

Modeling Effective Pedagogical Strategies for Teaching Mathematics

Robin A. Ward, Ph.D.
University of Arizona

K-8 preservice teachers enrolled in an elementary mathematics methods course engaged in a cross-curricular mathematics activity as a means to demonstrate effective pedagogical strategies. Teacher candidates experienced how to effectively integrate children's literature and technology into a mathematics lesson, which encouraged multiple approaches to problem solving, estimation, and collaboration.

Introduction

Once upon a time there lived a young map colorer. It was her job to color all the maps of the neighborhoods and regions of the kingdom...In those long ago days, crayons were rare and very expensive. So the young map colorer tried to use as FEW crayons as possible when she colored her maps...

These are the opening lines to the Internet-based book, *The Story of the Young Map Colorer*, a short but delightful problem-solving tale in which readers encounter the centuries-old mathematical conundrum commonly referred to as the "Four Color Theorem", a challenge to both mapmakers and mathematicians. A cohort of twenty K-8 preservice teachers enrolled in an elementary mathematics methods course recently donned their artistic thinking caps and engaged in an exploration of this famed theorem. Serving as the rationale for this activity, as well as for this article, is the importance of (1) modeling for preservice teachers how to develop cross-curricular activities; in particular, those that integrate mathematics, (2) engaging students in rich mathematical activities that feature problem-solving and children's literature; activities that they might implement in their future teaching, (3) demonstrating how to effectively use technology in one's teaching, and (4) articulating strategies and ideas by which K-12 teachers can enhance the teaching and learning of mathematics. The dilemma facing the story's young map colorer, which these preservice teachers were challenged to solve, was to determine the fewest number of colors needed to color a map such that regions sharing the same boundary have different colors. In short, the teacher candidates, unbeknownst to them, were about to explore the mathematically acclaimed and enigmatic Four Color Theorem.

Art Triggers Mathematics

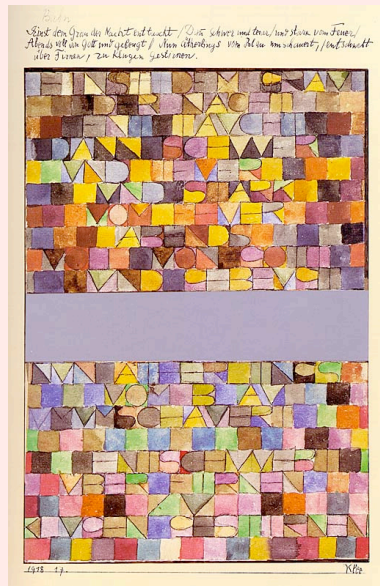
One day prior to the implementation of this activity, the teacher candidates engaged in an integrated art and social studies lesson in which they learned about the artwork and life of Paul Klee, a Swiss expressionist artist. The teacher candidates then dabbled in Klee's artistry, creating their own personalized name strips using watercolors and a 3" x 12" strip of white construction paper (see Figure 1), which mimicked one of Klee's masterpieces, *Once Emerged from the Gray of Night* (see Figure 2). Upon entering the classroom the next day, my eye caught the vibrantly colored "name strips" enveloping one of the classroom's bulletin boards and I was fascinated by their kaleidoscopic workmanship. I noticed how

many of the teacher candidates chose to alternate colors between adjoining regions within the letters of their name. Having studied topology decades past, this variance in color immediately summoned in my mathematical mind the renowned Four Color Theorem. Thus, I seized the opportunity to bridge the connection between art, mathematics, and what these teacher candidates had discussed in their social studies methods class, and I designed and implemented the following problem solving activity. While searching the Internet for relevant materials to assist me in the design of this cross-curricular activity, I stumbled across the internet-based book, *The Story of the Young Map Colorer*. I was captivated by the book's simplicity and how it so succinctly, and rather unassumingly, presented such a complex mathematical problem in a non-threatening manner, all couched within a real-life application to mapmaking. Having regularly integrated children's literature into my teaching and knowing the power of doing so, I felt confident in this book's promise to serve as the opening act in this problem solving exploration.

Figure 1. One teacher candidate's water colored name strip, painted in the spirit of Paul Klee



Figure 2. Paul Klee's masterpiece, "Once Emerged from the Gray of Night"



Posing the Problem

As I narrated, the teacher candidates listened intently as the story appeared on the screen in the front of their classroom. As the story unfolded, the teacher candidates saw examples of maps colored “well” and “badly” to visually assist them in formulating the rules of basic map-making. I then read the last passage of the story, posing the problem to be tackled and explored:

One day representatives from the Map-Making Committees of Boxville, Popdale, Swirltown, and Bumpington all showed up at once! Of course they wanted their maps in a hurry! How should the Young Map Colorer color each one?

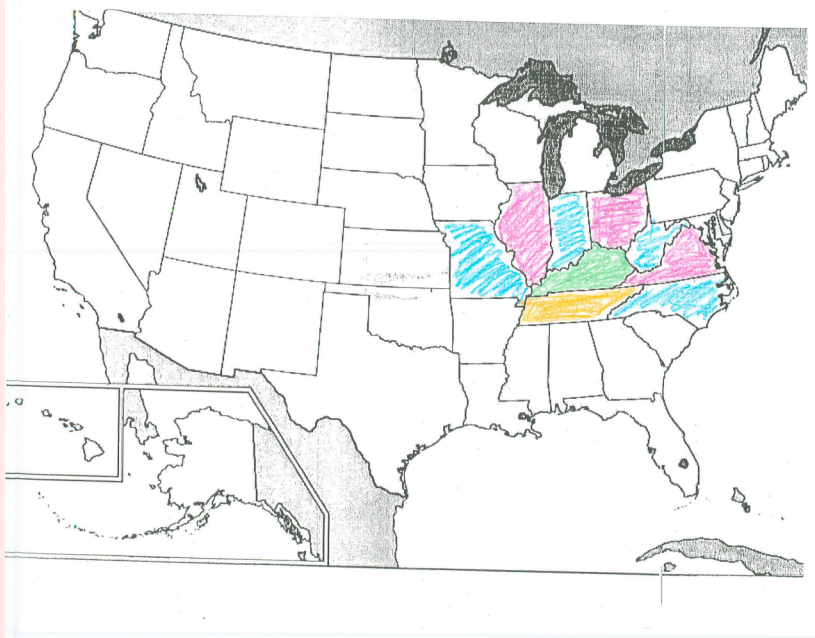
To assist the teacher candidates in solving this problem, I provided each of the teacher candidates with a map of the United States and a package of crayons. I asked my young map colorers to view the map and to first predict the least number of colors needed to color the map as previously described. One teacher candidate impetuously exclaimed, “Three colors...No wait, it has to be four.” Some of his classmates agreed; others estimated five colors. Before I could continue, a teachable moment arose when a teacher candidate interjected, “But why can’t two states, I think that’s what you mean by ‘regions,’ be the same color?” Why would it matter?” Instead of answering her question myself, I posed it to the class. One teacher candidate correctly responded “Having the same color next to each other might make it hard to tell different states apart. They’d blend together if you looked at it [the map] real quickly.” I further clarified, “It is okay for two regions that meet at a single point to be colored the same color, but if two regions that share a boundary are the same color, the map would look ambiguous from a distance.” Satisfied with this response, we began our map-coloring exploration.

The problem solving begins! Eager to solve the problem the teacher candidates, although seated in groups of four, worked independently to color their maps, using the least number of colors. As I navigated the classroom observing my neophyte mapmakers, I was amazed at the variety of problem solving strategies employed by the preservice teachers. Many began coloring the states moving from West-to-East (see Figure 3). Upon asking a teacher candidate why she chose to color in this directional fashion, she responded, “I guess it seemed natural, kinda like reading from left-to-right.” On the contrary, a handful of teacher candidates did the exact opposite, moving from East-to-West as they colored. When asked why she chose this right-to-left strategy, a different teacher candidate replied, “Well, I’m from back East so I figured I’d start there.”

Two other teacher candidates used what might be described as the “Polka Dot Strategy” whereby they used one color to exhaustion before selecting another one, placing a mark or some notation on each state as they progressed.

One teacher candidate explained, “I was afraid to just start coloring in case I made a mistake. So I only used two colors to start and when I couldn’t use them anymore I penciled in O’s and G’s on my map, for orange and green, so I could erase if I had to” (see Figure 4). Similarly, another teacher candidate used a pencil and notated the numerals 1, 2, 3, and 4 on her map placing the 1 on as many states as possible, followed by the 2, then the 3, and then the 4. This step-by-step recipe for solving a problem is also known as a “greedy algorithm” since it single-mindedly gobbles up all of its favorites first. The idea behind a greedy algorithm is to perform a single procedure repetitively until it cannot be performed again in an effort to see its results. Although employing this algorithm may not completely solve the problem or, if it does, it may not solve it efficiently, it still remains as one way of approaching the problem and, can sometimes yield very good or even the best possible results.

Figure 5. One teacher candidate's partially colored but complete solution to the Four Color Theorem



a potential solution to the problem to which she confidently responded, “I stopped coloring because for this map, you only need four colors.” She continued on to explain her strategy:

I started to move from the top left and go across and to mentally try to keep track of colors but then I thought, wait, I should start in the middle where there are lots of states. So I started with West Virginia, I mean Kentucky, because it touched so many other states and I just worked out from there.

Thus, unlike the vast majority of her other classmates, this teacher candidate realized that it was unnecessary to completely color the entire map to obtain a solution. Instead, by purposefully selecting a particular state, which bordered many other states, she very efficiently and elegantly obtained her solution.

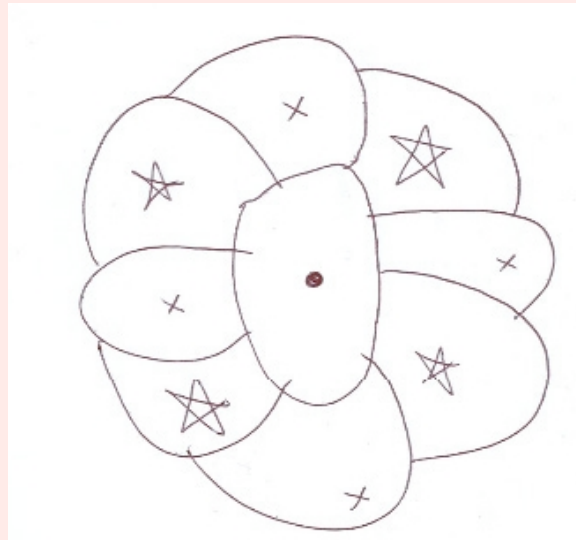
As each of the groups of teacher candidates colored, not only did they engage in the mathematics of the activity, modifying their original predictions and sharing their coloring strategies, but they also honed their geography skills, as some challenged their neighboring classmates to identify names of states and state capitals. After their maps were colored, I asked individual students to explain their coloring strategies to the class so the students could not only learn of the varied approaches to solving this

problem but also to model and emphasize the pedagogical importance of allowing students to articulate and defend their reasoning. The students were clearly impressed and amazed at all of the different strategies their classmates employed. I then asked the teacher candidates to compare their original predictions to their empirical results and they unanimously concurred that four was the minimum number of colors needed to color this map as prescribed.

Defending and Generalizing their Solutions. Following this, I asked the teacher candidates to reflect on their classmates' shared strategies and to discuss within their groups whether their results were generalizable to any and all maps; that is, will four always be the maximum number of colors needed? As the teacher candidates conversed and conjectured, I noticed one teacher candidate within a group begin to draw free-form shapes in his notebook and then tell his group members, "Let's see if we can all draw a map where lots of regions touch other regions and force five [colors] to be the answer. I'm trying to do it, but I can't seem to." Acting on his clever recommendation, all of the teacher candidates began sketching, but no one could design a map that would require five colors. At this point, they unanimously agreed that four must be the correct solution, but all were uncertain as to why. In another group, one teacher candidate, using her sketch to defend her solutions to the class, asserted that a map might only need two or three colors at most, depending on how it is drawn (see Figure 6).

Finally, to ease their curiosity, I then reported that finding a method to determine exactly how many colors is needed for any map continues to daunt mathematicians today and that some mathematicians contend that a fast method to find out the minimum number of colors needed to color a map does not exist.

Figure 6. A map needing only three colors



I then provided my young map colorers with the mathematics behind the famed Four Color Theorem and congratulated them on obtaining the correct solution. I shared with them some of the history of this perplexing theorem articulating how for centuries, it was a commonly known, but unproven rule among mapmakers that a map drawn on a flat surface or sphere requires no more than four colors to show distinct regions. This was important, as a mapmaker would want the user of a map to be able to

easily discern adjacent regions, and this was done by using different colors. The Four Color Theorem, sometimes also called Guthrie's Problem, was named after Francis Guthrie, a student of the famed logician, Augustus DeMorgan, who first conjectured the theorem in 1853 while trying to color a map of the counties of England and noticing that four colors sufficed. Members of the mathematical community debated and struggled for years to prove this seemingly unprovable theorem until 1976, when Kenneth Appel and Wolfgang Haken of the University of Illinois, with the aid of a computer, confirmed that four is indeed the magic number, thus ending over one hundred years of mathematical deliberations and proving the now famous Four Color Theorem. The program that these mathematicians wrote contained thousands of lines of code and took over 1200 hours to run. Since that time, a collective effort by interested mathematicians has been under way to check the program. To date, only minor errors have been detected and since corrected. Thus, most mathematicians accept the theorem as true, although some might argue otherwise.

Mathematical and Pedagogical Importance of the Four Color Theorem

The Four Color Theorem holds importance in mathematics because it was the first major theorem to be proved using a computer, having a proof that could not be verified directly by other mathematicians. Essentially, Appel and Haken used a computer to find roughly 1000 small maps and showed that if the theorem were false, one of the maps had to require five colors. Then they made the computer color the maps with four colors, showing that none required five colors, thus proving the theorem. Thus the Four Color Theorem can be considered as the birth of graph theory since, while trying to solve it, mathematicians invented many fundamental graph concepts and terminology.

From a mathematics teacher's perspective, the Four Color Theorem holds much promise as young students engaged in map coloring activities employ a variety of interesting strategies for finding a coloring of a map that uses a small number of colors. For example, younger students might employ the aforementioned greedy algorithm in search of a solution while older students might use deductive reasoning, where they begin with leads that are true and then follow them to a conclusion. Even older students might use proof by induction, which is a fundamental idea to which students are traditionally introduced in high school. The basic principle of proof by induction is to begin by looking at the simplest possible case and to demonstrate that what you are seeking to prove is true in the simplest case. Then you add elements to that simplest case in a step-by-step, orderly fashion, systematic enough to cover all possible combinations of elements in all of the cases. If you can show why, at each step, the truth that you demonstrated about the simplest case is preserved, you have completed a proof by induction. Thus, providing students with the opportunity to explore the Four Color Theorem allows a teacher to vividly demonstrate how varied approaches and strategies to problem solving exist and, further, how even sophisticated mathematical concepts can be presented to younger students.

Making Cross-curricular Connections. At the closing of this problem solving exploration, I connected their map-coloring experience with their prior watercolor activity completed in their social studies methods class, building a cross-curricular bridge between mathematics, art, and social studies. I articulated to these preservice teachers the importance of connecting new ideas to students' prior knowledge and that teaching mathematics well involves "creating, enriching, maintaining, and adapting instruction" in order to "capture and sustain interest and engage students in building mathematical understanding" (NCTM, 2000, p. 18).

Further, I also shared with them titles of other pieces of children's literature that engage readers in map-making and map exploration, that they might consider using in their future classrooms. These titles included: *As the Crow Flies: A First Book of Maps* (Hartman & Stevenson, 1993), *Maps and Globes* (Knowlton, 1986), *Mapping Penny's World* (Leedy, 2000), and *Me on the Map* (Sweeney, 1998). I then circulated the books among the teacher candidates allowing them to explore and discuss possibilities for integrating these books into their future teaching, focusing on the development of problem solving

activities. By bringing this activity to a closure in this fashion reminded and reinforced for these teacher candidates the pedagogical strategies and tools they should consider using once certified as classroom teachers.

Reflecting on Problem Solving. One week later, the teacher candidates shared their thoughts and reactions to this activity as chronicled in their journals. Several teacher candidates expressed surprise upon learning that one could access an entire storybook via the Internet. This discovery sparked an unplanned discussion of how the Internet has opened up many avenues to educators allowing them to obtain affordable and, most times, free learning resources and materials. Many teacher candidates expressed delight in engaging in mathematics but in a “non-threatening way,” or as one teacher candidate stated, “I was doing math but didn’t realize it. It was as if the book made the mathematics invisible so it didn’t seem scary or hard to solve.” Others additionally offered that it was helpful to their future pedagogy to experience first-hand how to integrate content areas. One teacher candidate stated, “I always heard there were connections between math and art, but now I finally know of one of them!” Finally, a few admitted their embarrassment over their deficient geography knowledge, expressing concern over the fact that they “better learn their states and state capitals fast” before their student teaching experience commences.

Final Remarks. In closing, I do not believe this problem solving activity would have been greeted with as much interest on behalf of the teacher candidates and have resulted in such success had *The Story of the Young Map Colorer* not been integrated, as this piece of children’s literature placed the mathematics to be explored into a real-life, plausible context and engaged the learners in meaningful conversations and authentic investigations in mathematics and geography. Unquestionably, this problem solving activity embodied the spirit of the NCTM’s problem-solving strand (NCTM, 2000, p. 52), as it enabled the teacher candidates to build mathematical knowledge, solve problems that arise in other contexts, apply and adapt a variety of strategies, and it allowed for reflection. Based on their positive, journaled reactions and shared reflections, the preservice teachers indicated that this cross-curricular, authentic mathematical investigation integrating art, technology, and children’s literature served as a model for how to design successful and engaging problem solving activities for their future K-8 students. Further, the vast majority of the preservice teacher candidates commented that they would implement this activity in the future, noting what one teacher candidate called its “curricular power” in tying together various content areas while “making mathematics fun and the focus.” In fact, two of the teacher candidates implemented this activity, one with fifth graders during her student teaching and one with fourth graders during her first year of teaching. Both of these teachers described how the online story, which depicted a real-life situation, “captured” their students’ attention and “helped students see there is more than one way to solve a problem.”

Implications

Educational organizations such as the National Council of Teachers of Mathematics (2000), the National Council for the Social Studies (1994), and the Consortium of National Arts Education Associations (1994) articulate the importance of integrating and making connections between and among mathematics, social studies, and the arts, as the arts can provide a meaningful and powerful foundation on which students can create or further enhance their understandings of mathematical and other curricular concepts. Shaffer (1997) asserts that “expressive activities are a powerful context for learning” (p. 110), and that one of the benefits of combining art and mathematics is that students are able to think about mathematical ideas in expressive ways and thus, can control their own learning. Other researchers such as Willet (1995) have noted that mathematical learning is more effective at the elementary school level when in the context of arts-based lessons, as opposed using traditional mathematics pedagogy, while Bickley-Green (1995) contends that when putting art together with math ideas, the art “constitutes an

illusionary base for thinking, interpreting, knowing, and problem-solving” (p. 15). Thus, she advocates for mathematics and art educators to provide more effective instruction through coordinated curricula.

Given the growing body of research and anecdotal evidence that supports the integration of mathematics across content areas as a means to enhance the learning process, the author emphasizes that it is these types of activities that not only teacher educators ought to engage in with their teacher candidates as a means to enhance and broaden their developing pedagogy, but also inservice teachers should implement in actual K-12 classroom settings in order to make the learning of mathematics applicable to other subject areas and more motivating. So, go observe your world! Look at a map and count the colors!

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About the Author:

Dr. Ward is an Assistant Professor of Mathematics Education at the University of Arizona. Her research interests include preservice teachers' technology training, mathematics content knowledge, and pedagogical content knowledge.