The rug has an area of 64 square feet and a perimeter of 40 feet.

- a. Sketch and label with proper units the dimensions of Kim's rug.
- b. What is the perimeter in inches? Yards?
- c. Kim decided a round living room table would go just perfectly. The table she found has a diameter of 34 cm. What would be the circumference of the table? Area?

Answers:

- a. Check student's sketch. Dimensions - 4 foot by 16 foot rectangle
- b. 480 in.; 13.3 yd.
- c. 106.76 cm; 907.46 sq. cm. (using  $\pi = 3.14$ ) \* if they use the  $\pi$  key on the calculator – 106.81 cm.; 907.92 sq. cm.

## Measurement: 5<sup>th</sup> and 6<sup>th</sup> Grade Rubric

- 4 Complete understanding of the concept, idea, and processes is shown by: using math words and symbols, showing all the work including labeled diagrams and sketches, answering the questions correctly using a solution sentence. No serious flaws in reasoning.
- 3 Almost complete understanding of the concept, ideas, and processes is shown. Some math words and symbols are used. Some basic diagrams and/or sketches were shown. Answers were generally correct, but may have had a few minor errors. No serious flaws in reasoning.
- 2 Some of the concept was understood. Some math words and symbols were used. Unclear or confusing diagram or sketch. The answers contained some serious errors. Flaws in reasoning.
- 1 Only the basic concept was understood. Math words and symbols were misused or not used at all. No diagram or sketch. The answers contain major errors. Serious errors in reasoning.
- 0 Shows no understanding of the concept or didn't attempt to answer the questions. No math words or symbols were used. No diagram or sketch. No attempt was made to find the solution or a reasonable answer. The answer was a guess.

# Bibliography

Alief Independent School District Mathematics Curriculum Guide

This web site offers examples of clarifying activities.

(<http://www.tea.state.tx.us>)

The Missouri Assessment Program Scoring Guide web site offers examples of math rubrics.

(<http://www.indep.k12.mo.us/pdc/maps/projectmap.htm>)

The EXEMPLARS web site offers examples of assessments and rubrics.

(<http://www.exemplars.com>)

This web site is a resource that can be used in conjunction with the Scott Foresman/Addison Wesley Course 1 textbook.

(<http://mathsurf.com/6/index.html>)

This is the Chicago Public Schools web site of performance assessments and rubrics.

([http://intranet.cps.k12.il.us/Assessments/Ideas\\_and\\_Rubrics\\_Bank/rubric\\_bank.html](http://intranet.cps.k12.il.us/Assessments/Ideas_and_Rubrics_Bank/rubric_bank.html))



**Alief ISD - RUSMP - SEDL**

**Mathematics Assessment Project**

**Assessments for Algebra I**

**Submitted by**

**Terrri Johnson**

**Monique Mays**

**Holub Middle School**

**Elsik Ninth Grade Center**

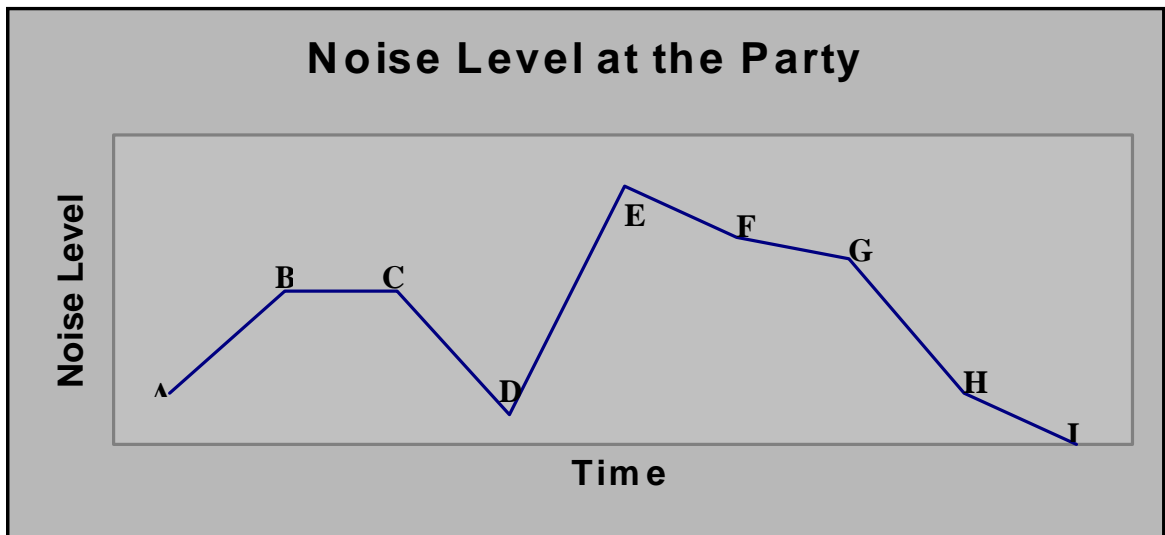
# Interpreting Graphs

# Interpreting Graphs

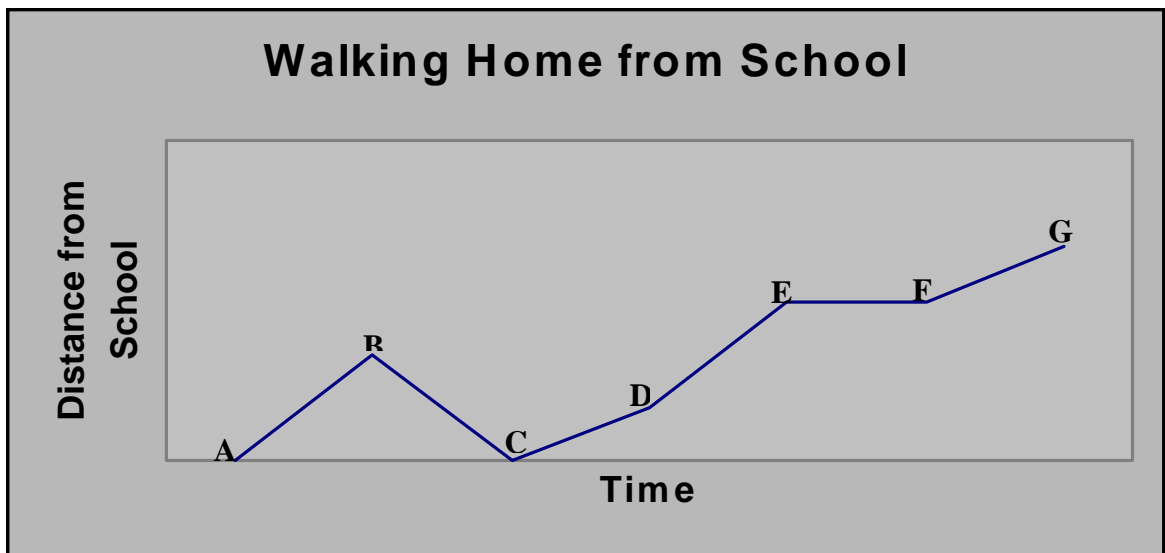
## Algebra I

### Example Assessment

Pretend you are at a party. The graph below shows noise level in the room versus time. On a separate sheet of paper, write a story that correctly reflects the information on the graph.



Pretend you are walking home from school. The graph below shows the distance from school versus time. On a separate sheet of paper, write a story that correctly reflects the information on the graph.



## Teacher Notes

**Course:** Algebra I

**Objective(s):** Interpreting Graphs

**Materials:** Interpreting Graphs Story Assessment

**Procedures:** Give each student a story assessment. Have the student create a story that properly matches the graph. There must be at least one sentence for each segment of the graph.

## Answer Key

### Sample Answers:

#### Noise Level at the Party:

I was having a party, and as my friends started arriving, the noise level started to rise. When the party reached capacity, I could not let in any more people so the noise level stayed the same. Then the police came by, so hush fell over the party. When the police left, we turned up the music and really started to party. We ran out of food, so some of the people started to leave, therefore reducing the amount of noise. As the night went on, some of my friends got tired and decided to go home. Later a fight broke out which ended the party and sent all the guests home except a few of my close friends who stayed behind to help me clean up. When we finished cleaning up, everyone went home.

#### Walking Home from School:

I was on my way home when I realized that I had left my math book in my locker. I had to go back to school and get it. I started on my way home again, this time walking a little slower because I was tired. Then a huge dog started chasing me, so I had to run. I jumped into the back of a parked truck to get away from the dog. I stayed there until the dog went away. I then finished my journey home, going a little slower than before because I was out of breath.

## Grading Scale for Interpreting Graphs

|   | 10  | 8  | 6  | 4  | 2  | 0 | Total |
|---|-----|----|----|----|----|---|-------|
| Criteria  | 5   | 4  | 3  | 2  | 1  | 0 |       |
| Accurate interpretation of the graph- Graph 1 ( x 3 ) |     |    |    |    |    |   |       |
| Accurate interpretation of the graph- Graph 2 ( x 3 ) |     |    |    |    |    |   |       |
| Spelling/Grammar/Structure- Graph 1 ( x 1 )           |     |    |    |    |    |   |       |
| Spelling/Grammar/Structure- Graph 2 ( x 1 )           |     |    |    |    |    |   |       |
| Sentences for each segment- Graph 1 ( x 1 )           |     |    |    |    |    |   |       |
| Sentences for each segment- Graph 2 ( x 1 )           |     |    |    |    |    |   |       |
|   | 100 | 80 | 60 | 40 | 20 | 0 |       |

**5 - Strong Understanding**  
**4 - Good Understanding**  
**3 - Basic Understanding**  
**2 - Limited Understanding**  
**1 - Minimal Understanding**  
**0 - No Understanding**

# Interpreting Graphs

## Algebra I

### Example Assessment

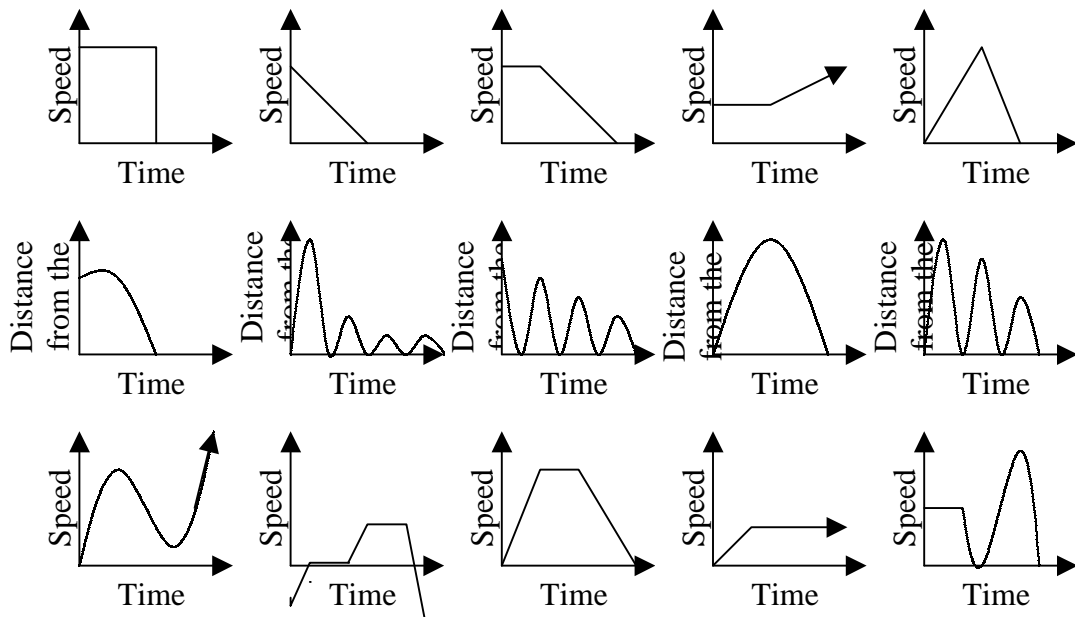
Create a game of memory for interpreting graphs where the player must match a given graph with the student created scenario. Using the graphs below, create a scenario to match each graph.

Your cards should not be any larger than 3” by 3”.

Create a design for the back of your card that says “Interpreting Graphs”

To receive extra credit, you may create your graphs as well as your own scenarios.

You will be given one week to complete this project.



## Teacher Notes

**Course:** Algebra I

**Objective(s):** Interpreting Graphs

**Materials:** Index Cards or Poster Board  
Crayons or Map Pencils  
Glue  
Scissors  
Memory Assessment

**Procedures:** Give each student a interpreting graphs assessment with graphs. The students will place the graphs on one index card and the student created matching scenario on another index card. The students will write the title "Interpreting Graphs" on the back of each index card. There should be twice as many index cards as graphs, one for the graph and one for the matching scenario.

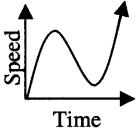
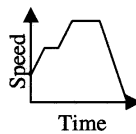
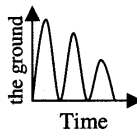
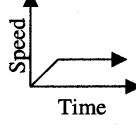
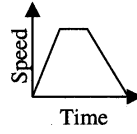
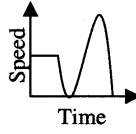
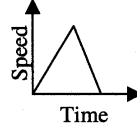
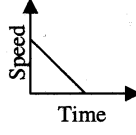
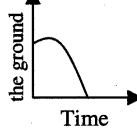
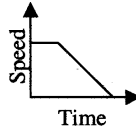
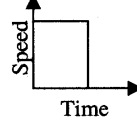
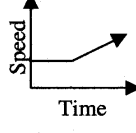
Each student should select a minimum of 6 graphs, 2 from each row. Which graphs do you think most students would select and why?  
What did students perceive as most difficult? Which, if any, did they avoid?

Extra credit grading: The student will do the required amount of work, but if he/she creates his own graphs, add extra points to his grade. For example, a student gets an 89, but created all his own graphs, add 10 points --  $89 + 10 = 99$ ; or a students gets an 89, but created half his own graphs, add 5 points --  $89 + 5 = 94$ .



**Interpreting Graphs Assessment – Answer Key**

Answers may vary, but here are some sample answers.

|   |   |   |   |
|---|---|---|---|
|    | <p>A driver increased his speed on the highway, ran into some traffic and had to slow down, then increased his speed again.</p> |    | <p>I started the race with a quick speed, then I had to steady the speed and pace myself, then I followed this pattern again. Finally, I slowed down.</p> |
|    | <p>The golfer hit the golf ball and it bounced to its spot.</p>   |    | <p>An airplane took off and then flew at a steady speed.</p>  |
|    | <p>A runner started the race at a quick steady pace, then leveled off and slowed down near the end of the race.</p>             |    | <p>A rollercoaster started at a steady speed, slowed down to climb the hill then increased coming down the hill and came to a slow stop.</p>              |
|  | <p>A car was speeding along and slammed on the brakes to stop for the red light.</p>  |  | <p>A ball was rolled and slowly came to a stop.</p>   |
|  | <p>A heavy weight was dropped from a tall building.</p>   |  | <p>A car was going at a steady pace and then slowed to a stop.</p>  |
|  | <p>I was riding in a car going a steady pace, then the car hit a wall.</p>  |  | <p>I was walking on a treadmill then I increased my speed so I could run.</p>   |

## Grading Scale for Interpreting Graphs “Memory”

|  | 10  | 8  | 6  | 4  | 2  | 0 |       |
|--|-----|----|----|----|----|---|-------|
| Criteria   | 5   | 4  | 3  | 2  | 1  | 0 | Total |
| Title on back of cards ( x 1 )   |     |    |    |    |    |   |       |
| Complete understanding of the graphs as explained by scenarios ( x 5 ) |     |    |    |    |    |   |       |
| Spelling/Grammar/Structure- ( x 1 )                                    |     |    |    |    |    |   |       |
| Required Number of Cards ( x 3 )                                       |     |    |    |    |    |   |       |
|  | 100 | 80 | 60 | 40 | 20 | 0 |       |
| <b>Extra Credit</b>  |     |    |    |    |    |   |       |
| Created own graphs (10 points)   |     |    |    |    |    |   |       |
| Created Scenarios (10 points)  |     |    |    |    |    |   |       |

|   |
|---|
| <b>5 - Strong Understanding</b><br><b>4 - Good Understanding</b><br><b>3 - Basic Understanding</b><br><b>2 - Limited Understanding</b><br><b>1 - Minimal Understanding</b><br><b>0 - No Understanding</b> |
|---|

# **Solving Linear Equations**

# Solving Linear Equations

## Algebra I

### Example Assessment

The students will form groups of two to complete the task. Each student will work a real world problem and a problem using algebra tiles on the assessing solving linear equations worksheet. Once they have completed both problems, they are to pass their work to their partner for them to check it. If the partner thinks the work is correct, they will circle the check at the right. If not, they will not circle anything, but will make a suggestion for correction in the box instead.

Use algebra tiles to solve the following problems. Be sure to draw a pictorial representation of each problem and solution.

1.  $4x - 2 = 10$
2.  $10 + 2x = 4 + x$
3.  $-2x + 6 = x - 2$
4.  $-2x - 8 = x - 2 + 6$

Real World Problems: Write an equation and solve it using the given information. Be sure to write down all the steps used in solving the problem.

1. The Student Council held a dance. The total amount made was \$400. Each student paid \$2 to get in the dance. How many students attended the dance?
2. New York is known for the stock market. You decided to buy into Alief Dough, a doughnut company. You spent \$200 buying stock. You paid \$25 per share plus a flat fee of \$50 to the stockbroker. How many shares did you buy?
3. Grant has eight less than four times the number of baseball cards as Chris. Grant has a total of 20 cards. How many cards does Chris have?
4. Mrs. Johnson works out on the stair master at the gym. There are 15 bars across the screen. Her workout is 30 minutes. How many seconds is each bar worth? (\*hint: pay attention to the units in which time is measured.)

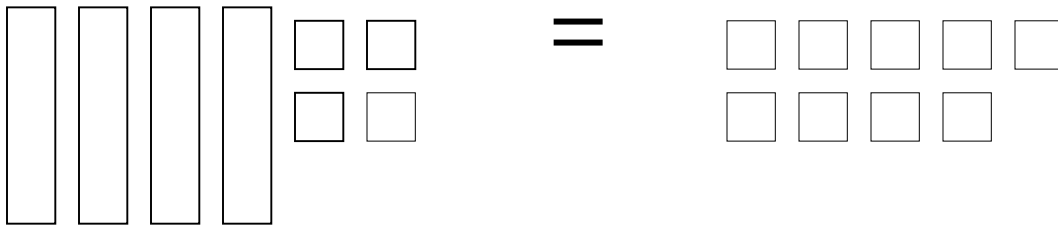
## Teacher Notes

- Course:** Algebra I
- Objective(s):** Solving Linear Equations
- Materials:** Algebra Tiles -- 1 set per student (at the least one set per pair)  
Solving Linear Equations Assessment  
Pairs Check Worksheet
- Procedures:** Group the students into pairs. Give each child one copy of the Solving Linear Equations Assessment and one copy of the Pairs Check Worksheet. Give each student (pair) a set of algebra tiles. One student will work the even numbered problems from both the algebra tiles and the real-world problem sections on the pairs check worksheet; while the other student will work the odd numbered problems from both the algebra tiles and the real-world problem sections on the pairs check worksheet. The students will switch papers and check their partners worksheet. For every problem that is correct, the student will circle the check at the right. If the problem is incorrect, the student must make a suggestion for correction in the box with the problem. The students switch papers again. If any problems were incorrect, the student must try the problem again in the rework section of the pairs check worksheet. Continue this cycle until all the problems are correct.

# Solving Linear Equations Assessment

## Challenge

Challenge the student to create a story problem given the following algebra tiles. Have them solve the equations and write the steps they used in the process of finding their solution.



## Assessing Solving Linear Equations Worksheet

| <i>Real World Problems</i> | <i>Algebra Tile Problems</i> |
|----------------------------|------------------------------|
| √                          | √                            |
| √                          | √                            |
| <b>Rework Section</b>      |                              |
| √                          | √                            |
| √                          | √                            |

### Solving Linear Equations – Answer Key

#### Algebra Tiles

1.  $4x - 2 = 10$

$X = 3$

2.  $10 + 2x = 4 + x$

$X = -6$

3.  $-2x + 6 = x - 2$

$-X = -2 \frac{2}{3}$   
 $X = 2 \frac{2}{3}$

4.  $-2x - 8 = x - 2 + 6$

$-X = 4$   
 $X = -4$



## Real World Problems- Answer Key

1.  $\frac{2x}{2} = \frac{400}{2}$

$x = 200$  students

2.  $25x + 50 = 200$   
 $\frac{-50}{25} \quad \frac{-50}{25}$   
 $\frac{25x}{25} = \frac{150}{25}$

$x = 6$  shares

3.  $4x - 8 = 20$   
 $\frac{+8}{4} \quad \frac{+8}{4}$   
 $\frac{4x}{4} = \frac{28}{4}$

$x = 7$  cards

4.  $\frac{15x}{15} = \frac{30}{15}$

$x = 2$  minutes or 120 seconds

## Grading Scale for Solving Equations

|  | 10  | 8  | 6  | 4  | 2  | 0 |       |
|--|-----|----|----|----|----|---|-------|
| Criteria   | 5   | 4  | 3  | 2  | 1  | 0 | Total |
| Comprehension of how to set up and solve the real world problems ( x 3 ) |     |    |    |    |    |   |       |
| Completely understanding of the process used to solve equations ( x 2 )  |     |    |    |    |    |   |       |
| Pictorial drawing to represent the problem ( x 2 )                       |     |    |    |    |    |   |       |
| Ability to correctly work with Algebra tiles ( x 2 )                     |     |    |    |    |    |   |       |
| Ability to check answers correctly ( x 1 )                               |     |    |    |    |    |   |       |
|  | 100 | 80 | 60 | 40 | 20 | 0 |       |
| <b>Extra Credit</b>  |     |    |    |    |    |   |       |
| Story correctly matches the title (5 points)                             |     |    |    |    |    |   |       |
| Ability to correctly solve the problem (5 points)                        |     |    |    |    |    |   |       |

|   |
|---|
| <b>5 - Strong Understanding</b><br><b>4 - Good Understanding</b><br><b>3 - Basic Understanding</b><br><b>2 - Limited Understanding</b><br><b>1 - Minimal Understanding</b><br><b>0 - No Understanding</b> |
|---|

# Graphing Linear Functions

# Graphing Linear Functions Algebra I Amusement Park Assessment

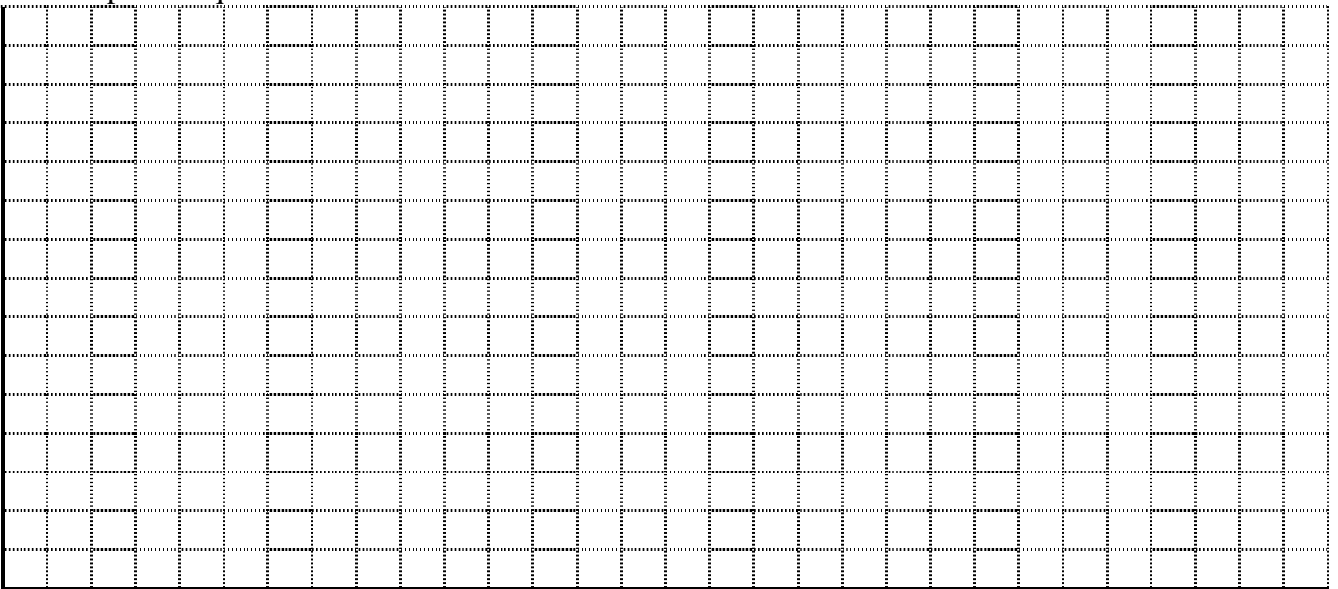
Terri wants to go to Astroworld. She lives 30 miles away from the amusement park. She will travel to the park at a rate of 1 mile per minute.

1. Complete the following table.

| Minutes traveled | Distance from the park |
|------------------|------------------------|
| 0                |                        |
| 5                |                        |
| 10               |                        |
| 15               |                        |
| 20               |                        |
| 25               |                        |
| 30               |                        |

2. Write an equation that represents this situation.

3. Graph the equation from #2.



4. What is the slope of this line?
5. What does the slope mean in this situation?
6. What is the y-intercept?
7. What does the y-intercept mean in this situation?
8. What is the x-intercept?
9. What does the x-intercept mean in this situation?
10. When will Terri reach Astroworld?
11. If Terri's family moved 60 miles from the park, how would the graph change? Write the new equation.
12. If Terri's family drives in car at a rate of 60 miles per hour, would the graph in #3 above change? If so, explain how. If not, explain why?

## Teacher Notes

- Course:** Algebra I
- Objective(s):** Graphing Linear Functions  
Determining the x and y intercepts  
Translating between different forms of equations, graphs, tables, & charts  
Slope  
Estimating from graph
- Materials:** Amusement Park Assessment
- Procedures:** Give each student a worksheet. The student will answer all the questions on the assessment.

## Amusement Park Assessment – Answer Key

Terri wants to go to Astroworld. She lives 30 miles away from the amusement park. She will travel to the park at a rate of 1 mile per minute.

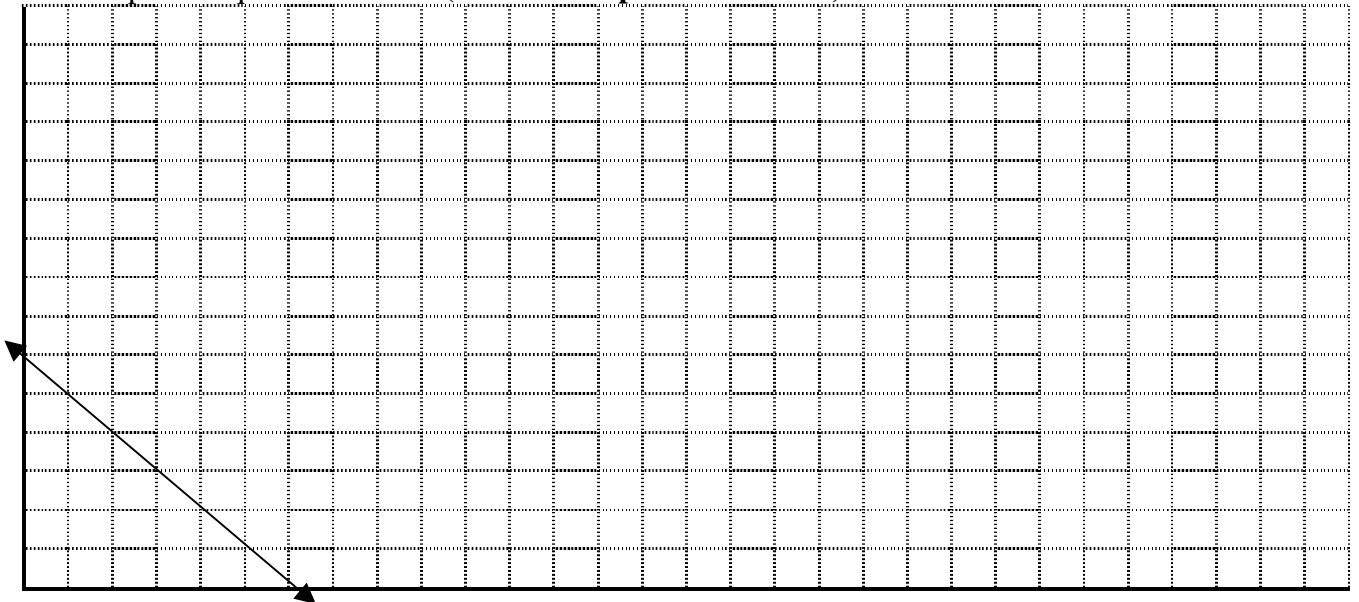
1. Complete the following table.

| Minutes traveled | Distance from the park |
|------------------|------------------------|
| 0                | 30                     |
| 5                | 25                     |
| 10               | 20                     |
| 15               | 15                     |
| 20               | 10                     |
| 25               | 5                      |
| 30               | 0                      |

2. Write an equation that represents this situation.

$$Y = 30 - X$$

3. Graph the equation from #2.( each line represents 5 units)



4. What is the slope of this line?

**-1**

5. What does the slope mean in this situation?

**It is the rate Terri is traveling.**

6. What is the y-intercept?

**(0,30)**

7. What does the y-intercept mean in this situation?

**She is 30 miles from the amusement park when she begins her trip.**

8. What is the x-intercept?

**(30,0)**

9. What does the x-intercept mean in this situation?

**She reached Astroworld in 30 minutes.**

10. When will Terri reach Astroworld?

**30 minutes**

11. If Terri's family moved 60 miles from the park, how would the graph change? Write the new equation.

**The graph will shift up 30 units.**

12. If Terri's family drives in car at a rate of 60 miles per hour, would the graph in #3 above change? If so, explain how. If not, explain why?

**No, because 60 miles per hour is equivalent to 1 mile per minute.**



## Grading Scale for Astroworld Trip

|  | 10  | 8  | 6  | 4  | 2  | 0 |       |
|--|-----|----|----|----|----|---|-------|
| Criteria   | 5   | 4  | 3  | 2  | 1  | 0 | Total |
| Knowledge of how to complete a table given a situation ( x .5 )            |     |    |    |    |    |   |       |
| Generate a written equation from the given information ( x 2 )             |     |    |    |    |    |   |       |
| Correctly graph the equation ( x 2 )                                       |     |    |    |    |    |   |       |
| Comprehension of the meaning of slope ( x .5 )                             |     |    |    |    |    |   |       |
| Ability to find the slope ( x 1 )  |     |    |    |    |    |   |       |
| Ability to find the x-intercept ( x 1 )                                    |     |    |    |    |    |   |       |
| Ability to find the y-intercept ( x 1 )                                    |     |    |    |    |    |   |       |
| Comprehension of the y-intercept ( x .5 )                                  |     |    |    |    |    |   |       |
| Ability to write a new equation given new information ( x .5 )             |     |    |    |    |    |   |       |
| Ability to describe the effect on the graph given new information ( x .5 ) |     |    |    |    |    |   |       |
| Comprehension of the x-intercept ( x .5 )                                  |     |    |    |    |    |   |       |
|  | 100 | 80 | 60 | 40 | 20 | 0 |       |

**5 - Strong Understanding**  
**4 - Good Understanding**  
**3 - Basic Understanding**  
**2 - Limited Understanding**  
**1 - Minimal Understanding**  
**0 - No Understanding**

# **Graphing Linear Equations**

## **Algebra I**

### **Graphing Lines Assessment**

Each group of two students will receive a set of clues about a line. Place all the important information from the clues into the correct place on the Big Picture worksheet.

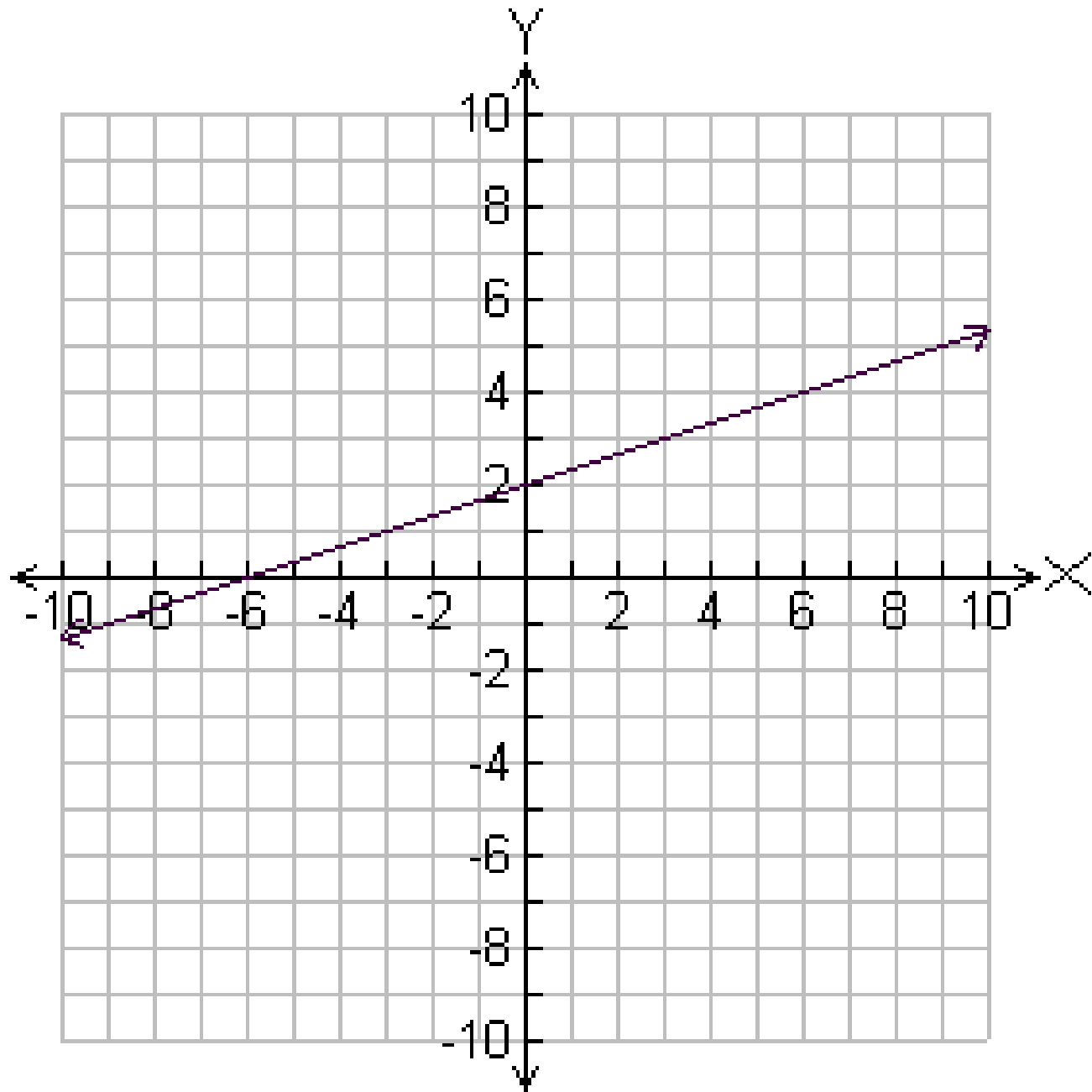
Use the clues to figure out the equation of the line and graph it.

Look around the room and write the number of the graph that matches your line.

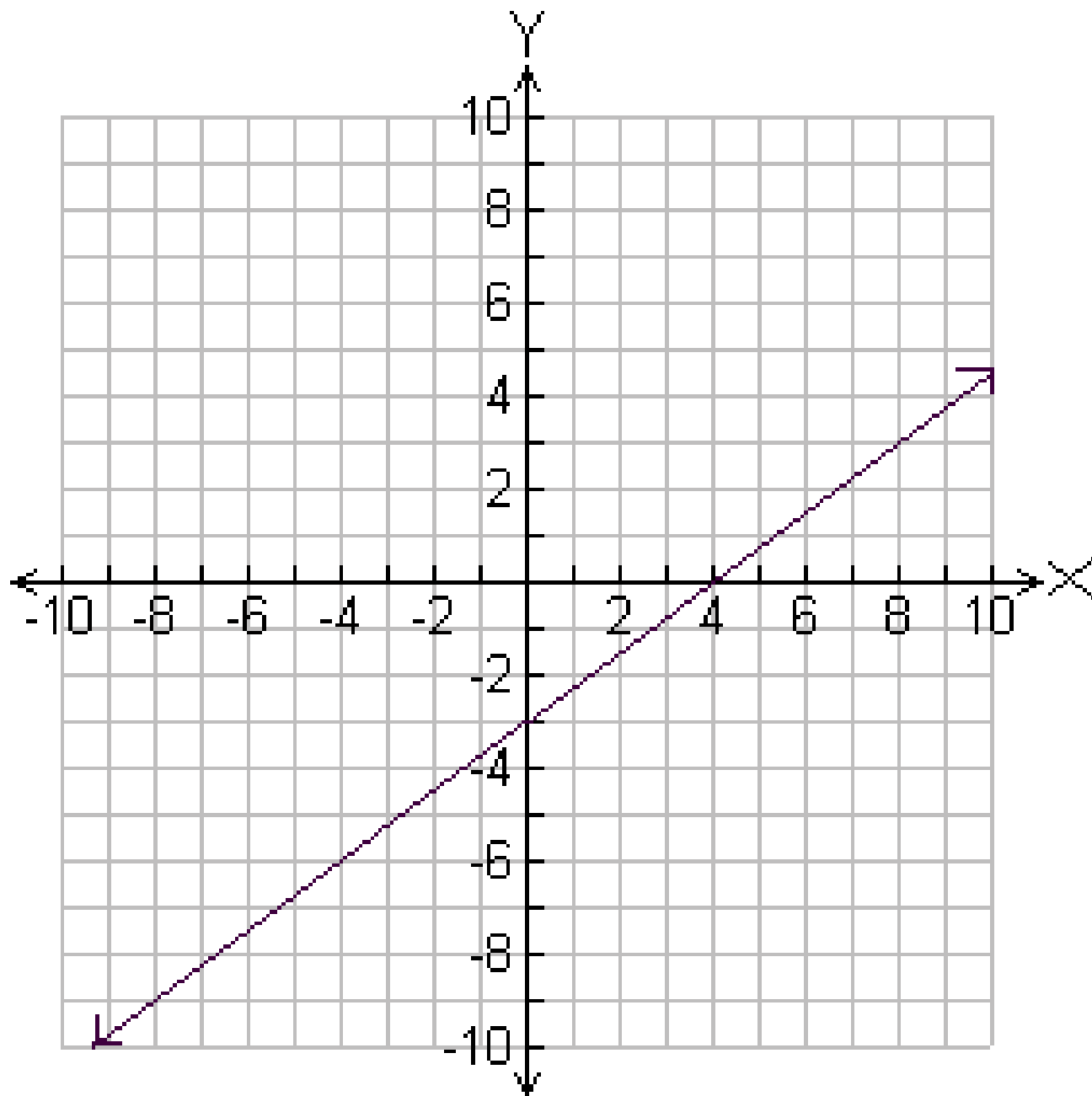
Then check another group's answer. Write on your paper if their equation is correct or incorrect. Explain why or why not.

## Teacher Notes

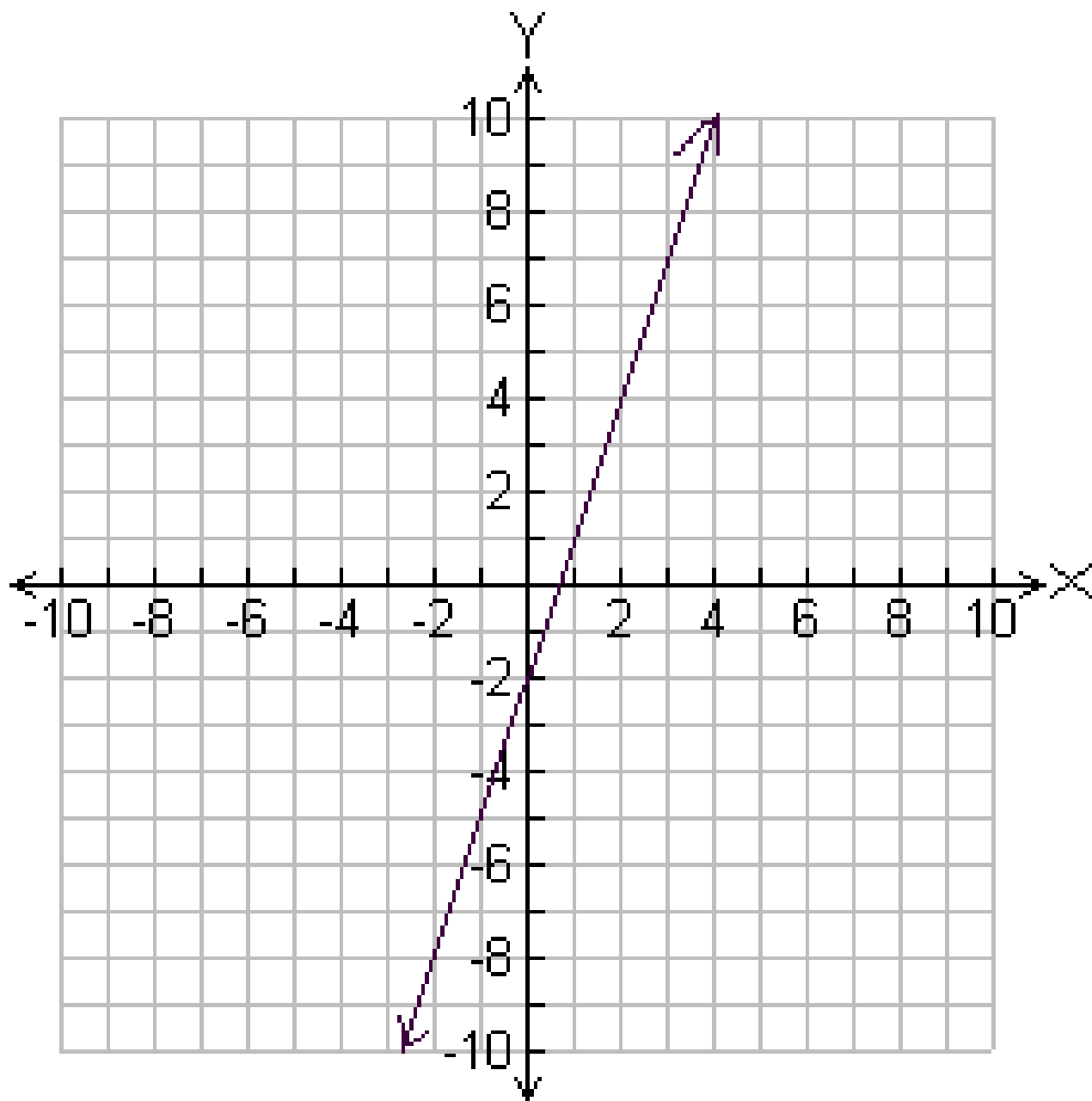
- Course:** Algebra I
- Objective(s):** Graphing Linear Functions  
Determining the x and y intercepts  
Translating between different forms of equations, graphs, tables, & charts  
Slope
- Materials:** Graphs to place around the room (included following the teacher notes)(The scale for each graph is 1. Each dot represents one unit)  
Graphing Lines Assessment Instructions  
Big Picture Assessment  
Clue Sets  
Graphing Calculator (optional)
- Procedures:** The teacher will place the 12 (included) numbered line graphs around the room. Give each pair of students one set of clues about a line, and a Big Picture Assessment Sheet. The students will use the clues to fill in as much information as possible on the Big Picture Assessment. Based on the information provided by the clues, the students will fill in the rest of the blank spaces, especially the graph and the equation of the line. Once the students have drawn the graph, they will look around the room at the graphs on the wall and match their graph to one on the wall and write that number on the Big Picture Assessment. Once everything is complete the pair will switch papers and clues with another pair and check their work. In the boxes at the bottom, they must write if the group they checked is correct or incorrect and explain why or why not.
- |                        |                         |
|------------------------|-------------------------|
| Clue 1 matches Line 2  | Clue 7 matches Line 1   |
| Clue 2 matches Line 4  | Clue 8 matches Line 3   |
| Clue 3 matches Line 6  | Clue 9 matches Line 5   |
| Clue 4 matches Line 8  | Clue 10 matches Line 7  |
| Clue 5 matches Line 10 | Clue 11 matches Line 9  |
| Clue 6 matches Line 12 | Clue 12 matches Line 11 |



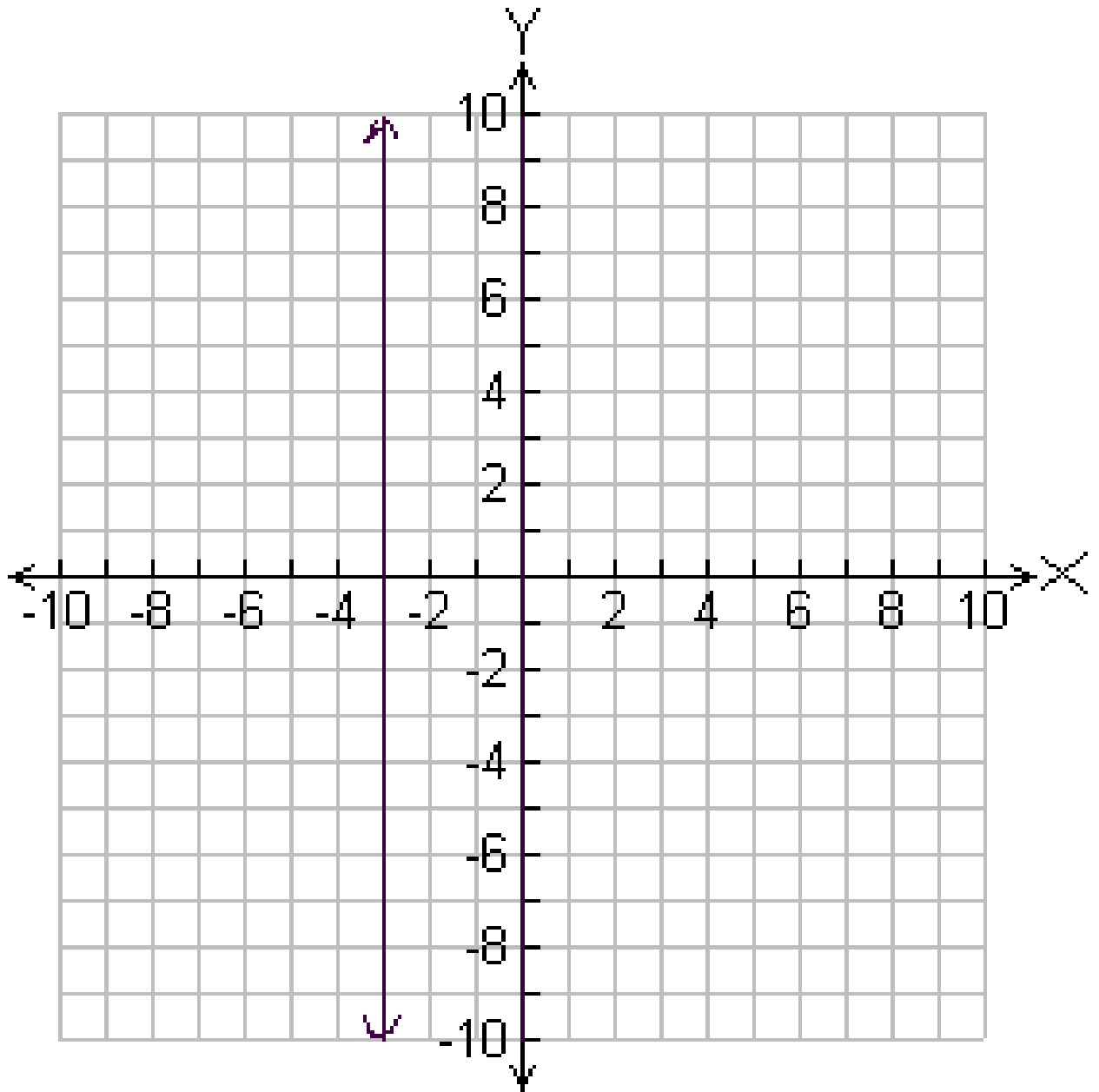
Line #1



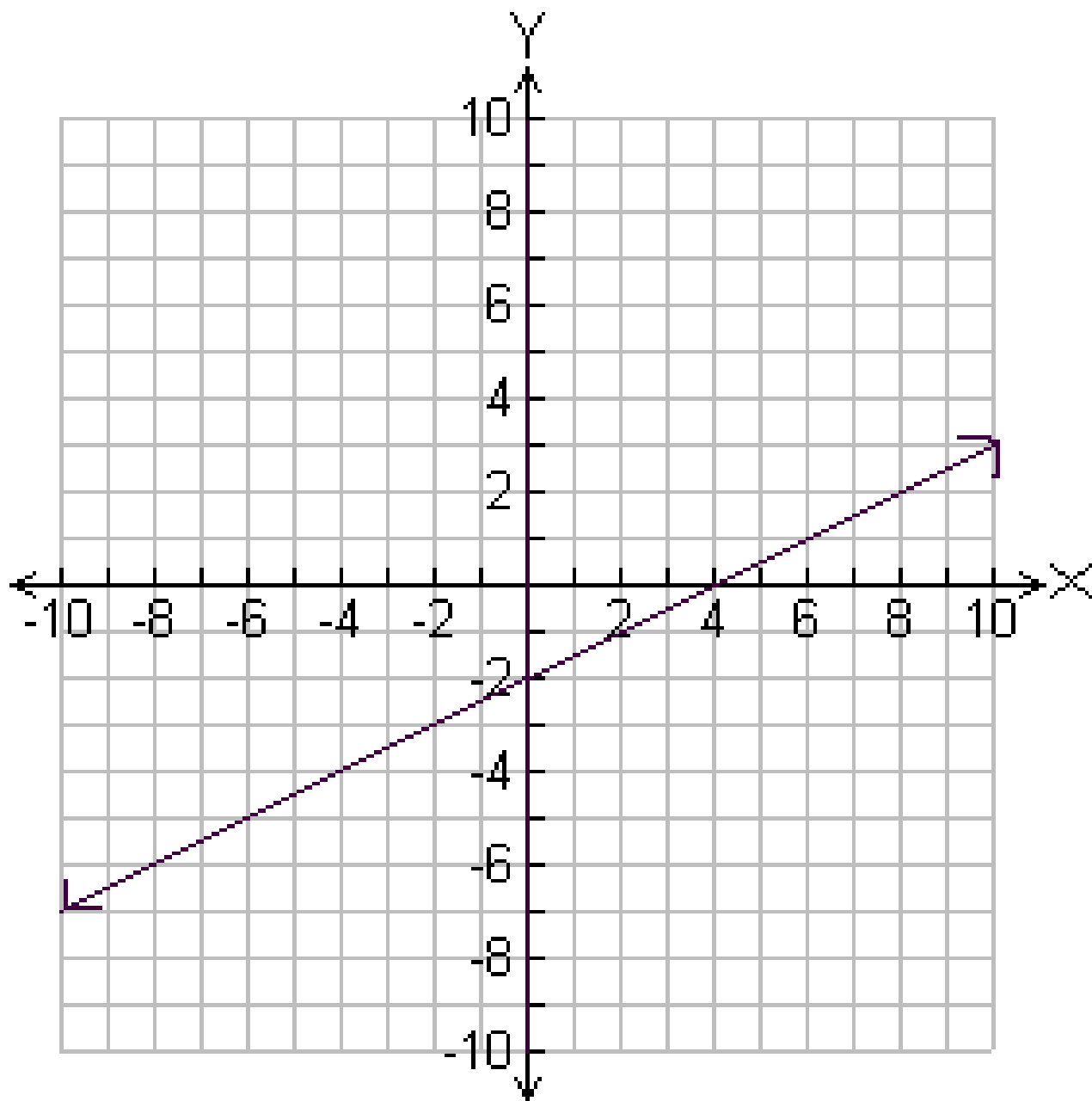
Line #2



Line #3

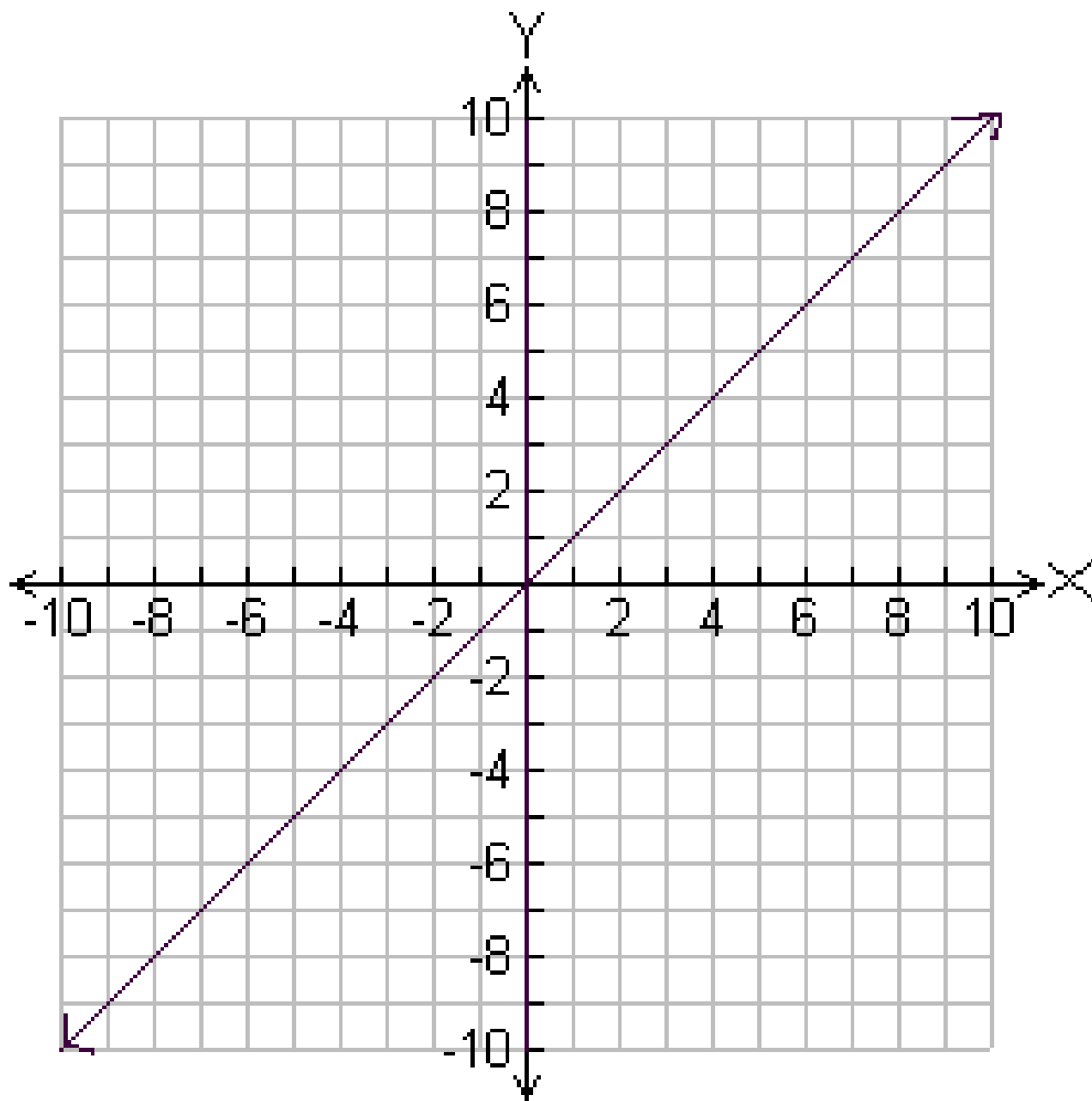


Line #4

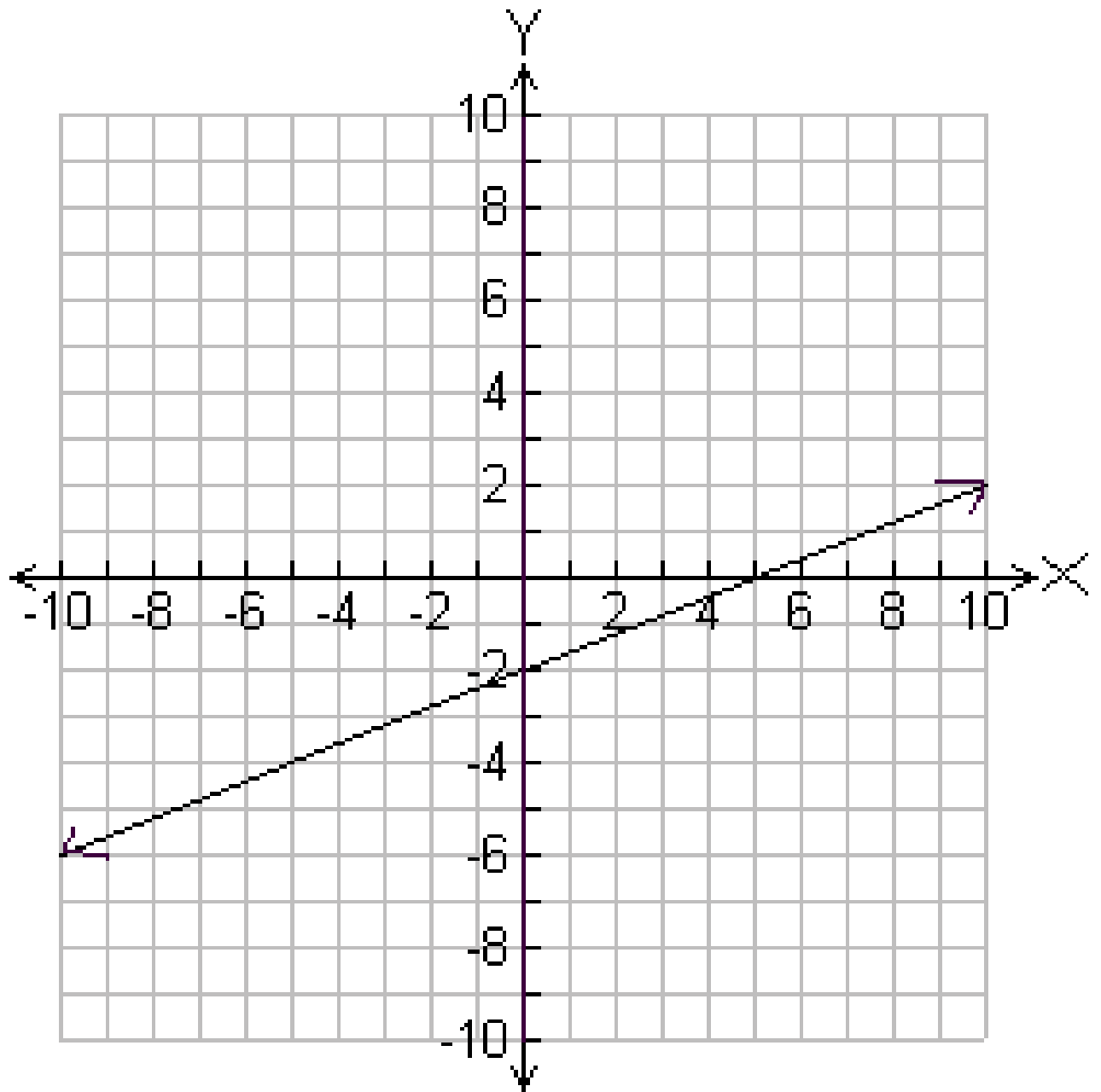


Line #5

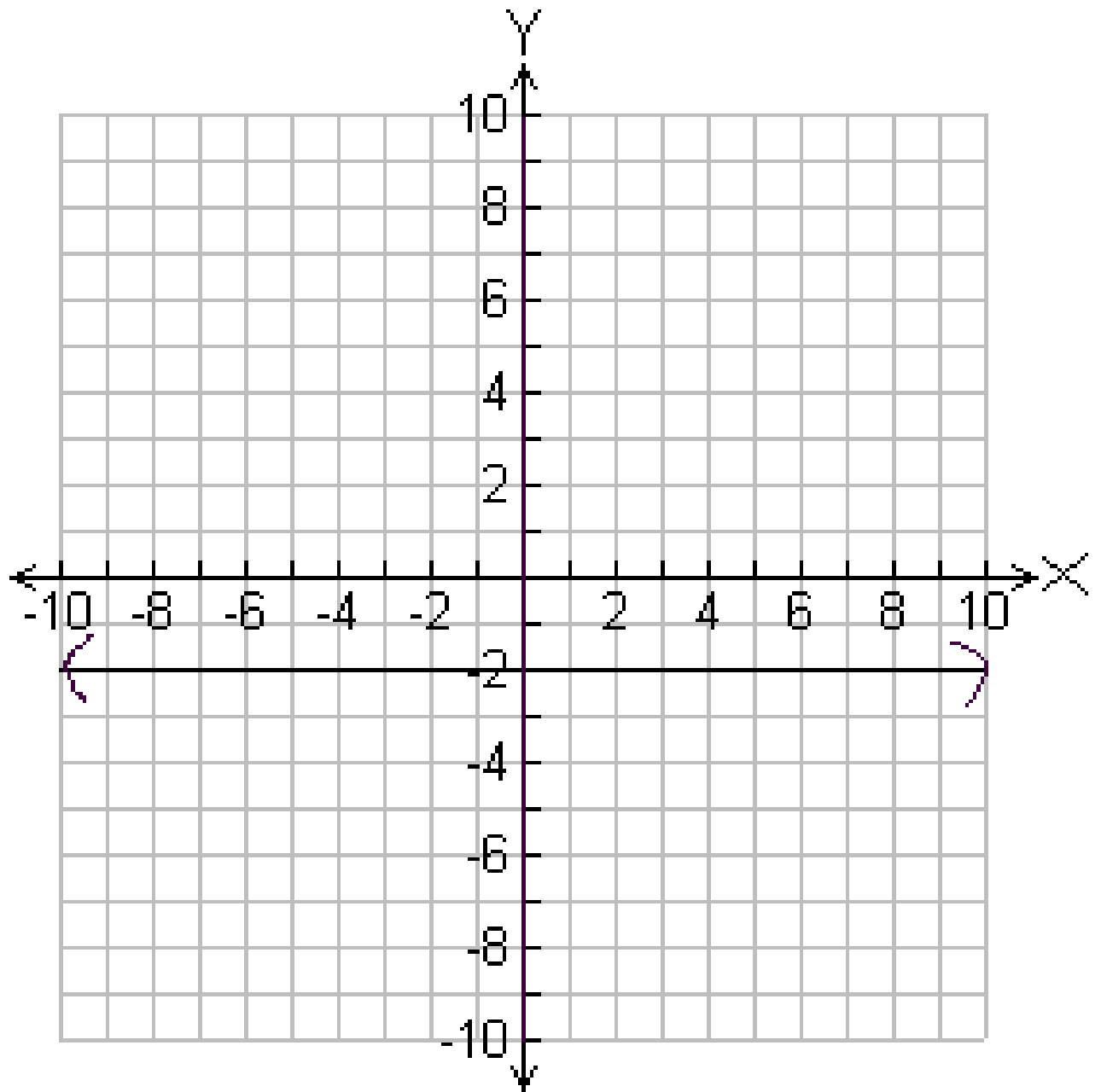




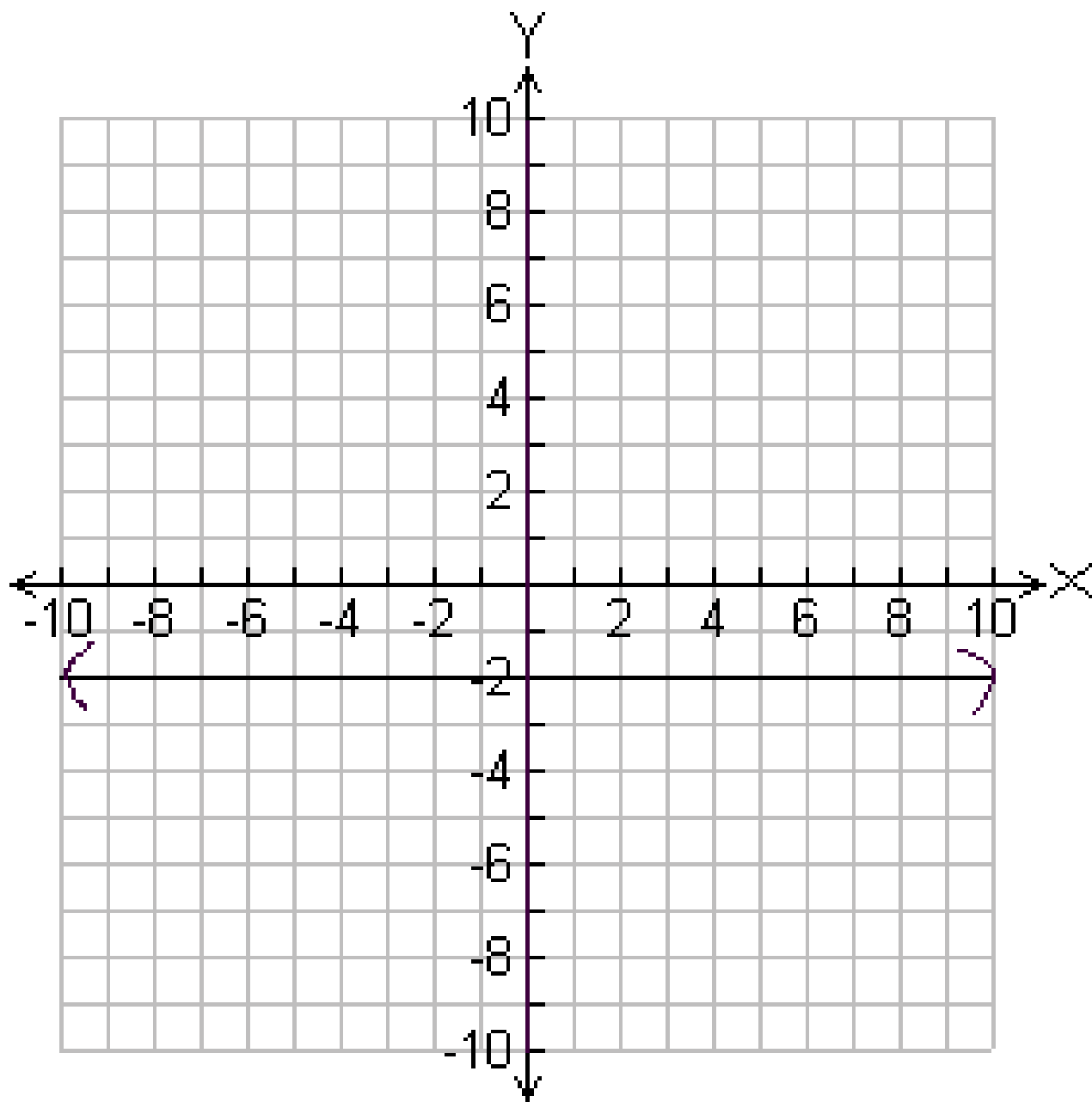
Line #6



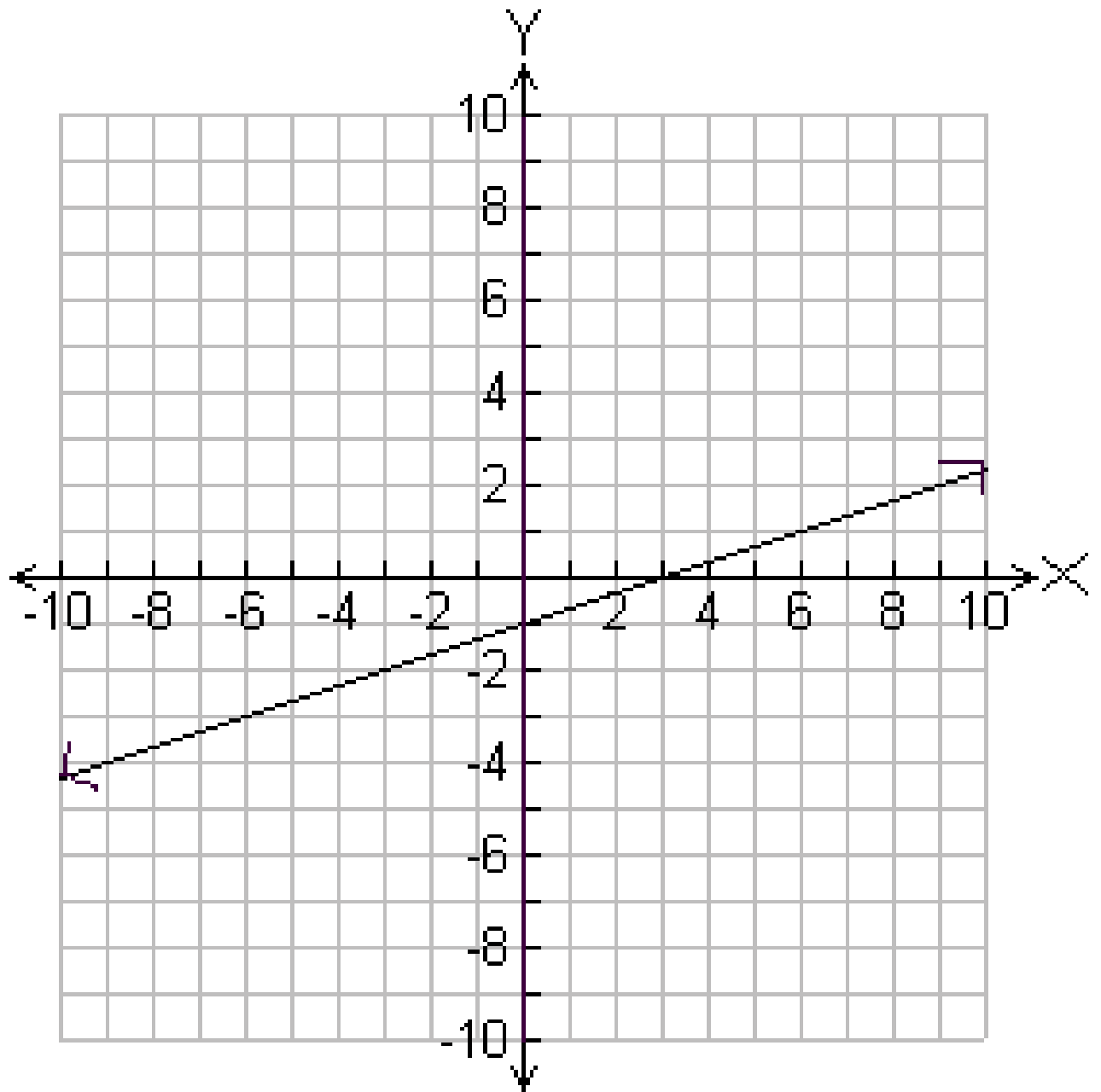
Line #7



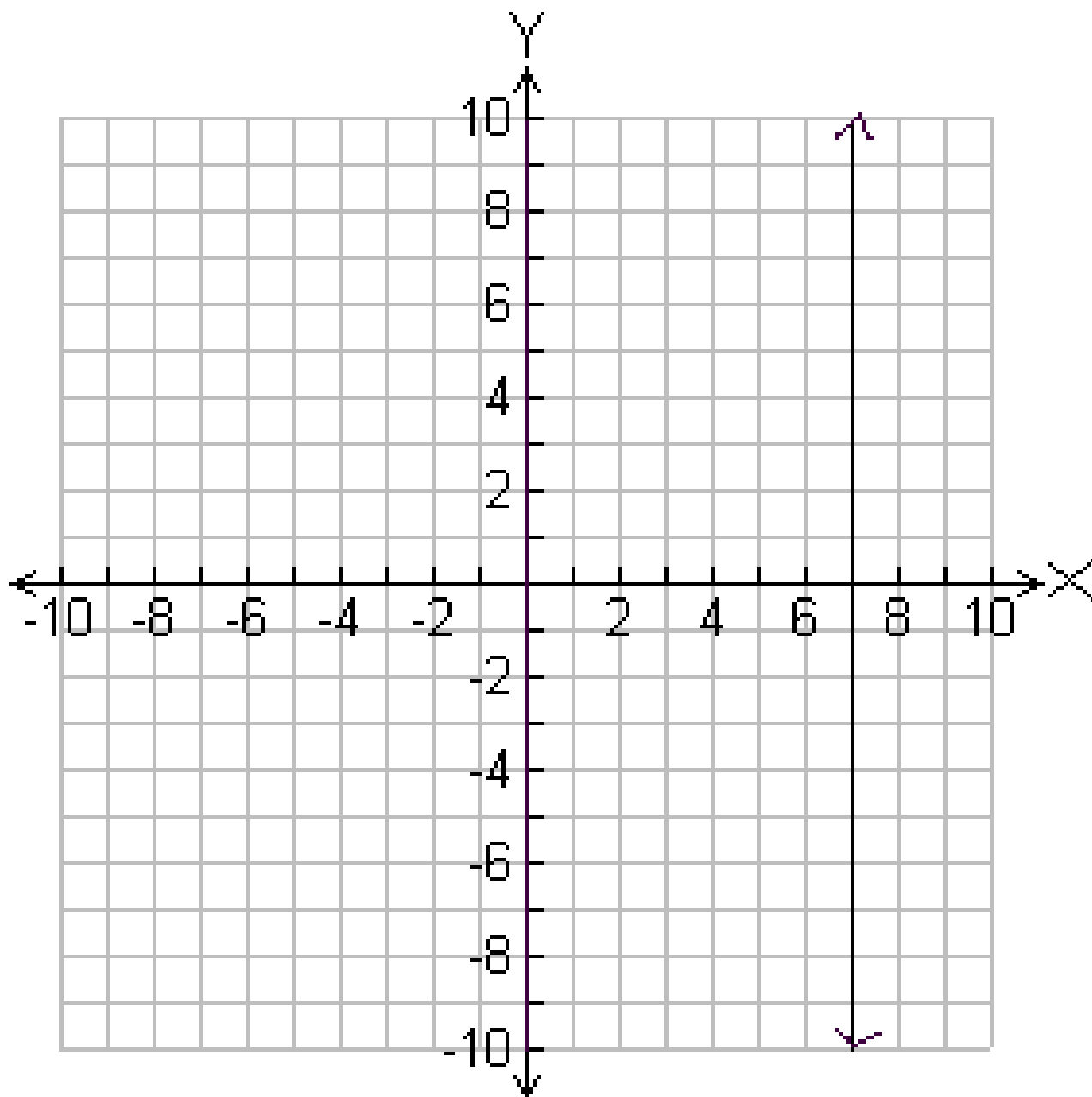
Line #8



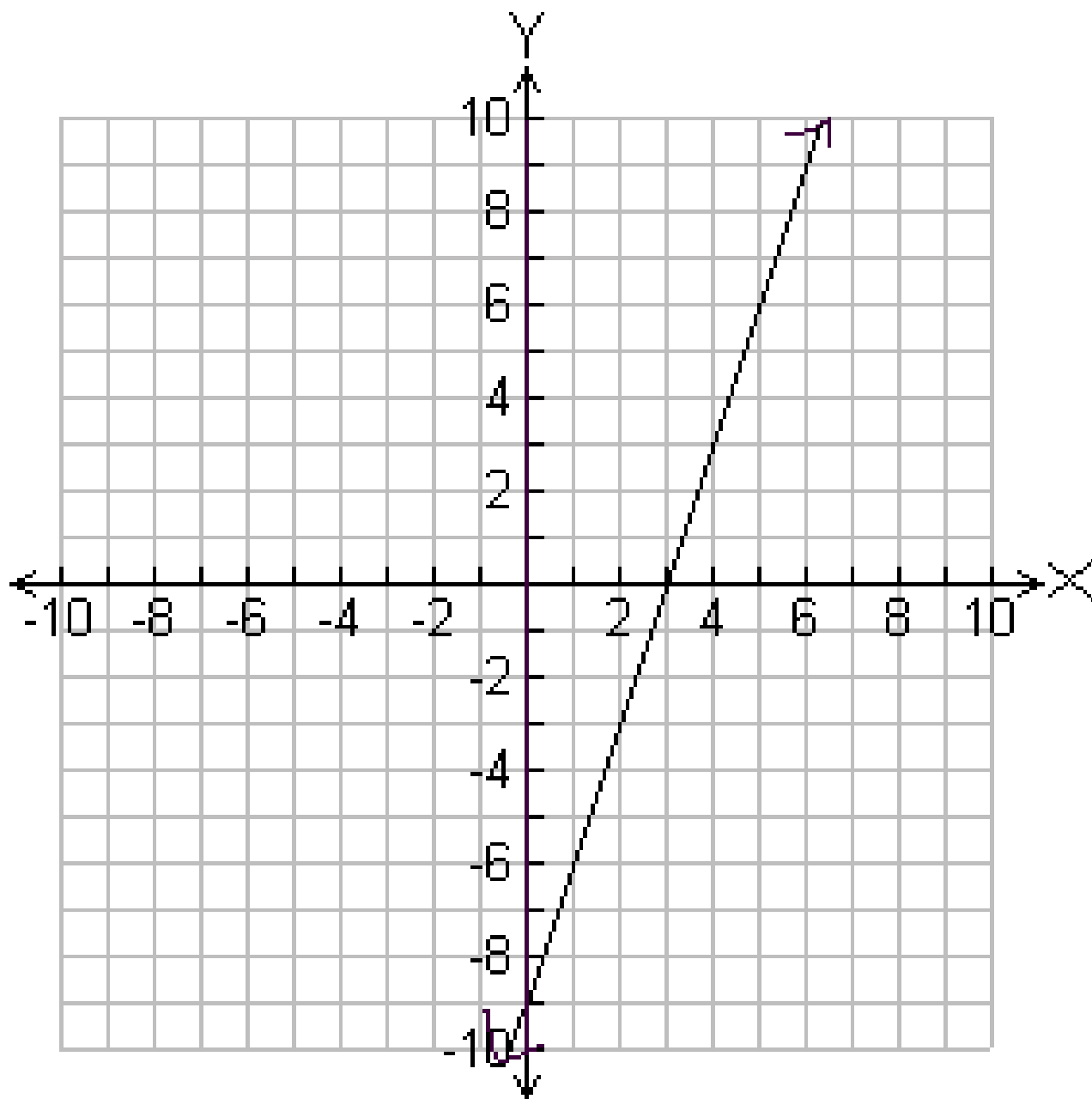
## Line #9



Line #10



Line #11



## Line #12

## Clues for graphs for “The Big Picture”

### Clue 1:

- The y-intercept is  $-3$
- The x-intercept is  $4$
- The slope is  $(3 / 4)$
- The line passes through the point  $(8, 3)$
- The line passes through the point  $(12, 6)$
- The line is parallel to the line  $y = (3 / 4) x + 7$
- The line is perpendicular to the line  $y = (-4 / 3) x + 8$

### Clue 2

- The line passes through the point  $(-3, 8)$
- The line passes through the point  $(-3, -9)$
- The line is perpendicular to the line  $y = 7$
- The line has undefined slope
- The x-intercept is  $-3$
- The line is parallel to the y-axis

### Clue 3

- The line passes through the origin
- The line has a slope of  $1$
- When  $x = 7, y = 7$
- The line passes through the point  $(-12, -12)$
- The line is perpendicular to the line  $y = -x + 6$
- The line is parallel to the line  $2x - 2y = 5$

### Clue 4

- The line is parallel to the x-axis
- The line has a zero slope
- The line passes through the point  $(4, -2)$
- The line intercepts the y-axis at  $-2$
- The line is perpendicular to the line  $x = 11$
- The line passes through the point  $(-8, -2)$



Clue 5

- The x-intercept is 3
- The y-intercept is  $-1$
- The slope is  $(1 / 3)$
- The line is perpendicular to the line  $6x + 2y = 8$
- The line is parallel to the line  $12y - 4x = 6$
- The line passes through the point  $(9,2)$
- The line passes through the point  $(-6, -3)$

Clue 6

- The x-intercept is 3
- The y-intercept is  $-9$
- The slope is 3
- The line is perpendicular to the line  $y = (-1 / 3) x + 8$
- The line is parallel to the line  $6x - 2y = 10$
- The line passes through the point  $(6,9)$
- The line passes through the point  $(-2, -15)$

Clue 7

- The y-intercept is 2
- The x-intercept is 6
- The slope is  $(1 / 3)$
- The line is perpendicular to the line  $y = 3x - 8$
- The line is parallel to the line  $-x - 3y = 17$
- The line passes through the point  $(12, -2)$
- The line passes through the point  $(-6, 4)$

Clue 8

- The y-intercept is  $-2$
- The x-intercept is  $(2 / 3)$
- The slope is 3
- The line is perpendicular to the line  $y = (-1 / 3) x + 8$
- The line is parallel to the line  $y = 3x + 15$
- The line passes through the point  $(5,13)$
- The line passes through the point  $(-6, -20)$

Clue 9

- The y-intercept is  $-2$
- The x-intercept is  $-4$
- The slope is  $(1 / 2)$
- The line is perpendicular to the line  $2x + y = 7$
- The line is parallel to the line  $-5x + 10y = 4$
- The line passes through the point  $(8, 2)$
- The line passes through the point  $(-6, -5)$

Clue 10

- The y-intercept is  $-2$
- The x-intercept is  $5$
- The slope is  $(2 / 5)$
- The line is perpendicular to the line  $y = (-5 / 2)x + 4$
- The line is parallel to the line  $2x - 5y = 31$
- The line passes through the point  $(10, 2)$
- The line passes through the point  $(-5, -4)$

Clue 11

- The y-intercept is  $1$
- The x-intercept is  $(-1 / 3)$
- The slope is  $3$
- The line is perpendicular to the line  $x + 3y = 15$
- The line is parallel to the line  $y = 3x + (4 / 5)$
- The line passes through the point  $(5, 16)$
- The line passes through the point  $(-2, -5)$

Clue 12

- The x-intercept is  $7$
- The line has an undefined slope
- The line is perpendicular to the line  $y = 6$
- The line is parallel to the y-axis
- The line passes through the point  $(7, -13)$
- The line passes through the point  $(7, (2 / 3))$

| Table |   |
|-------|---|
| X     | Y |
|       |   |
|       |   |
|       |   |

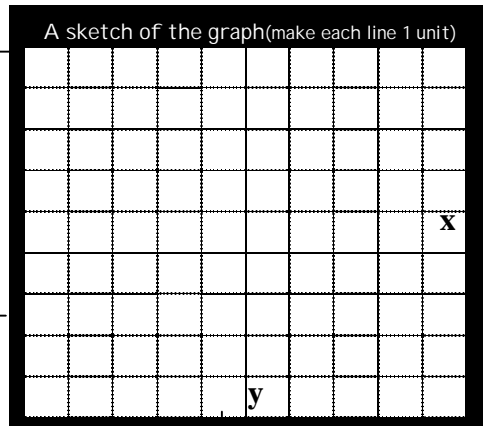
**Some Solutions are:**  
 (Coordinate notation)

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_



**No. of graph it matches in classroom**

\_\_\_\_\_

**The equation in Standard Form**

\_\_\_\_\_

**The equation in Slope-Intercept Form**

\_\_\_\_\_

**The slope of a parallel line is**

\_\_\_\_\_

**Slope**

\_\_\_\_\_

**The slope of a line perpendicular is**

\_\_\_\_\_

**The line crosses the y-axis at**

\_\_\_\_\_

**The line crosses the x-axis at**

\_\_\_\_\_

**Calculations**

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

**Clue # of the group you checked**

\_\_\_\_\_

**Correct or Incorrect**

\_\_\_\_\_

**Explain how you know correct or incorrect**

\_\_\_\_\_

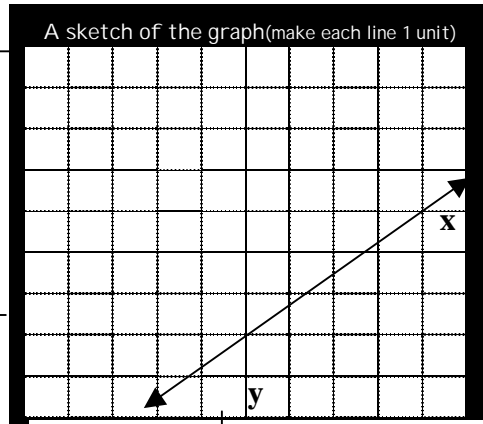
\_\_\_\_\_

\_\_\_\_\_

| Table |    |
|-------|----|
| X     | Y  |
| 8     | 3  |
| 12    | 6  |
| 0     | -3 |

Some Solutions are:  
 (Coordinate notation)

\_\_\_\_\_ (8,3) \_\_\_\_\_  
 \_\_\_\_\_ (12,6) \_\_\_\_\_  
 \_\_\_\_\_ (0,-3) \_\_\_\_\_  
 \_\_\_\_\_ (4,0) \_\_\_\_\_



No. of graph it matches in classroom

\_\_\_\_\_ 2 \_\_\_\_\_

The equation in Standard Form

\_\_\_\_\_  $-3x + 4y = -12$  \_\_\_\_\_

The equation in Slope-Intercept Form

\_\_\_\_\_  $y = (3/4)x - 3$  \_\_\_\_\_

The slope of a parallel line is

\_\_\_\_\_  $3/4$  \_\_\_\_\_

Slope

\_\_\_\_\_  $3/4$  \_\_\_\_\_

The slope of a line perpendicular is

\_\_\_\_\_  $-4/3$  \_\_\_\_\_

The line crosses the y-axis at

\_\_\_\_\_  $-3$  \_\_\_\_\_

The line crosses the x-axis at

\_\_\_\_\_  $4$  \_\_\_\_\_

Calculations

$Y = (3/4)X - 3$   
 $-(3/4)X \quad -(3/4)X$   
 $(-3/4)X + Y = -3$   
 $4 * [(-3/4)X + Y] = -3 * 4$   
 $-3X + 4Y = -12$

Clue # of the group you checked

\_\_\_\_\_

Correct or Incorrect

\_\_\_\_\_

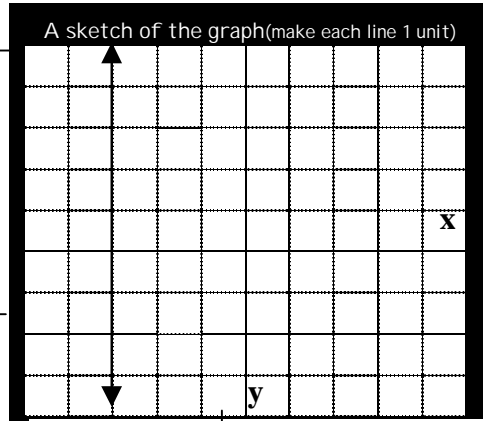
Explain how you know correct or incorrect

\_\_\_\_\_

| Table |    |
|-------|----|
| X     | Y  |
| -3    | 8  |
| -3    | -9 |
|       |    |

Some Solutions are:  
 (Coordinate notation)

\_\_\_\_\_ (-3,8) \_\_\_\_\_  
 \_\_\_\_\_ (-3,-9) \_\_\_\_\_  
 \_\_\_\_\_ (-3,0) \_\_\_\_\_  
 \_\_\_\_\_



No. of graph it matches in classroom

4

The equation in Standard Form

$X = -3$

The equation in Slope-Intercept Form

$X = -3$

The slope of a parallel line is

undefined

Slope

undefined

The slope of a line perpendicular is

0

The line crosses the y-axis at

Does not cross the y-axis

The line crosses the x-axis at

-3

Calculations

Clue # of the group you checked

Correct or Incorrect

Explain how you know correct or incorrect



| Table |    |
|-------|----|
| X     | Y  |
| 4     | -2 |
| 0     | -2 |
| -8    | -2 |

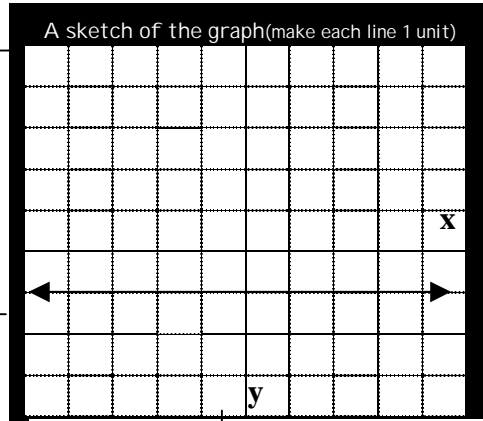
Some Solutions are:  
 (Coordinate notation)

(4,-2)

(0,-2)

(-8,-2)

\_\_\_\_\_



No. of graph it matches in classroom

8

The equation in Standard Form

$Y = -2$

The equation in Slope-Intercept Form

$Y = -2$

The slope of a parallel line is

0

Slope

0

The slope of a line perpendicular is

undefined

The line crosses the y-axis at

-2

The line crosses the x-axis at

Does not cross the x-axis

Calculations

Clue # of the group you checked

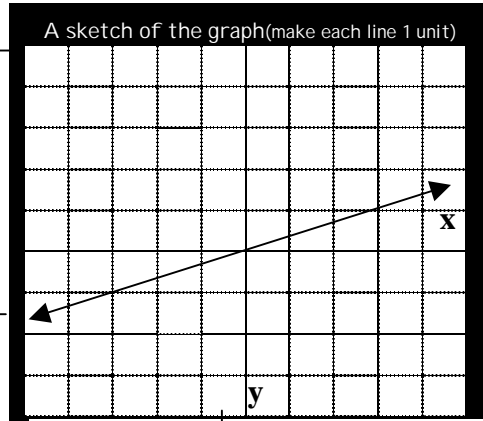
Correct or Incorrect

Explain how you know correct or incorrect

| Table |    |
|-------|----|
| X     | Y  |
| 9     | 2  |
| -6    | -3 |
| 0     | -1 |

Some Solutions are:  
 (Coordinate notation)

\_\_\_\_\_(9,2)\_\_\_\_\_  
 \_\_\_\_\_(-6,-3)\_\_\_\_\_  
 \_\_\_\_\_(0,-1)\_\_\_\_\_  
 \_\_\_\_\_(3,0)\_\_\_\_\_



No. of graph it matches in classroom  
 10

The equation in Standard Form  
 $-X + 3Y = -3$

The equation in Slope-Intercept Form  
 $Y = (1/3)X - 1$

The slope of a parallel line is  
 $1/3$

Slope  
 $1/3$

The slope of a line perpendicular is  
 $-3$

The line crosses the y-axis at  
 $-1$

The line crosses the x-axis at  
 $3$

**Calculations**

$Y = (1/3)X - 1$   
 $-(1/3) \quad -(1/3)$   
 $3 * [-(1/3)X + Y] = -1 * 3$   
 $-X + 3Y = -3$

Clue # of the group you checked

Correct or Incorrect

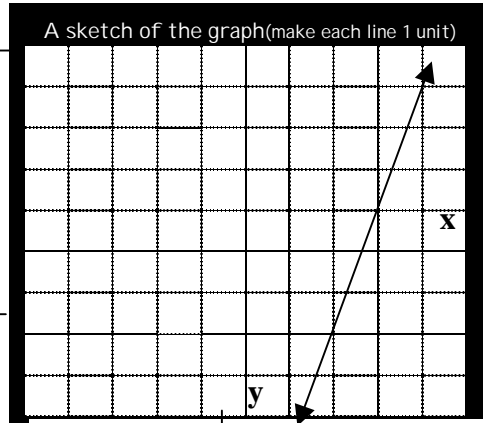
Explain how you know correct or incorrect



| Table |     |
|-------|-----|
| X     | Y   |
| 6     | 9   |
| -2    | -15 |
| 0     | -9  |

Some Solutions are:  
 (Coordinate notation)

\_\_\_\_\_  
 (6,9)  
 \_\_\_\_\_  
 (-2,-15)  
 \_\_\_\_\_  
 (0,-9)  
 \_\_\_\_\_  
 (3,0)  
 \_\_\_\_\_



No. of graph it matches in classroom  
 12

The equation in Standard Form  
 $-3X + Y = -9$

The equation in Slope-Intercept Form  
 $Y = 3X - 9$

The slope of a parallel line is  
 3

Slope  
 3

The slope of a line perpendicular is  
 $-(1/3)$

The line crosses the y-axis at  
 -9

The line crosses the x-axis at  
 3

**Calculations**

$$Y = 3X - 9$$

$$-3X \quad -3X$$

$$-3X + Y = -9$$

Clue # of the group you checked

Correct or Incorrect

Explain how you know correct or incorrect



| Table |     |
|-------|-----|
| X     | Y   |
| 5     | 13  |
| -6    | -20 |
| 0     | -2  |

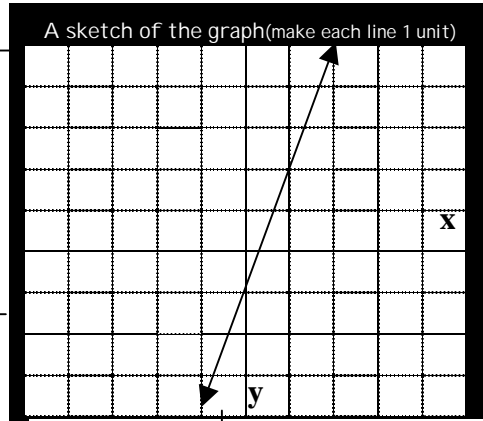
Some Solutions are:  
 (Coordinate notation)

(5,13)

(-6,-20)

(0,-2)

((2/3),0)



No. of graph it matches in classroom

3

The equation in Standard Form

$-3X + Y = -2$

The equation in Slope-Intercept Form

$Y = 3X - 2$

The slope of a parallel line is

3

Slope

3

The slope of a line perpendicular is

$-1/3$

The line crosses the y-axis at

-2

The line crosses the x-axis at

$2/3$

**Calculations**

$$\begin{array}{r}
 Y = 3X - 2 \\
 -3X \quad -3X \\
 \hline
 -3X + Y = -2
 \end{array}$$

Clue # of the group you checked

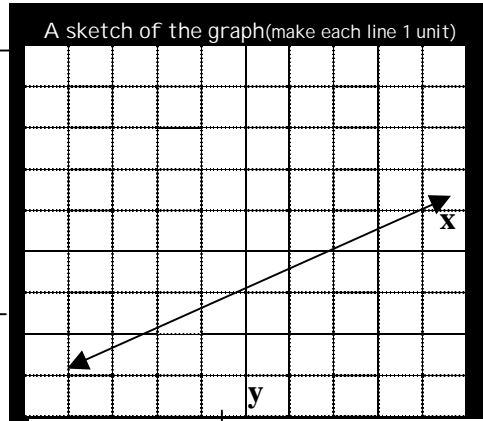
Correct or Incorrect

Explain how you know correct or incorrect

| Table |    |
|-------|----|
| X     | Y  |
| 8     | 2  |
| -6    | -5 |
| 0     | -2 |

Some Solutions are:  
 (Coordinate notation)

\_\_\_\_\_ (8,2) \_\_\_\_\_  
 \_\_\_\_\_ (-6,-5) \_\_\_\_\_  
 \_\_\_\_\_ (0,-2) \_\_\_\_\_  
 \_\_\_\_\_ (-4,0) \_\_\_\_\_



No. of graph it matches in classroom  
 $-X + 2Y = -4$

The equation in Standard Form  
 $1/2$

The equation in Slope-Intercept Form  
 $Y = (1/2)X - 2$

The slope of a parallel line is  
 $1/2$

Slope  
 $1/2$

The slope of a line perpendicular is  
 $-2$

The line crosses the y-axis at  
 $-2$

The line crosses the x-axis at  
 $-4$

**Calculations**

$Y = (1/2)X - 2$   
 $-(1/2)X \quad -(1/2)X$   
 $2 * [-(1/2)X + Y] = -2 * 2$   
 $-X + 2Y = -4$

Clue # of the group you checked

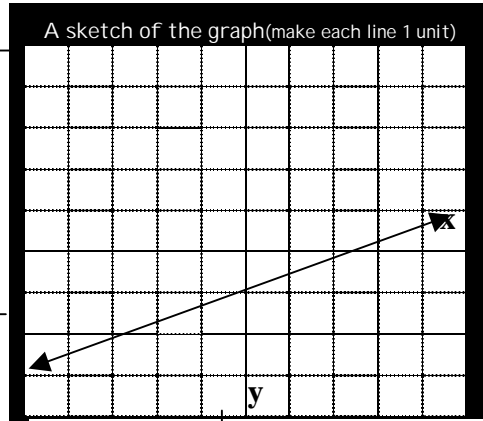
Correct or Incorrect

Explain how you know correct or incorrect

| Table |    |
|-------|----|
| X     | Y  |
| 10    | 2  |
| -5    | -4 |
| 0     | -2 |

Some Solutions are:  
 (Coordinate notation)

\_\_\_\_\_  
 (10,2)  
 \_\_\_\_\_  
 (-5,-4)  
 \_\_\_\_\_  
 (0,-2)  
 \_\_\_\_\_  
 (5,0)  
 \_\_\_\_\_



No. of graph it matches in classroom  
 7

The equation in Standard Form  
 $-2X + 5Y = -10$

The equation in Slope-Intercept Form  
 $Y = (2/5)X - 2$

The slope of a parallel line is  
 $2/5$

Slope  
 $2/5$

The slope of a line perpendicular is  
 $-5/2$

The line crosses the y-axis at  
 -2

The line crosses the x-axis at  
 5

Calculations

$$Y = (2/5)X - 2$$

$$-(2/5)X \quad -(2/5)X$$

$$5 * [-(2/5)X + Y] = -2 * 5$$

$$-2X + 5Y = -10$$

Clue # of the group you checked

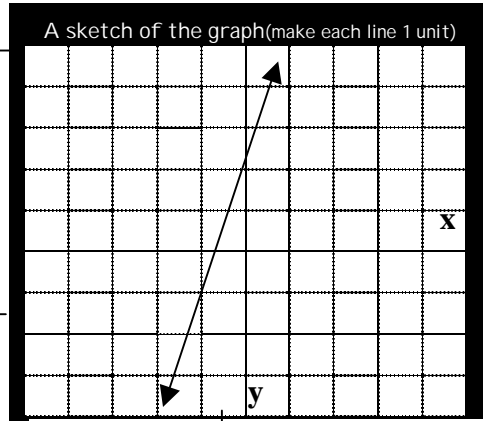
Correct or Incorrect

Explain how you know correct or incorrect

| Table |    |
|-------|----|
| X     | Y  |
| 5     | 16 |
| -2    | -5 |
| 0     | 1  |

Some Solutions are:  
 (Coordinate notation)

      (5,16)        
      (-2,-5)        
      (0,1)        
      ((-1/3),0)      



No. of graph it matches in classroom

9

The equation in Standard Form

$-3X + Y = 1$

The equation in Slope-Intercept Form

$Y = 3X + 1$

The slope of a parallel line is

3

Slope

3

The slope of a line perpendicular is

$-1/3$

The line crosses the y-axis at

1

The line crosses the x-axis at

$-1/3$

**Calculations**

$$\begin{array}{r}
 Y = 3X + 1 \\
 -3X \quad -3X \\
 \hline
 -3X + Y = 1
 \end{array}$$

Clue # of the group you checked

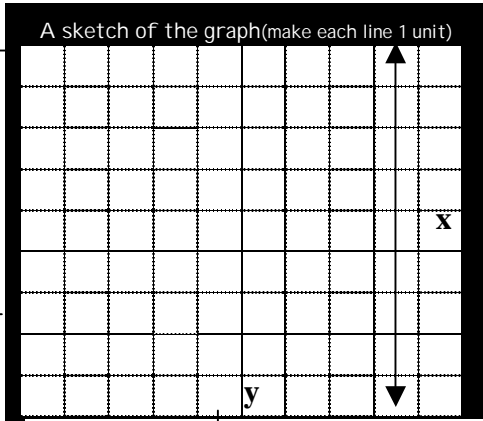
Correct or Incorrect

Explain how you know correct or incorrect

| Table |     |
|-------|-----|
| X     | Y   |
| 7     | -13 |
| 7     | 2/3 |
| 7     | 0   |

Some Solutions are:  
 (Coordinate notation)

      (7,-13)        
      (7,(2/3))     
      (7,0)         
 \_\_\_\_\_



The scale on the graph is by two

No. of graph it matches in classroom  
 11

The equation in Standard Form  
 $X = 7$

The equation in Slope-Intercept Form  
 $X = 7$

The slope of a parallel line is  
 undefined

Slope  
 undefined

The slope of a line perpendicular is  
 0

The line crosses the y-axis at  
 Does not cross the y-axis

The line crosses the x-axis at  
 7

Calculations

Clue # of the group you checked

Correct or Incorrect

Explain how you know correct or incorrect

## Grading Scale for “The Big Picture”- Graphing Linear Functions

|  | 10  | 8  | 6  | 4  | 2  | 0 |       |
|--|-----|----|----|----|----|---|-------|
| Criteria   | 5   | 4  | 3  | 2  | 1  | 0 | Total |
| Information from clues in the correct place on picture worksheet ( x .5 )    |     |    |    |    |    |   |       |
| Able to translate between the different forms of equations of a line ( x 1 ) |     |    |    |    |    |   |       |
| Ability to find the equation ( x 2 )   |     |    |    |    |    |   |       |
| Generate at least two more points on the line ( x 1 )                        |     |    |    |    |    |   |       |
| Rewrite the table’s values as coordinate points ( x .5 )                     |     |    |    |    |    |   |       |
| Ability to graph the line ( x 2 )  |     |    |    |    |    |   |       |
| Can explain why the other group is correct ( x 1 )                           |     |    |    |    |    |   |       |
| Ability to describe the effect on the graph given new information ( x 1 )    |     |    |    |    |    |   |       |
| Comprehension of the x-intercept ( x 1 )                                     |     |    |    |    |    |   |       |
|  | 100 | 80 | 60 | 40 | 20 | 0 |       |

**5 - Strong Understanding**  
**4 - Good Understanding**  
**3 - Basic Understanding**  
**2 - Limited Understanding**  
**1 - Minimal Understanding**  
**0 - No Understanding**



# **Systems of Linear Equations**

**Systems of Linear Equations**  
**Algebra I**  
**Example Assessment-“Which Cellular Phone Plan is Best for You?”**

Your parents are planning to buy you a cellular phone, but you must pay the monthly bill. You must go out and research at least two different cellular phone plans that different companies offer. Based on your research, you will decide which plan is best for you.

- a. Predict which plan you think will be best for you.
- b. Collect data by recording how much time you spend talking on the phone each night for one week.
- c. Write an equation for the plans that you researched. (If the plan, you researched has a bundle of minutes for a given price and does not charge per minute, you must divide the price by the number of minutes. Also, be sure to include the price of the phone as the start up cost.)
- d. Graph the equations from part (c) above.
- e. Using the data you collected, average the number of minutes talked each night. Now multiply the average by 30 to find how many minutes you might talk in a month. This number becomes your “monthly total”.
- f. Use the different strategies learned in solving systems of equations to figure out when the companies are charging the same price.
- g. Use your “monthly total” to decide which plan is truly best for you.
- h. State the reasons why your prediction was right or wrong.
- i. Be sure to turn in your pamphlets or any papers you received from the different phone companies, as well as, your graphs, predictions, conclusion and data collections.

**Power Point Presentation over Unit Project**

Have the students create a power point presentation about the unit project. Have them include the different strategies learned in solving a system of equations. Create a title slide, table of contents, and reference sources. Have them provide personal data, graphs, and a conclusion from their research. Finally, have them write a brief summary about how their conclusion was reached.

## Teacher Notes

**Course/Grade Level:** Algebra I

**Objective(s):** Systems of Equations

**Materials:** Which Cell Phone Plan Assessment  
Access to information on Cellular Phone Plans  
Computers(optional)  
Graphing calculators (optional)

**Procedures:** Give the students the “Which Cell Phone Plan Assessment”. Go over the instructions together. Allow the students one week to collect all data (time spent on the phone and cellular phone plans). Allow one more week to put all the information together.

**Grading Scale for “Which Cellular Phone Plan is Best for You?”**

|   | <b>10</b>  | <b>8</b>  | <b>6</b>  | <b>4</b>  | <b>2</b>  | <b>0</b> |              |
|---|------------|-----------|-----------|-----------|-----------|----------|--------------|
| <b>Criteria</b>   | <b>5</b>   | <b>4</b>  | <b>3</b>  | <b>2</b>  | <b>1</b>  | <b>0</b> | <b>Total</b> |
| <b>Prediction is clearly stated ( x 1 )</b>   |            |           |           |           |           |          |              |
| <b>Able to collect and organize data ( x 1 )</b>  |            |           |           |           |           |          |              |
| <b>Complete understandings of the strategies used to solve systems of equations ( x 3 )</b> |            |           |           |           |           |          |              |
| <b>Use of technology to illustrate the different strategies ( x 1 )</b>                     |            |           |           |           |           |          |              |
| <b>Correct monthly total ( x 1 )</b>  |            |           |           |           |           |          |              |
| <b>Reference page ( x 1 )</b>   |            |           |           |           |           |          |              |
| <b>Written conclusion is stated clearly using correct grammar and spelling ( x 1 )</b>      |            |           |           |           |           |          |              |
| <b>Research materials handed in with project ( x 1 )</b>                                    |            |           |           |           |           |          |              |
|   | <b>100</b> | <b>80</b> | <b>60</b> | <b>40</b> | <b>20</b> | <b>0</b> |              |

**5 - Strong Understanding**  
**4 - Good Understanding**  
**3 - Basic Understanding**  
**2 - Limited Understanding**  
**1 - Minimal Understanding**  
**0 - No Understanding**



Alief ISD - RUSMP - SEDL  
Mathematics Assessment Project

Assessments for High School Mathematics

Submitted by

Amy Brown  
Mary Ann Frazee  
EuRhonda Morgan  
Cindy Scott  
Heather Webb

Elsik High School  
Hastings High School  
Hastings High School  
Elsik High School  
Hastings High School

# **Systems of Linear Equations**

# Systems of Linear Equations

## High School Algebra II

### Hit or Miss Assessment

Your company has been hired by a Hollywood director to plan a near collision stunt for a movie. The director has limited funds and cannot afford to do multiple takes during filming. Your task is to plan and then stage a two-vehicle near-collision stunt where no one is injured and the vehicles do not actually crash. This would be too costly for the director and the producer of the movie if he had to purchase more vehicles or pay hospital bills for injured stunt men. The following are conditions to take into consideration when planning your stunt and documenting your stunt for posterity.

1. Assume that both roads are straight and that they intersect at right angles.
2. Describe your two vehicles and **record** any dimensions that you think will be important to the success of the stunt.
3. Use the CBR and the program RANGER in conjunction with your calculator to gather information about the rate of speed for each of your vehicles. (See teacher for instructions on use of CBR.)
4. Use the upper and lower bound coordinates to sketch a distance vs. time graph for each vehicle.
5. Write an equation for each distance vs. time (rate) graph and explain how your group decided to arrive at those equations.
6. Explain the real meaning of the slope and y-intercept for each graph.
7. **REMEMBER...**your key to this stunt is to cause some anxiety for those watching. The vehicles should pass through the intersection as closely as possible without crashing. **You must start your vehicles at least 20 inches away from the intersection.**
8. Write a paragraph explaining your decisions regarding the starting point of each vehicle from the intersection. Also, explain whether or not the cars will begin simultaneously, and why or why not.
9. You will not be allowed to practice the stunt due to cost expenditures; therefore, your mathematics in calculating the stunt must be accurate.

GOOD LUCK!



# Teacher Notes

**Course/Grade Level:** Algebra II, Pre-AP Algebra II

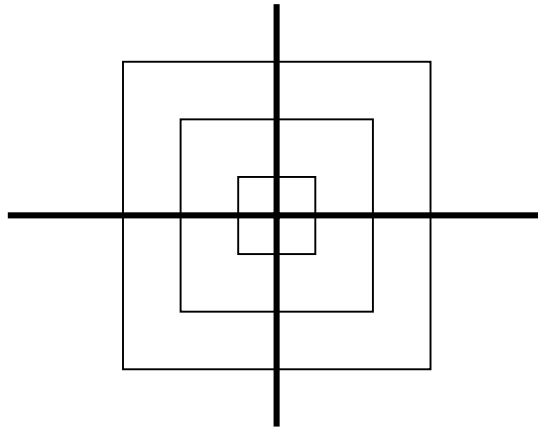
**Objective(s):** The learner will determine through collection and analysis of data, with the aid of technology, a near-miss car stunt.

**Materials:** Battery operated toy cars(enough for 2 cars/group)  
TI 82/83's  
CBRs(or CBLs)  
Tape measures  
Batteries  
Masking tape  
Graph paper  
Paper  
Pencil

**Procedures:** Prior to the student work it is advisable to demonstrate the use of the technology by conducting a sample run of a vehicle to demonstrate how the graphs will look. Explain to students the use of the upper and lower bound coordinates to achieve a more presentable graph.

When purchasing vehicles for use, opt for cars that would have continual motion when turned on and others that have a delayed start. These vehicles are usually emergency vehicles that don't move until the sirens have run. It is also worthwhile for each group of students to have its own set of batteries so the rate of speed doesn't fluctuate.

Place a target on the floor using masking tape that will simulate the intersection through which the vehicles will pass.



## Possible Answers

2. Vehicle 1: Police SUV  
Length: 5.5 in  
Width: 2.25 in  
Siren delays take off

Vehicle 2: Beetle  
Length: 5 in  
Width: 2.375 in

3. Draw the graph for each vehicle.  
The rate of change for each vehicle can best be described as the slope of each of the graphs. In each graph, distance is a function of time. As time goes by the vehicle moves farther and farther away from the CBR, therefore creating a constant rate of change.

4. Draw graph of distance versus time from the CBR, using lower bound and upper bound features, arrive at two points which represent vehicle 1 in motion.

Vehicle one, points: (3.9, .47) and (8.5, 1.35).

Use the slope formula to calculate the slope of the line:  $m = \frac{1.35 - .47}{8.5 - 3.9} = .19\text{m/s}$

Use point-slope form of the equation of a line to generate the equation of the line in Slope-intercept form:  $y = .47 + .19(x - 3.9)$

$$y = .19x - .27 \text{ ( The graph indicated a delay in the movement of police vehicle so we assumed that the vehicle was in continual motion, therefore we have a negative y-intercept.)}$$

Draw graph of distance versus time from the CBR, using lower bound and upper bound features, arrive at two points which represent vehicle 2 in motion.

Vehicle two, points: (.2, .45) (3.3, 1.48)

Use the slope formula to calculate the slope of the line:  $m = \frac{1.48 - .45}{3.3 - .2} = .3\text{m/s}$

Use point-slope form of the equation of a line to generate the equation of the line in Slope-intercept form:  $y = .4 + .3(x-.2)$

$$y = .3x + .39$$

5. The slope of each vehicle represents the speed of each vehicle. The y-intercept for the Beetle is where the vehicle began from the CBR. We assumed for the police car continual motion so using the point-slope equation resulted in this vehicle appearing to have a negative y-intercept.
  
6. We used the slope to calculate how many inches the vehicle could cover in a certain amount of time. Therefore we are starting the vehicles at the same time but at different distances from the intersection.
  - Police vehicle: starting 20 inches from the center
  - Beetle: starting 72 inches from the center.

## Analytic Rubric

| Criteria  | 4 | 3 | 2 | 1 | Total |
|---|---|---|---|---|-------|
| Graphs of linear equations with appropriate labels x 4                          |   |   |   |   |       |
| Equations for each graph determined mathematically (show all work) x 4          |   |   |   |   |       |
| Description of the real meaning of the slope and y-intercept for each graph x 4 |   |   |   |   |       |
| Presentation of the staged near-miss car stunt x 2                              |   |   |   |   |       |
| Aesthetics of entire presentation x 2   |   |   |   |   |       |
| <b>TOTALS</b>   |   |   |   |   |       |

- 4** Keen insight and understanding of criteria is communicated effectively with near-perfect detail.
- 3** Complete insight and understanding of criteria is communicated effectively with less perfect detail.
- 2** Partial insight and understanding of criteria is communicated with inaccurate detail.
- 1** Very little insight and understanding is communicated with major inaccurate detail.

| <b>TOTALS</b> | Grade Conversion | SCORE |
|---------------|------------------|-------|
| 54 - 64       | 90 – 100         | A     |
| 43 – 53       | 80 – 89          | B     |
| 36 – 42       | 75 – 79          | C     |
| 23 – 35       | 70 – 74          | D     |
| 16 – 22       | 0 – 69           | F     |

# **Volume and Surface Area**

# Volume and Surface Area Geometry Cans, Cans, Cans Assessment

All calculations are to be in centimeters.

1. You are given a can. Measure your can and calculate its volume. Include a sketch of the can with its dimensions, the formula you are using and necessary calculations.

| Calculations | Sketch |
|--------------|--------|
|              |        |
|              |        |
|              |        |
|              |        |
|              |        |

2. Now calculate how much material it took to make your can. Draw a net of the can. Show the calculation for the area of the net labeling all dimensions. Clearly show how you are doing the calculation and any formula you are using.

| Calculations | Sketch |
|--------------|--------|
|              |        |
|              |        |
|              |        |
|              |        |
|              |        |

## Teacher Notes

**Course/Grade Level:** Geometry/8-10

**Objective(s):** Find the volume of a cylinder and find the surface area of a cylinder using a net.

**Materials:** Tin Can  
Ruler (cm)

**Procedures:** Students are to work individually in class.

# Cans, Cans, and More Cans - Rubric

1)

| Criteria                                     | 4 | 3 | 2 | 1 | Total |
|--|---|---|---|---|-------|
| Sketch of tin can with accurate dimensions   |   |   |   |   |       |
| Correct formula for the volume of a cylinder |   |   |   |   |       |
| Calculations with accurate units             |   |   |   |   |       |

2)

| Criteria                                  | 4 | 3 | 2 | 1 | Total |
|---|---|---|---|---|-------|
| Sketch of net with accurate dimensions    |   |   |   |   |       |
| Correct formula for the net of a cylinder |   |   |   |   |       |
| Calculations with accurate units          |   |   |   |   |       |

4 – Accurate

3 – Minor Error

2 – Major Error

1 – Little or no attempt



# Quadratic Equations

**Quadratic Equations**  
**Algebra II**  
**Sample-Assessment-FOUL PLAY?**

**PROBLEM SITUATION:**

Mark McGwire hits a foul ball. It had an initial velocity of 100 ft/sec and an initial height of 4 ft.

Directions: Write the equation that models the height of the ball with respect to time.

Equation: \_\_\_\_\_

What method will you use to solve the equation? Why did you choose this method?

---

---

---

---

---

Solve:

What are the x-intercepts and describe the meaning of each?

---

---

---

---

---

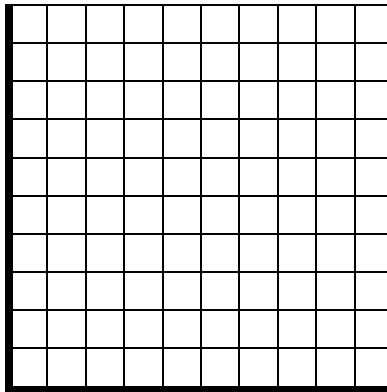
What is the y-intercept and what does it represent?

---

---

---

---



Use complete sentences to answer the following questions. Show your work.

1. How long did it take the ball to reach the ground?
2. When is the ball at its highest point? How do you know? What is the point called?
3. What is the height of the ball 1.5 sec. after it is hit? When will the ball be this height again?

# Teacher Notes

**Course/Grade Level:** Algebra II/ 10-12

**Objective(s):** To present students with a real world situation involving quadratic equations that will allow them to select and apply the appropriate method(s) to solve.

**Skills:** Evaluation of formulas, solving equations, factoring, working knowledge of methods to solve quadratic equations, critical thinking, higher order and problem solving skills

**Materials:** Graphing calculators, graph paper, handout

**Procedures:** Students will work in groups of three. Students will formulate an equation that models the problem situation, select an appropriate method and solve.\* When each student is finished, have them exchange papers, check each other's work and make the necessary corrections.

\* Teachers may use discretion in providing the formula  $h = -16t^2 + v_0t + h_0$ , where  $-16$  represents half of the acceleration due to gravity in ft/second<sup>2</sup>,  $v_0$  is the initial velocity, and  $h_0$  is the initial height.

## Answer Key- Foul Play?

### PROBLEM SITUATION:

Mark McGwire hits a foul ball. It had an initial velocity of 100 ft/sec and an initial height of 4 ft.

Directions: Write the equation that models the height of the ball with respect to time.

Equation:  $-16t^2 + 100t + 4 = 0$

What method will you use to solve the equation? Why did you choose this method?

*Possible methods for solving the quadratic are:*

- *Completing the square*
- *Graphing*
- *Using quadratic formula*

Solve:

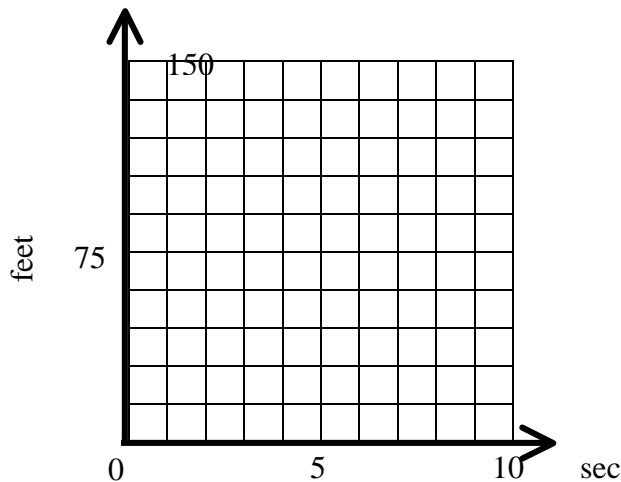
*Answers will vary.*

What are the x-intercepts and describe the meaning of each?

*The x-intercepts are 6.29 and -0.03. The 6.29 represents the amount of time, in seconds, it took for the ball to reach the ground. The -0.03 is meaningless in this problem situation since time cannot be negative.*

What is the y-intercept and what does it represent?

*The y-intercept is 4. It represents the initial height, in feet, of the ball at 0 seconds.*



Use complete sentences to answer the following questions. Show your work.

1. How long did it take the ball to reach the ground?

*The baseball will reach the ground in about 6.29 seconds.*

2. When is the ball at its highest point? How do you know? What is the point called?

*The ball is at its highest point after 3.13 seconds when it has reached a maximum height of 160.25 feet. After this point in time, the height of the ball decreases. This point is called the vertex of the graph.*

3. What is the height of the ball 1.5 sec. after it is hit? When will the ball be this height again?

*The ball is 118 feet off the ground at 1.5 seconds. It will reach this distance again after 4.75 seconds.*

## Rubric for “Foul Play” Assessment

|  | 10 | 8 | 6 | 4 | 2 | 0 | TOTAL |
|--|----|---|---|---|---|---|-------|
| Criteria                                       | 5  | 4 | 3 | 2 | 1 | 0 |       |
| Formulation of Equation<br>x 1                 |    |   |   |   |   |   |       |
| Selection of Method and Solution<br>x 1        |    |   |   |   |   |   |       |
| Identification & meaning of x-intercept<br>x 2 |    |   |   |   |   |   |       |
| Identification & meaning of y-intercept<br>x 2 |    |   |   |   |   |   |       |
| Answers to extension questions<br>x 4          |    |   |   |   |   |   |       |
| <b>TOTAL</b>                                   |    |   |   |   |   |   |       |

## RUBRIC CRITERIA

### FIVE

- Acceptable method is chosen and executed correctly with justification
- Specific relationships and appropriate connections are identified and defined
- Correct use of equation and computations that are organized and easy to follow
- Strong mathematical communication
- Correct pictorial representation

### FOUR

- Acceptable method is chosen and executed with minor errors with justification
- Most relationships and appropriate connections are identified
- Correct use of equation, however there are some computational errors but there is organization
- Some mathematical communication
- Pictorial representation

### THREE

- Acceptable method is chosen, however partially executed with justification
- Some relationships are stated but with a partial connection
- Equation is present, however there are many computation errors with little organization
- Little mathematical communication
- Pictorial representation

### TWO

- Acceptable method chosen with no justification
- Relationships and connections are present but justification to concept is weak
- Partial equation present and computational errors are significant
- No pictorial representation

### ONE

- Acceptable method chosen with no justification
- No relationships or connections are stated
- Some computation is present, but lacks accuracy and organization
- No mathematical communication
- No pictorial representation

### ZERO

Task not attempted



# Parametric Equations

**Parametric Equations**  
**Pre-Calculus**  
**Sample Assessment-Ships in the Fog**  
**Version 1**

Two ships in the fog are being monitored at ten-second intervals by Ensign Pulver. As they enter his radar screen at 6:00 a.m., the Minnow is 1000 meters due north on Ensign Pulver's radar screen. The Good Ship Lollipop is 8000 meters due east. After ten seconds, the Minnow is 40 meters east and 10 meters north of its original position. The Lollipop is 30 meters west and 20 meters north of its original position. Assume that the ships maintain their courses and speeds.

- Write parametric equations to represent the course of each ship. Use meters per minute.

Minnow

$$x_1T = \underline{\hspace{2cm}}$$

$$y_1T = \underline{\hspace{2cm}}$$

Lollipop

$$x_2T = \underline{\hspace{2cm}}$$

$$y_2T = \underline{\hspace{2cm}}$$

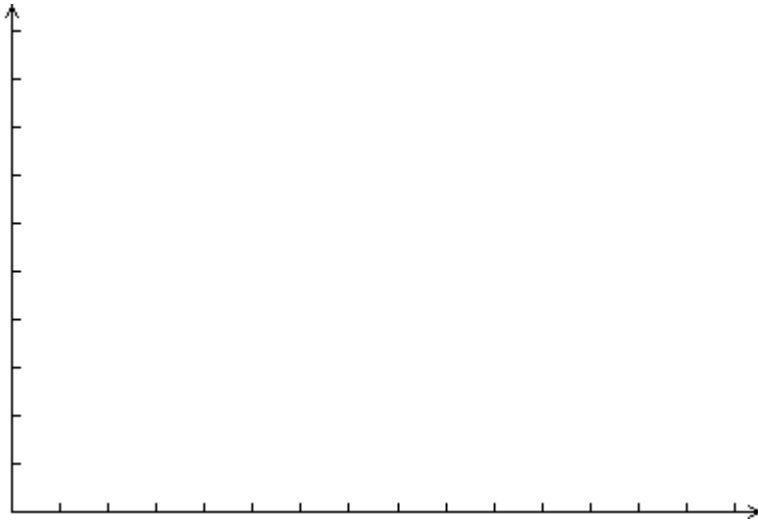
- Put your calculator in parametric and simultaneous mode and graph.
- Establish a suitable window for this problem. Record your chosen entries below.

|            |       |            |       |            |       |
|------------|-------|------------|-------|------------|-------|
| $T_{\min}$ | _____ | $X_{\min}$ | _____ | $Y_{\min}$ | _____ |
|------------|-------|------------|-------|------------|-------|

|            |       |            |       |            |       |
|------------|-------|------------|-------|------------|-------|
| $T_{\max}$ | _____ | $X_{\max}$ | _____ | $Y_{\max}$ | _____ |
|------------|-------|------------|-------|------------|-------|

|                  |       |                  |       |                  |       |
|------------------|-------|------------------|-------|------------------|-------|
| $T_{\text{scl}}$ | _____ | $X_{\text{scl}}$ | _____ | $Y_{\text{scl}}$ | _____ |
|------------------|-------|------------------|-------|------------------|-------|

4. Graph your equations. Record your graph in the grid provided below. Name and label both axes. Identify each ship clearly.



5. Use trace to locate the approximate coordinates of the intersection of the 2 courses. Record the result here. (      ,      ) Also record the time of this intersection. T

\_\_\_\_\_

6. Write another set of parametric equations to represent the distance between the 2 ships. Record the equations here and enter them into your calculator as  $x_3T$  and  $y_3T$ .

$$x_3T = \underline{\hspace{2cm}}$$

$$y_3T = \underline{\hspace{2cm}}$$

7. Turn off the equations representing the ships' courses.

8. Establish a new window to represent the distance between the 2 ships.

To do this, recall the coordinates of the approximate intersection of the 2 courses.

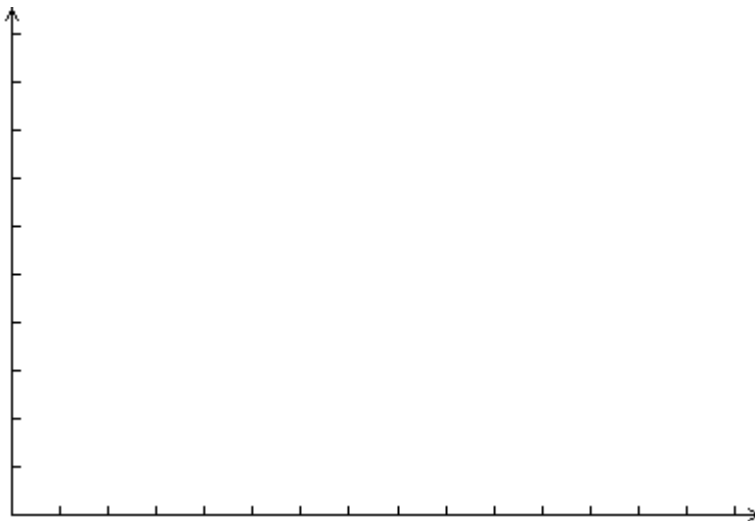
Refer to your answer from question #5. Set your time within a small interval of the time of intersection. Set your y-values to represent the approximate distance between the 2 ships. Record your entries here. Be careful. Remember the 2 ships could collide and this would be a distance of zero meters between them.

$$T_{\min} = \underline{\hspace{2cm}} \quad X_{\min} = \underline{\hspace{2cm}} \quad Y_{\min} = \underline{\hspace{2cm}}$$

$$T_{\max} = \underline{\hspace{2cm}} \quad X_{\max} = \underline{\hspace{2cm}} \quad Y_{\max} = \underline{\hspace{2cm}}$$

$$T_{\text{scl}} = \underline{\hspace{2cm}} \quad X_{\text{scl}} = \underline{\hspace{2cm}} \quad Y_{\text{scl}} = \underline{\hspace{2cm}}$$

9. Graph the distance between the 2 ships. Record your graph in the grid provided below. Name and label both axes.



10. Use trace to determine whether the 2 ships collide or if they narrowly miss. Record the closest distance between them. Write a sentence to indicate this distance and whether they collide or not.

---

---

---

11. Set up a table using your calculator with  $x_3T$  and  $y_3T$  to determine a better approximation for the closest the ships are to each other. Record your result here.

---

## Ships in the Fog- Version 2

Two ships in the fog are being monitored at ten-second intervals by Ensign Pulver. As they enter his radar screen at 6:00 a.m., the Minnow is 1000 meters due north on Ensign Pulver's radar screen. The Good Ship Lollipop is 8000 meters due east. After ten seconds, the Minnow is 40 meters east and 10 meters north of its original position. The Lollipop is 30 meters west and 20 meters north of its original position. Assume that the ships maintain their courses and speeds.

You are to advise Ensign Pulver on this potentially dangerous situation. You are to rely upon your understanding of parametric equations and the use of graphs with the aid of a graphing calculator to communicate to Ensign Pulver whether or not the two ships collide.

You are to include a graph that reflects the original positions of the ships and their paths with appropriate labels and units of measure.

You are to determine the time and the location of the collision with respect to the origin of your graph, if the ships collide.

If the ships do not collide, you are to determine the time that they are closest to each other and the distance that separates them at that time.

You are to develop a plan and execute this plan to effectively communicate your analysis of this situation with sound, convincing mathematics to Ensign Pulver.

## Teacher Notes

**Course/Grade Level:** Pre-calculus mathematics, (Honors Pre-calculus, Pre-A.P. Pre-calculus)

Two versions are included to allow teacher discretion due to students' abilities, group or individual activity, and time allotted for parametric study.

**Objectives:** The learner will determine mathematically whether or not two ships collide. The learner will implement the following concepts or skills.

- (1) Use of parametric equations to reflect time as well as the movement of ships.
- (2) Development of parametric equations to represent horizontal and vertical movement.
- (3) Development of parametric equations to represent the distance between two objects.
- (4) Appropriate use of a graphing calculator to show the paths and collision or non-collision of the two ships.
- (5) Use of a graphing calculator to show the distance between the two ships at any time.

**Materials:**

- (1) Ships in the Fog handout (Version 1 or Version 2)
- (2) Straight edge
- (3) Graphing calculator
- (4) Paper and pencil

**Procedures:** Students will work independently or within groups to complete the assessment. The assessment should take approximately 90 minutes of class time to complete.

# Ships in the Fog

(Version 1)

Two ships in the fog are being monitored at ten-second intervals by Ensign Pulver. As they enter his radar screen at 6:00 a.m., the Minnow is 1000 meters due north on Ensign Pulver's radar screen. The Good Ship Lollipop is 8000 meters due east. After ten seconds, the Minnow is 40 meters east and 10 meters north of its original position. The Lollipop is 30 meters west and 20 meters north of its original position. Assume that the ships maintain their courses and speeds.

- Write parametric equations to represent the course of each ship. Use meters per minute.

Minnow

$$x_1T = \underline{0 + 240T}$$

$$y_1T = \underline{1000 + 60T}$$

Lollipop

$$x_2T = \underline{8000 - 180T}$$

$$y_2T = \underline{0 + 120T}$$

- Put your calculator in parametric mode with the equations graphed simultaneously.
- Establish a suitable window for this problem. Record your chosen entries below.

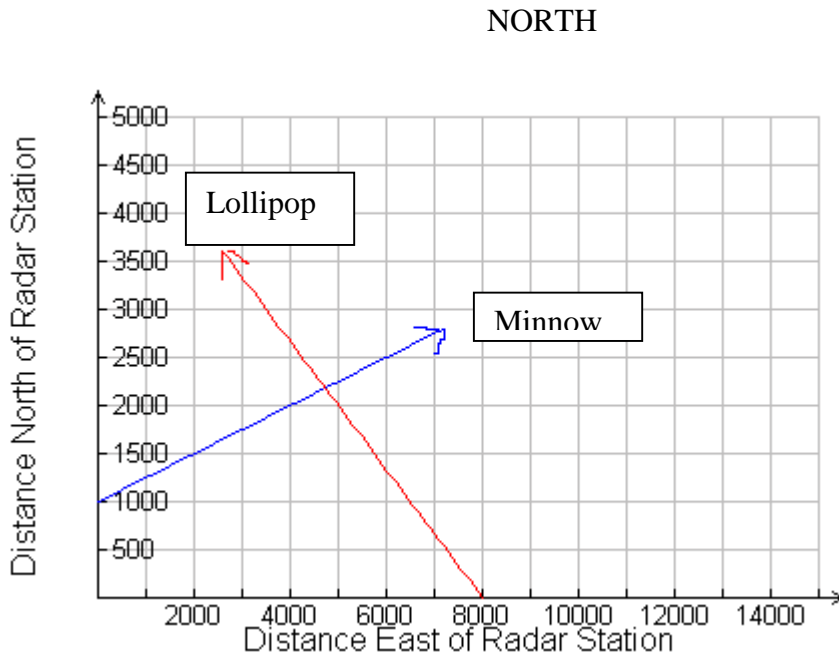
|            |          |            |          |            |          |
|------------|----------|------------|----------|------------|----------|
| $T_{\min}$ | <u>0</u> | $x_{\min}$ | <u>0</u> | $y_{\min}$ | <u>0</u> |
|------------|----------|------------|----------|------------|----------|

|            |           |            |             |            |             |
|------------|-----------|------------|-------------|------------|-------------|
| $T_{\max}$ | <u>30</u> | $x_{\max}$ | <u>9000</u> | $y_{\max}$ | <u>2500</u> |
|------------|-----------|------------|-------------|------------|-------------|

|                  |           |                  |             |                  |            |
|------------------|-----------|------------------|-------------|------------------|------------|
| $T_{\text{scl}}$ | <u>.5</u> | $x_{\text{scl}}$ | <u>1000</u> | $y_{\text{scl}}$ | <u>500</u> |
|------------------|-----------|------------------|-------------|------------------|------------|



5. Graph your equations. Record your graph in the grid provided below. Name and label both axes. Identify each ship clearly.



**Note: Origin represents the radar station**

5. Use trace to locate the approximate coordinates of the intersection of the 2 courses.  
Record the result here.  $(4680, 2170)$  Also record the time of this intersection.  $T$  19.5
6. Write another set of parametric equations to represent the distance between the 2 ships. Record the equations here and enter them into your calculator as  $x_3$  and  $y_3$ .

$$x_3 = \underline{18}$$

$$y_3 = \underline{\sqrt{((8000 - 420T)^2 + (60T - 1000)^2)}}$$

7. Turn off the equations representing the ships' courses.

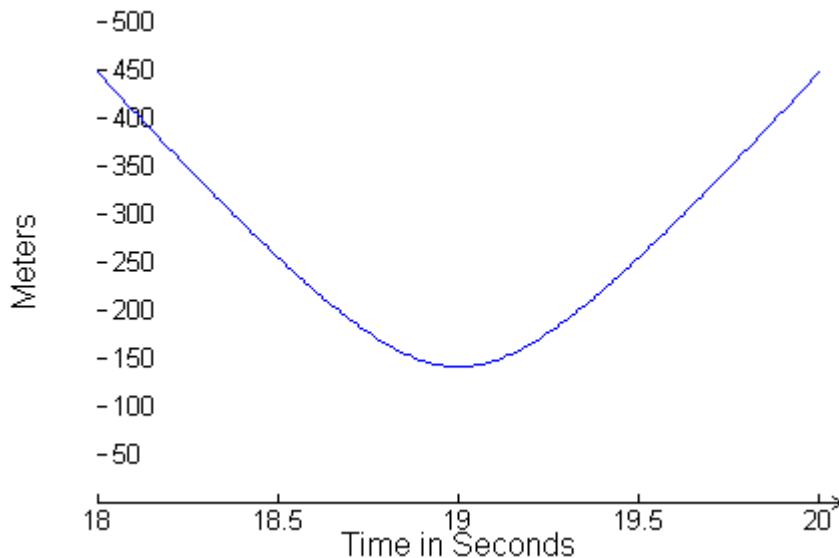
8. Establish a new window to represent the distance between the 2 ships.

To do this recall the coordinates of the approximate intersection of the 2 courses.

Refer to your answer from question #5. Set your time within a small interval of the time of intersection. Set your y-values to represent the approximate distance between the 2 ships. Record your entries here. Be careful. Remember the 2 ships could collide and this would be a distance of zero meters between them.

|                                    |                                   |                                    |
|------------------------------------|-----------------------------------|------------------------------------|
| $T_{\min} = \underline{18}$        | $x_{\min} = \underline{18}$       | $y_{\min} = \underline{0}$         |
| $T_{\max} = \underline{20}$        | $x_{\max} = \underline{20}$       | $y_{\max} = \underline{500}$       |
| $T_{\text{scl}} = \underline{.01}$ | $x_{\text{scl}} = \underline{.5}$ | $y_{\text{sscl}} = \underline{50}$ |

9. Graph the distance between the 2 ships. Record your graph in the grid provided below. Name and label both axes.



10. Use trace to determine whether the 2 ships collide or if they narrowly miss. Record the closest distance between them. Write a sentence to indicate this distance and whether they collide or not.

The ships miss each other by 141.42 meters

11. Set up a table using your calculator with  $x_3T$  and  $y_3T$  to determine a better approximation for the closest the ships are to each other. Record your result here.

(19, 141.42)

**Ships in the Fog  
(Version 2)  
Analytic Rubric**

| Criteria  | 4 | 3 | 2 | 1 | TOTAL |
|---|---|---|---|---|-------|
| Parametric equations used to represent the motion<br>x3               |   |   |   |   |       |
| Graphs of parametric equations with appropriate labels provided<br>x3 |   |   |   |   |       |
| Collision or non-collision of ships determined mathematically<br>x4   |   |   |   |   |       |
| Presentation and clarity of mathematics<br>x4                         |   |   |   |   |       |
| Aesthetics of entire presentation<br>x2                               |   |   |   |   |       |
| Totals  |   |   |   |   |       |

Keen insight and understanding of criteria is communicated effectively with near perfect detail -4-

Complete insight and understanding of criteria is communicated effectively with less perfect detail -3-

Partial insight and understanding of criteria is communicated with inaccurate detail -2-

Very little insight and understanding is communicated with major inaccurate detail -1-

| <u>TOTALS</u> | <u>Grade Conversion</u> | <u>Score</u> |
|---------------|-------------------------|--------------|
| 54-64         | 90-100                  | A            |
| 43-53         | 80-89                   | B            |
| 36-42         | 75-79                   | C            |
| 23-35         | 70-74                   | D            |
| 16-22         | 0-69                    | F            |

# Trigonometric Ratios

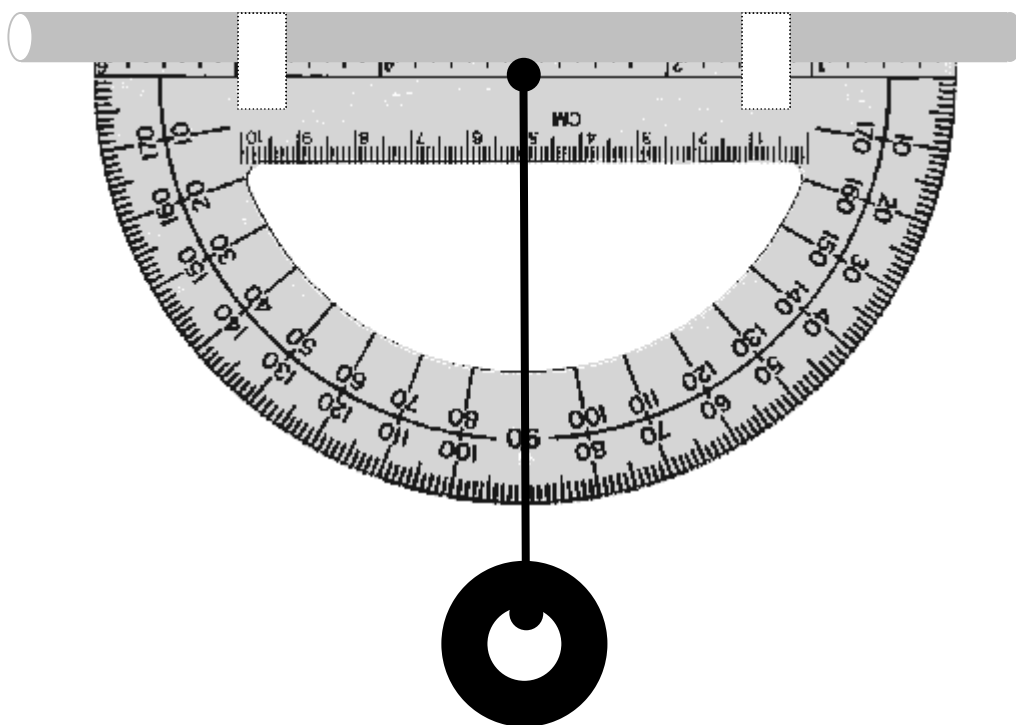
# Trigonometric Ratios

## Pre-Calculus

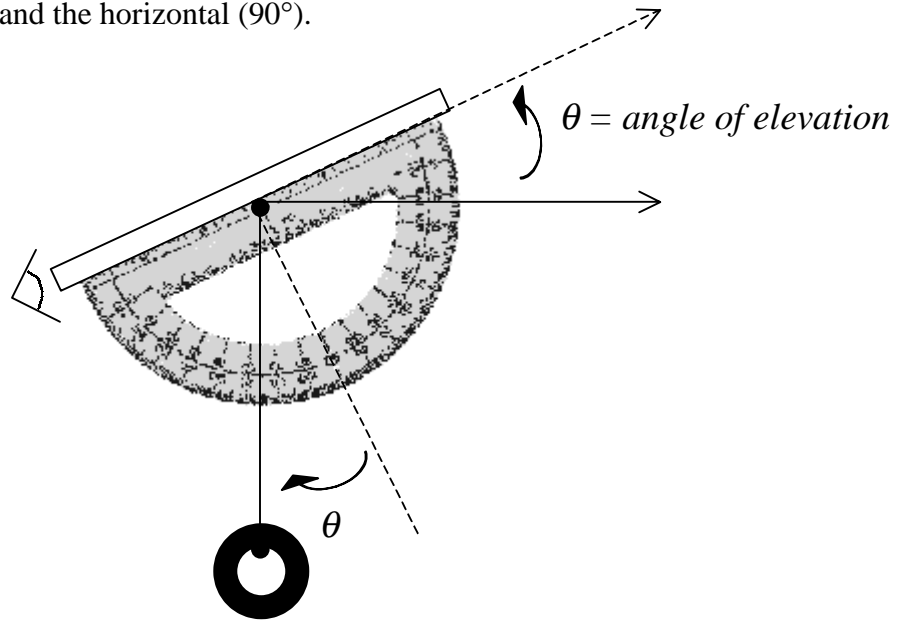
### Sample Assessment-Indirect Measurement

Procedure:

1. Building an Inclinometer - Tape the straw to the straight edge of the protractor. Thread one end of the string through the hole of the protractor and tie it securely. Tie a washer to the free end of the string.



- Using the Inclinometer – The inclinometer is an instrument used to determine the angle of elevation using the line of sight to some point. Hold the inclinometer in the horizontal position to see that the string will hang across the  $90^\circ$  mark. To measure the angle of elevation, look through the straw to the top of the object being measured. The washer, responding to gravity, will pull the string across the protractor at some angle. The angle of elevation is the difference between this measurement and the horizontal ( $90^\circ$ ).



- Use only a tape measure, the inclinometer and your knowledge of trigonometric ratios to find the height of a tree.
- ABC Construction has to run a guy wire from the top of a tower to a point on the ground, 7 meters from the tower. You have been asked to help. You have been informed by ABC Construction that the tower is the same height as the flagpole in front of your school. Find the length of guy wire needed to complete this task.
- Write a report of your findings. Include illustrations and a complete mathematical description of your procedure.

## Teacher Notes

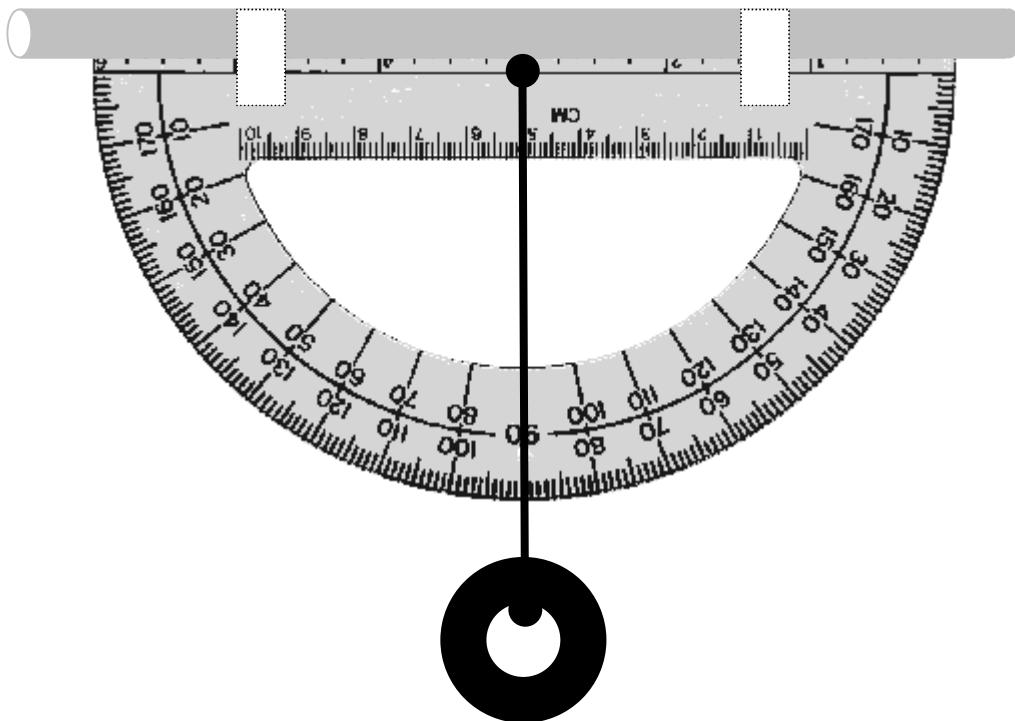
**Course:** Pre-Calculus

**Objectives:** The student will use his knowledge of trigonometric ratios and angle measurement to complete two tasks.

**Materials:** Protractor  
Straw  
Tape  
String (approximately 15 cm)  
Washer  
Tape Measure

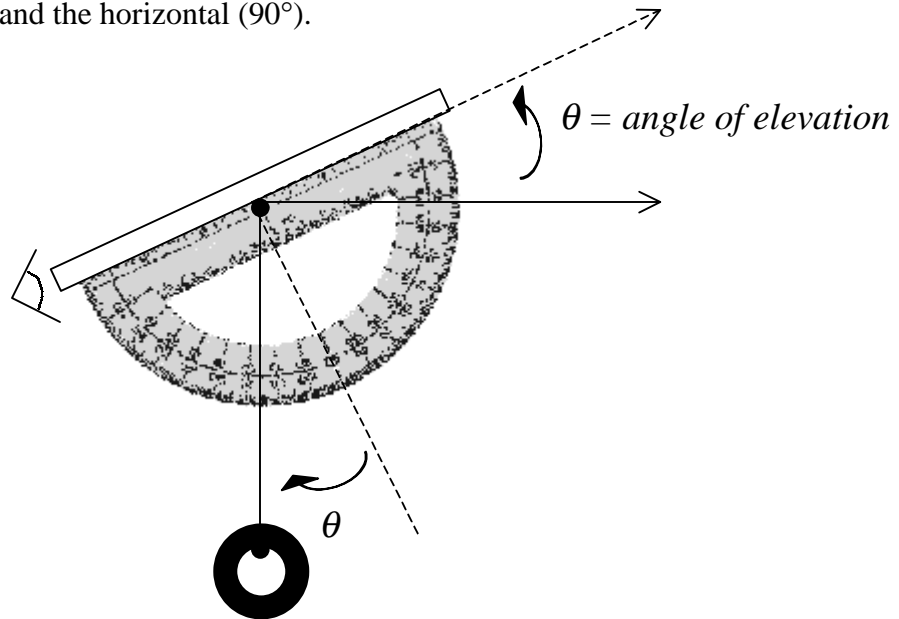
**Procedure:**

1. Building an Inclinator - Tape the straw to the straight edge of the protractor. Thread one end of the string through the hole of the protractor and tie it securely. Tie a washer to the free end of the string.

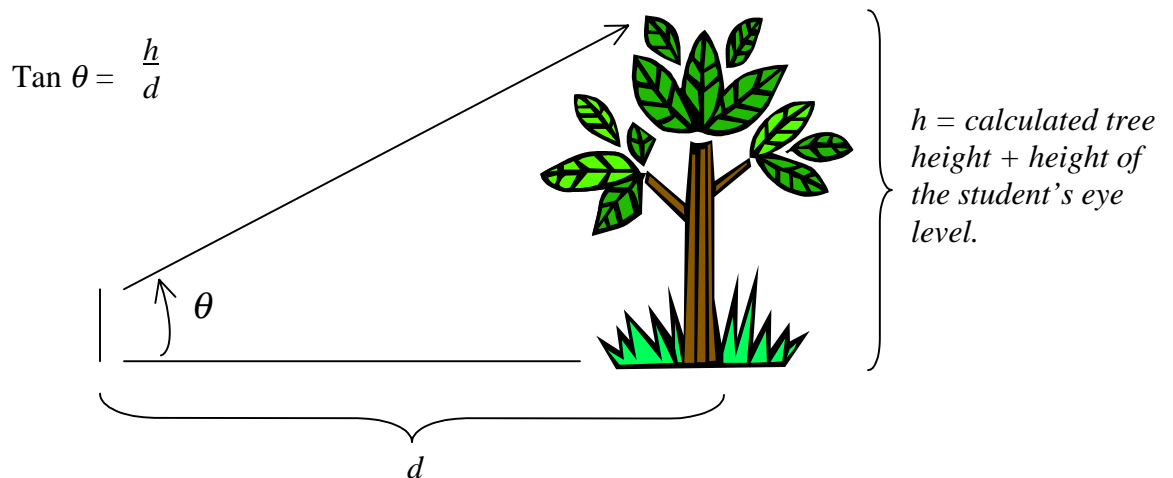




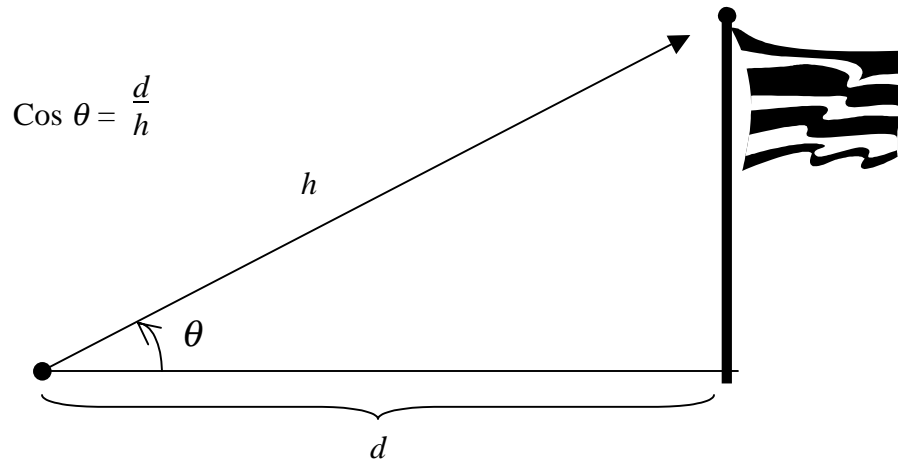
2. Using the Inclinometer – The inclinometer is an instrument used to determine the angle of elevation using the line of sight to some point. Hold the inclinometer in the horizontal position to see that the string will hang across the  $90^\circ$  mark. To measure the angle of elevation, look through the straw to the top of the object being measured. The washer, responding to gravity, will pull the string across the protractor at some angle. The angle of elevation is the difference between this measurement and the horizontal ( $90^\circ$ ).



3. Measuring the height of a tree – Using the tape measure, measure the distance,  $d$ , between the student and the tree. Use the inclinometer to find the angle of elevation,  $\theta$ , to the highest point on the tree. The tangent function can be used to find the height. Since the angle measurement is taken at eye-level and not the ground, you must take into account the height of the person taking the angle measurement. This number can be added to the calculated height of the tree at the end of the problem.



4. Measuring the Length of the Guy Wire – Using the tape measure, measure the distance,  $d = 7$  meters, from the flagpole. Use the inclinometer to find the angle of elevation,  $\theta$ , to the highest point on the flagpole. The best way to find the angle of elevation is by getting the inclinometer as close to the ground as possible. This will eliminate the problem of the different students' heights affecting the outcome. The cosine ratio can be used to find the length of the guy wire,  $h$ .



# Indirect Measurement Using Trig Ratios Scoring Rubric

| Criteria  | 4 | 3 | 2 | 1 | Total |
|---|---|---|---|---|-------|
| Correct trig functions used<br>(5 points)         |   |   |   |   |       |
| Equation and ratio set-up<br>(5 points)           |   |   |   |   |       |
| Calculations with accurate<br>units<br>(4 points) |   |   |   |   |       |
| Accurate procedure<br>description<br>(7 points)   |   |   |   |   |       |
| Neat and accurate<br>illustrations<br>(4 points)  |   |   |   |   |       |
| Total Possible Points = 100                       |   |   |   |   |       |

Accurate – 4  
 Minor Error – 3  
 Major Error – 2  
 Little or no attempt – 1

## **Annotated Bibliography of Assessment References**

Ainsworth, L., & Christinson, J. (1998). Student generated rubrics: An assessment model to help all students succeed. Orangeburg, NY: Dale Seymour.

The book provides a guide for creating effective rubrics and allowing students to use them to evaluate their peers. Examples of focus questions some involving mathematics, performance tasks, student rubrics, and student work are also given.

Association for Supervision and Curriculum Development (1989). Redirecting Assessment [Special issue] Educational leadership, 46(7).

This special issue deals with many facets of assessment, including case studies of implementation of alternative assessment practices. Although no articles focus specifically on mathematics much of the information is applicable to K-12 mathematics.

Balanced Assessment for the Mathematics Curriculum. (1999). Advanced high school assessment. White Plains, NY: Dale Seymour.

This book contains activities of various length that cover knowledge and problem solving based on the National Council of Teachers of Mathematics (NCTM) Standards. A description, a solution, suggestions for evaluation, and examples of student work are provided for each activity.

Barton, J. & Collins, A. (Eds.). (1997). Portfolio assessment: A handbook for educators. White Plains, NY: Dale Seymour.

This book demonstrates how portfolios can enhance instruction and provides guidelines for integrating them into the classroom along with examples of their use in all subjects including mathematics.

Bush, W. S., & Greer, A. S. (Eds.). (1999). Mathematics assessment: A practical handbook for grades 9-12. Reston, VA: The National Council of Teachers of Mathematics.

This book explains the necessity for novel approaches to assessment and gives instructions for creating and modifying assessment tasks. It also discusses scoring and how assessment data can be used.

This project was sponsored by the Southwest Consortium for the Improvement for Mathematics and Science Teaching (SCIMAST). 132

Clarke, D. (1997). Constructive assessment in mathematics: Practical steps for classroom teachers. Berkeley, CA: Key Curriculum Press.

This book guides mathematics teachers in developing a comprehensive assessment plan including a variety of assessment instruments and advice on how to use assessment information for both instructional improvement and grade assignment.

Danielson, C., & Marquez, E. (1998). A collection of performance tasks and rubrics: High school mathematics. Larchmont, NY: Eye on Education.

The book provides an introduction to and overview of performance assessment as well as guidelines for creating and adapting performance tasks and rubrics. It also contains twenty-one performance tasks and rubrics along with examples of student work.

Freedman, R. L. H. (1994). Open-ended questioning: A handbook for educators. Menlo Park, CA: Addison-Wesley.

This book provides instructions and examples for creating and assessing open-ended questions, which stimulate thought and enhance assessment in all subjects and grade levels.

Hart, D. (1994). Authentic assessment: A handbook for educators. Menlo Park, CA: Addison-Wesley.

The book provides a history and overview of the assessment movement and practical guidelines for designing assessment. It also contains a glossary and a list of resources.

Kane, M., & Apple, M. (1993). Alternative assessment: Evaluating student performance in elementary mathematics. Palo Alto, CA: Dale Seymour.

This book explores interviews, observations, portfolios, student self-assessment, performance tasks, and student writing as a means of aligning instruction and assessment. It provides over ninety lessons that model these assessment strategies.

Kuhs, T. (1997). Measure for measure: Using portfolios in K-8 mathematics. Portsmouth, NH : Heinemann.

This book provides guidelines for the implementation of portfolio assessment in mathematics for elementary and middle school classrooms, including the development of a portfolio culture in the classroom

Kulm, G. (Ed.) (1991). Assessing higher order thinking in mathematics. Washington DC: American Association for the Advancement of Science.

This book contains various articles pertaining to the use of assessment that measure higher order thinking. The use of technology in assessment and the role research plays in developing assessment instruments are major themes of several articles.

Lambdin, D. V., Kehle, P. E., & Preston, R. V. (Eds.). (1996). Emphasis on assessment: Readings from NCTM's school-based journals. Reston, VA: The National Council of Teachers of Mathematics.

This collection of articles provides a rationale for changing assessment and discusses alternative assessment methods, offering tips for getting started. It concludes with discussions on teachers' self-evaluation. The book also includes valuable bibliographies on assessment and matrices for searching the bibliographies by topic.

Madfes, T. J., & Muench, A. (1999). Learning from assessment: Tools for examining assessment through standards. San Francisco, CA: WestEd.

This resource attempts to link assessment to standards for instruction. It consists of three modules: Aligning Assessment to Standards, Collecting Standards-based Evidence, and Planning Instruction to Support Standards, each of which includes various assessment techniques.

Mathematical Sciences Education Board. (1993). Measuring up: Prototypes for mathematics assessment. Washington, DC: National Academy Press.

This book provides a specific example of an assessment approach based on NCTM's recommendations for curriculum and teaching standards. It addresses the questions of whether assessments should provide goals for teachers and students, whether assessments should gauge what students can do or what students cannot do, whether they should assess both individual and group work, and how assessment can stimulate invention and creativity.

Moon, J., & Schulman, L. (1995). Finding the connections: Linking assessment, instruction and curriculum in elementary mathematics. Portsmouth, NH: Heinemann.

This book demonstrates the integration of assessment, instruction, and curriculum. It provides suggestions for communicating with students, parents and administrators and helps readers develop original assessment plans.

National Council of Teachers of Mathematics. (1992). Assessment [Focus issue] . Arithmetic teacher, 39(6).

This issue contains a variety of articles discussing assessment of students and of teaching geared to K-8 grade teachers. The main focus of most articles is the development and use of appropriate alternative assessment instruments.

National Council of Teachers of Mathematics. (1992). Alternative assessment [Theme issue]. Mathematics teacher, 85(8).

This issue contains a variety of articles discussing assessment in the high school classroom. Articles address the assessment of cooperative work, the evaluation of problem solving and the alignment of assessment to the NCTM Standards.

Perrone, P. (Ed.) (1991). Expanding student assessment. Alexandria, VA: Association for Supervision and Curriculum Development.

This set of articles on various aspects of alternative assessment includes an article focusing on the need for a clear understanding of what is being assessed in standardized mathematics tests.

This project was sponsored by the Southwest Consortium for the Improvement for Mathematics and Science Teaching (SCIMAST). 135

Silver, E. A., Kilpatrick, J., & Schlesinger, B. (1990). Thinking through mathematics: Fostering inquiry and communication in mathematics classrooms. New York: College Entrance Examination Board.

This book explains the importance of activity, both individual and social, in mathematics learning. It also demonstrates how increasing communication in the mathematics classroom enhances the students' and teachers' understanding of the material. It provides suggestions for increasing communication and encouraging students to move from simply absorbing the material to actually thinking about it.

Stenmark, J. K. (1989). Assessment alternatives in mathematics: An overview of assessment techniques that promote learning. Berkeley, CA: Regents, University of California.

The book reviews mathematics assessment methods including student products and performance. It also addresses student self-assessment and provides sample problems.

Stenmark, J. K. (Ed.). (1991). Mathematics assessment: Myths, models, good questions, and practical suggestions. Reston, VA: The National Council of Teachers of Mathematics.

This book gives overviews and examples of the use of performance assessment; observations, interviews and conferences; and portfolios in mathematics. It also provides models for implementing these modes of assessment.

Webb, N. L., & Coford, A. F. (Eds.). (1993). Assessment in the mathematics classroom. Reston, VA: National Council of Teachers of Mathematics.

This book contains different perspectives on classroom assessment, a variety of assessment techniques and suggestions for managing assessment. It contains examples for assessing problem solving, communication, and reasoning for students at all grade levels.



## Websites with Assessment Information

The National Council of Teachers of Mathematics: [www.nctm.org](http://www.nctm.org)

This website contains the electronic version of Principles and Standards of School Mathematics with links to material that clarifies all standards and principles including the assessment principle.

The Eisenhower National Clearinghouse: [www.enc.org](http://www.enc.org)

This website's searchable database includes references to a variety of assessment resources.

Southwest Educational Development Laboratory: [www.sedl.org](http://www.sedl.org)

This website includes a variety of resources for teachers, including a guide to making assessment meaningful for everyone.

The Missouri Assessment Program Scoring Guide rubrics:  
[www.indep.k12.mo.us/pdc/maps/projectmap.htm](http://www.indep.k12.mo.us/pdc/maps/projectmap.htm)

This website offers samples of mathematics scoring rubrics.

The EXEMPLARS: [www.exemplars.com](http://www.exemplars.com)

This website offers examples of mathematics assessment instruments and scoring rubrics.