

Using Technology to Make Connections in Algebra



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Problem 1 – Intersecting Lines

1. What is the slope of each line on page 1.2?
2. What is the point of intersection of the two lines?
3. What is the slope of each line on page 1.3?
4. What is the point of intersection of the two lines?
5. Two lines with different slopes will intersect in one point.
 Always Sometimes Never

Problem 2 – Parallel Lines

6. What is the slope of each line on page 2.1?
7. Parallel lines intersect.
 True False
8. Solve the equations on the left side on page 2.2 for y . What is the slope of each line?
9. The lines $x + 3y = 1$ and $x - 3y = 1$ are parallel.
 True False
10. What kind of lines are $y = 4$ and $x = 4$?
11. What is another way to describe or name that pair of lines?



Problem 3 – Same two lines

12. Solve the equations on page 3.1 for y . What is the slope of each line?

13. How are the two lines related to each other?

14. Are the two lines on page 3.2 the same? How do you know?

15. The slope of both lines is -3 .

True

False

Homework – Word problems (Problems 4 and 5)

16. Write two equations that represent the problem given on page 4.1.

17. Enter three pairs of numbers that add up to 12 on page 4.2. What are your three pairs?

18. Using the graph on page 4.3, determine the solution to the problem.

19. Write two equations that represent the problem given on page 5.1.

20. Enter three pairs of ages on page 5.2. What are your three pairs?

21. Using the graph on page 5.3, determine the solution to the problem.



Problem 1 – Observe motion and graphically solve

Read the problem on page 1.2. Press Play to view the motion of boats traveling downstream and upstream on the Mississippi River.

1. How long does it take the boat to travel *downstream* from towns A to B?
2. How long does it take the boat to travel *upstream* from towns B to A?
3. Let r be the rate of the boat in still water. How could the rate upstream and the rate downstream be expressed?
4. Use the above information to fill in the blank spaces.

distance = rate \times time

down d = _____ \times _____

up d = _____ \times _____

5. Set the equations equal to each other and solve for r algebraically. Show your work here.
6. On page 1.9, graph the two equations. Solve the system of equations graphically. Press **MENU > Points and Lines > Intersection Point**. Then, click both lines. The coordinates of the intersection point will appear.

Does this agree with your solution from Question 5? Record the rate of the boat in still water and the distance between the towns. Include units.



Problem 2 – Distance-time graph, explore slopes

Read the problem on page 2.1. On the page 2.3, click the arrows to graphically explore values for the rate s of the steam engine locomotive and v of Velma walking that satisfy the given information. Notice the two different slopes.

7. What does the slope of the distance-time graph represent? _____

8. Draw a diagram for the situation similar to the boat animation.

9. Apply $d = r \cdot t$ to this situation. $r = ?$

(Hint: r depends on s , the speed of the steam engine, and v , the velocity of Velma.)

distance = rate \times time

forward ___ = _____ \times _____

back ___ = _____ \times _____

10. Algebraically solve the equation.

Problem 3 – Extension/Homework

Complete the problems on pages 3.1 and 3.2.



Problem 1 – Number patterns can be revealed in a scatter plot.

On page 1.2, you see the ordered pairs that will be plotted on the next page. Focus on the y -values.

- Can you figure out a mathematical pattern for these values? Write an explanation if you can.

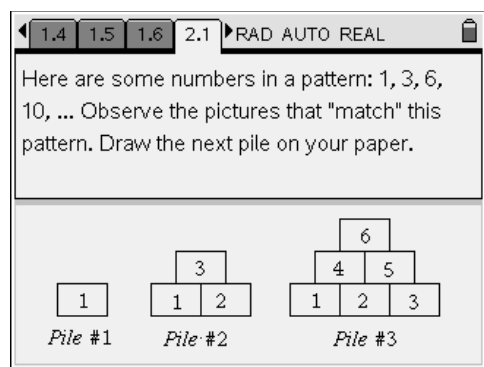
On page 1.3, the spreadsheet is shown again. Enter the next three y -values that follow the same pattern. You will need to scroll down to the empty cells in the **yval** column. The scatter plot will reveal your new ordered pairs immediately.

- Do you see a visual pattern or shape? What is it?
- On page 1.5, a function is graphed over the scatter plot. Do you agree that it matches the data? Why or why not?
- Now, use the matching function to predict a future term of the pattern. What is $f(7)$?

Problem 2 – A pattern that has a visual model with it

On page 2.1, you will see stacks of bricks. Observe the pattern.

- Predict what the next pile will look like and sketch it below.



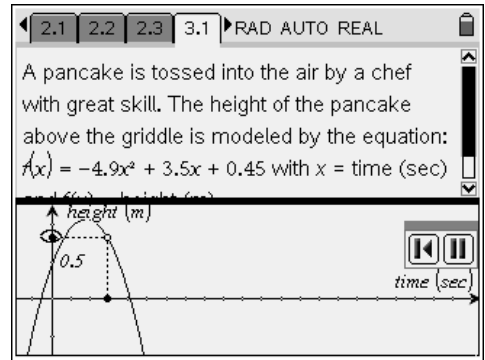
On page 2.2, enter the number you think will continue the pattern, representing how many bricks will be in the next larger “pile.” You can check your answer by pressing MENU , and selecting **Check Answer** before you enter that number into the spreadsheet column labeled **bricks**.

- What shape does the data form?

Problem 3 – A pancake in the air? An application of a parabola at breakfast time!

Look at how the pancake rises slightly before it falls back to land on the griddle. Have you ever flipped a pancake? Have you ever seen a chef toss a pizza crust?

On page 3.2, use **Graph Trace** to find how high the pancake is above the griddle after several different lengths of time. Select **MENU > Trace, Graph Trace**, and then use the right/left arrows on the NavPad to trace along the graph. Ordered pairs will appear that represent a time in seconds, and the height of the pancake above the griddle at that time.



- Record several ordered pairs here: (you should round to the nearest tenth)

Time (sec)	Height (cm)

- On page 3.3, you will use **Graph Trace** to find the maximum height of the pancake, and when it occurs. Using **Graph Trace** will give you both the ordered pair, and an indication that this point is a maximum.

Time (sec):_____ Maximum height (cm):_____

- At what time does the pancake land on the hot griddle? As you trace left and right, watch for the ordered pair to appear with an accompanying message that says “zero.” This is your solution.

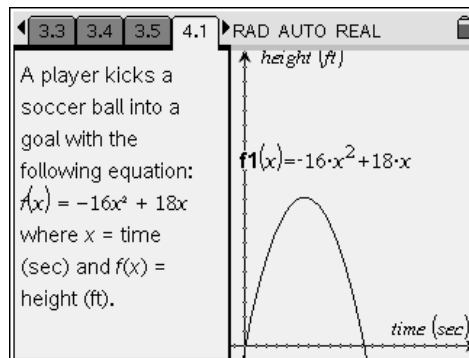
Time (sec):_____ Height (cm): 0 (sizzle...Yum!)

- The pancake graph is a parabola because “what goes up must come down.” True or false?

Homework Problem – Soccer Ball

A new scenario is presented as an application of parabolas. This time, the height of the object is measured in feet and time is still measured in seconds.

On page 4.2, you will repeat the procedure using **Graph Trace**. Answer the following questions.



- Time (sec): 0.2 sec Height (ft): _____
- Time: _____ Maximum height (ft): _____
- Time: _____ Height: 0 ft. (ball lands on the ground, perhaps inside the goal net!)
- The soccer ball flies toward a goal that is 6 feet high. Could the ball go over the top bar of the goal?

Extensions

- Two different equations were introduced that actually have the effect of “gravity” built into them. In the pancake problem, the equation contains $-4.9x^2$, and in the soccer problem, the equation contains $-16x^2$. Investigate why there are two different numbers used in these equations. You may need to search for information about “acceleration due to gravity” on the internet in order to answer.
- Explain the entire formula for projectile motion: $h(t) = -0.5gt^2 + v_0t + h_0$. Your previous answer will help to explain the variable “ g ” in the formula.
- Write your own projectile motion problem using $g = 32$, initial velocity of 28 ft/sec and an initial height of 3 ft. Choose an object to be launched, and graph the equation. Make sure the window shows the parabola in the first quadrant so that you can see both the maximum (vertex) and where the parabola intersects the x-axis. Explain the meaning of the vertex and the x-intercept. Find several heights at different times.

Notes



Problem 1 – Introduction to area of a rectangle

View the rectangle on page 1.3.

1. What are the lengths of the sides of the rectangle?
2. What is the area of the rectangle when $a = 6$?

Now, change the length of the side by clicking on the arrows next to **a: = 6**.

3. What is the area of the rectangle when $a = 4$? When $a = 11$?
4. How is the expression for the area simplified?

Problem 2 – Areas of small rectangles

On page 2.1, you see a rectangle of dimensions $(x + 7)$ and $(x + 2)$. Each piece of the rectangle is a different color so that you can focus on its area.

5. What is the area of each small rectangle?
6. What is the total area of the rectangle?

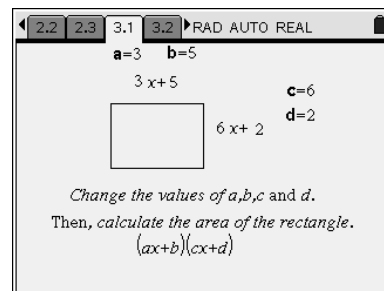
Problem 3 – FOIL method

The rectangle on pages 3.1 and 3.2 has the same dimensions as the rectangle from Problem 2. The FOIL method is shown on page 3.2.

7. How do the areas of the small rectangles in Problem 2 relate to the expression shown on page 3.2?

You can change the dimensions of the rectangle on page 3.1 by double-clicking on **a**, **b**, **c**, or **d**. Practice finding the area of the rectangle and then check your answers on page 3.2.

8. What is the area of a rectangle with dimensions $(3x + 5)$ and $(6x + 2)$?



9. **a.** $(4x + 1)(3x + 9) =$ **b.** $(x + 8)(7x + 3) =$ **c.** $(2x + (-3))(5x + 8) =$



Homework/Extensions

On page 4.1, there is another opportunity for you to practice finding area. Record your answer to the first problem here. Show each step of your work. Advance to page 4.2 to check your answer.

1. a. $(4x + 2)(x + 7) =$

b. $(3x - 7)(2x + 4) =$

c. $(2x + 5)(6x + 1) =$

d. $(5x + 3)(9x - 2) =$

On pages 5.1 and 5.2, you will be multiplying a trinomial (3 terms) times a binomial (2 terms) to find the area of a rectangle.

2. What method can you use to find the simplified expression for the area?

3. Use the letters **a**, **b**, **c**, **d**, and **e** to determine the formula used to find the 6 terms of area shown on page 5.2.

4. What is the area of the rectangle with dimensions $(1x^2 + 3x + 4)$ and $(5x + 6)$?

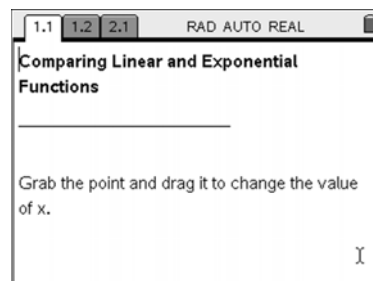
5. a. $(2x^2 + 1x + 7)(3x + (-6)) =$

b. $(4x^2 + 3x + 8)(x + 3) =$

c. $(2x^2 + 6x + 4)(-3x + 9) =$

Comparing Linear and Exponential Expressions STUDENT ACTIVITY

This activity explores the values of the expressions $3x$ and 3^x as x changes from 0 to 5. You will compare the two expressions by investigating patterns in how their values change both in a table and graphically.



- **Open the TI-Nspire document** *Compare_Linear_Exponential*.
- **Move to page 1.2 to begin the lesson.**

1. Complete the table below by moving the point. Which column is growing faster?

x	$3x$	3^x
0		
1		
2		
3		
4		
5		

2. a. As x increases from 2 to 3 in the table, how does the value of $3x$ change?

b. As x increases by 1 in the table, describe the pattern you notice in the numbers in the $3x$ column of the table.

3. a. As x increases from 2 to 3 in the table, how does the value of 3^x change?

b. As x increases from 3 to 4 in the table, how does the value of 3^x change?

Comparing Linear and Exponential Expressions STUDENT ACTIVITY

c. As x increases by 1 in the table, describe the pattern you notice in the numbers in the 3^x column of the table.

4. Complete the bottom row of the table in for $x = 6$. How did you determine the values for $3x$ and 3^x ?

5. Why are the values for 3^x increasing faster than the values for $3x$?

6. The function $f(x) = 3^x$ is called an *exponential function*, while the function $f(x) = 3x$ is a *linear function*. Describe the difference in the two functions.

➤ **Move to page 2.1 and explore the graphs of $f(x) = 3x$ and $f(x) = 3^x$**

7. Drag the point on the arrow to the right to produce two graphs- one solid, one dashed. Use the information from the table to identify which graph represents an exponential function and which graph represents a linear function. Justify your answer.

8. How do the graphs of $f(x) = 3x$ and $f(x) = 3^x$ support your response to #5?

9. Aaron says that the values of $f(x) = 5^x$ will increase faster than the values of the linear function $f(x) = 5x$. Do you agree or disagree? Support your answer.

Notes

Notes