

## **Tapia Camp Calculus Course Evaluation Report**

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### **Preparing Success in Calculus**

High school students from the Houston Independent School District (HISD) representing more than 10 HISD high schools participated in a three-week Calculus course on the Rice University campus between July 10 - 28, 2023. Dr. Papakonstantinou, clinical professor of mathematics and director of the Rice University School Mathematics Project (RUSMP) taught the course entitled "Preparing for Success in Calculus."

More than 20 students convened in the Humanities building on Rice campus in a room that had an ideal setup for collaborative and active learning from 9:00 a.m. – noon, Mondays through Fridays for three weeks (except for the first Wednesday when students had a whole-day field trip to NASA—Space Center Houston). See Appendix A for the Calendar of Topics. Dr. Papakonstantinou also held office hours every morning before class (except for the first day) from 8:15 a.m. – 9:00 a.m. All course materials used during the course were provided to students as hard copies in a binder. An electronic version of the same materials as well as additional resources (useful web links, readings, videos, sample A. P. Calculus test items, I. B. Mathematics test items, information about the A.P. Calculus exams) were shared with students in a Google Drive folder.

### Pre- and Post-Survey and Post-Subject-Matter Test

Students were administered the short form of Attitudes Toward Mathematics Inventory (ATMI; Lim & Chapman, 2013) before 9 AM on the first day of the course (July 10) as a presurvey. ATMI is a previously developed and validated scale that is composed of 19 five-point Likert-scale items (1=strongly disagree to 5=strongly agree) related to four areas (i.e., constructs): enjoyment in doing mathematics, motivation towards mathematics, self-confidence in mathematics, and value (outcome expectancy; Eccles, 2009) of mathematics. Each item had a statement for which students stated the degree of their agreement. Twenty-three students took the pre-survey. On the last day of the course (July 28), students took the same survey with the additional three open-ended questions regarding their personal evaluation of the course. See Appendix B for the surveys. Twenty-one students completed the survey as two students dropped out after the first week due to transportation problems to Rice campus. Students took these surveys on paper. Their responses were then transferred to an electronic data sheet for analysis by Rice student research assistants.

Students were also administered a paper-pencil subject-matter test on the last day of the course. The subject-matter test had seven multi-part written-response questions. Students had about an hour and a half to complete their subject-matter tests. The 21 test papers were graded by Dr. Papakonstantinou. Grades ranged from 79 to 98. There were 13 grades in the 90s, 6 in the 80s, and two in the very high 70s.

#### **Survey Results**

To compare the scores for student attitudes towards mathematics on the pre-survey and post-survey, paired-samples *t*-test was conducted on each item and on each of four constructs that were composed of four to five items. Enjoyment, self-confidence, and value constructs had five items each; and the motivation construct had four items. Self-efficacy items have reverse

statements—agreement with the statement would imply low-self-confidence and disagreement would imply high-self-confidence. Before the item-by-item analysis and aggregate analysis (by construct), self- confidence items are reverse-coded so that the higher and lower scores would mean higher and lower self-confidence, respectively.

Item by item analysis indicates that students had more positive attitudes at the end of the Calculus course than they had at the onset of the course. More specifically, students had higher agreements with 17 out of the 19 positive statements on the post-survey than they did in the presurvey (see Table 1).

### Table 1.

Item by item analysis of ATMI statements from the onset and end of the course.

| S4-4- 4                                                             | Correct t    | Pre-                         | Post-                        | <b>™</b> ⊺                       | Difference                            |  |
|---------------------------------------------------------------------|--------------|------------------------------|------------------------------|----------------------------------|---------------------------------------|--|
| Statement                                                           | Construct    | Survey                       | Survey                       | N                                | (post-pre)                            |  |
| I have usually enjoyed studying mathematics in school.              | Enjoyment    | 3.90                         | 4.24                         | 21                               | 0.33                                  |  |
| I like to solve new problems in                                     | Enjoyment    | 4.10                         | 4.33                         | 21                               | 0.24                                  |  |
| mathematics.                                                        | Lijoyment    | 4.10                         | 4.55                         | 21                               | 0.24                                  |  |
| I really like mathematics.                                          | Enjoyment    | 3.80                         | 4.35                         | 20                               | 0.55                                  |  |
| I am happier in a mathematics class than                            | Enjoyment    | 3.14                         | 3.62                         | 21                               | 0.48                                  |  |
| in any other class.                                                 |              |                              |                              |                                  |                                       |  |
| Mathematics is a very interesting subject.                          | Enjoyment    | 4.29                         | 4.57                         | 21                               | 0.29                                  |  |
| I am confident that I could learn advanced                          | Motivation   | 4.14                         | 4.38                         | 21                               | 0.24                                  |  |
| mathematics.                                                        |              |                              |                              |                                  |                                       |  |
| I am willing to take more than the                                  | Motivation   | 4.48                         | 4.57                         | 21                               | 0.10                                  |  |
| required amount of mathematics.                                     |              | 4.20                         | 4.25                         | 20                               | 0.05                                  |  |
| I plan to take as much mathematics as I                             | Motivation   | 4.30                         | 4.35                         | 20                               | 0.05                                  |  |
| can during my education.<br>The challenge of mathematics appeals to | Motivation   | 3.95                         | 4.15                         | 20                               | 0.20                                  |  |
| me.                                                                 | Wouvation    | 5.95                         | 4.15                         | 20                               | 0.20                                  |  |
| Studying mathematics makes me feel                                  | Self-        | 3.29                         | 3.62                         | 21                               | 0.33                                  |  |
| nervous.                                                            | Confidence   | 5.27                         | 5.02                         | 21                               | 0.55                                  |  |
| I am always under a terrible strain in a                            | Self-        | 3.38                         | 3.67                         | 21                               | 0.29                                  |  |
| mathematics class.                                                  | Confidence   |                              |                              |                                  |                                       |  |
| It makes me nervous to even think about                             | Self-        | 3.81                         | 4.14                         | 21                               | 0.33                                  |  |
| having to do a mathematics problem.                                 | Confidence   |                              |                              |                                  |                                       |  |
| I am always confused in my mathematics                              | Self-        | 3.48                         | 3.86                         | 21                               | 0.38                                  |  |
| class.                                                              | Confidence   |                              |                              |                                  |                                       |  |
| •                                                                   |              | 3.24                         | 3.67                         | 21                               | 0.43                                  |  |
|                                                                     |              |                              |                              |                                  |                                       |  |
| •                                                                   | Value        | 4.71                         | 4.76                         | 21                               | 0.05                                  |  |
| • •                                                                 | Value        | 1 24                         | 1 20                         | 21                               | 0.05                                  |  |
|                                                                     |              |                              |                              |                                  |                                       |  |
| ±                                                                   | <i>varue</i> | т <b>.</b> т <i>.</i> ј      | т.ЈЈ                         | <i>4</i> 1                       | 0.10                                  |  |
|                                                                     |              | 3.24<br>4.71<br>4.24<br>4.43 | 3.67<br>4.76<br>4.29<br>4.33 | 21<br>21<br>21<br>21<br>21<br>21 | 0.38<br>0.43<br>0.05<br>0.05<br>-0.10 |  |

| Statement                                                                                           | Construct | Pre-<br>Survey | Post-<br>Survey | N  | Difference<br>(post-pre) |
|-----------------------------------------------------------------------------------------------------|-----------|----------------|-----------------|----|--------------------------|
|                                                                                                     |           | v              | e e             |    | u i /                    |
| College mathematics lessons would be<br>very helpful no matter what I decide to<br>study in future. | Value     | 4.33           | 4.38            | 21 | 0.05                     |
| A strong mathematics background could help me in my professional life.                              | Value     | 4.71           | 4.57            | 21 | -0.14                    |

Examination of the aggregated scores on four constructs revealed that enjoyment of and self-confidence in doing mathematics significantly improved by participating in the Calculus course (see Table 2). The effect size for these statistically significant improvements is moderate to high with as indicated by the Cohen's *d* values ranging from 0.43 to 0.66.

|                     | Time | Time 1 (Pre) |      | 2 (Post) | _                    |       |      |    |                  |
|---------------------|------|--------------|------|----------|----------------------|-------|------|----|------------------|
| Construct           | Μ    | SD           | М    | SD       | Mean ∆<br>(post-pre) | t     | р    | df | Cohen's <i>d</i> |
| Motivation          | 4.21 | 0.67         | 4.38 | 0.93     | 0.16                 | 1.07  | .15  | 20 | 0.23             |
| Enjoyment           | 3.86 | 0.79         | 4.22 | 0.89     | 0.36**               | 3.04  | <.01 | 20 | 0.66             |
| Self-<br>Confidence | 3.44 | 0.96         | 3.79 | 0.98     | 0.35*                | 1.99  | .03  | 20 | 0.43             |
| Value               | 4.49 | 0.62         | 4.47 | 0.94     | -0.02                | -0.18 | .43  | 20 | -0.04            |

Table 2.

Open responses indicate the positive impact that this course had on the students. In terms of the extent of how this course met students' needs, many students referred to the foundational aspect of this course and its value for their future mathematics courses. They believed this course gave them a "head start" and provided a good foundation and "preparation" for their future calculus classes in high school. Samples quotes from students' open responses are as follow:

"This calculus course met my needs by providing a great introduction into calculus that puts me ahead for the school year. This course helped me feel confident and prepare me for my calculus course in school."

"[This course] helped get a head start on a very important math subject."

"I found that my pre-calculus course was not sufficient enough to give me a proper foundation for calculus. Dr. Papa's class helped me realize that I have many basics to practice to ensure my success in calculus."

Several students also confirmed how much they "learned" and the extent of "resources" for everyone as the following sample quotes indicate:

"... I was able to learn so much additional math."

"[The course had] [g]ood pacing, lots of resources, [and] good teaching style."

"Dr. Papa provided me with extensive notes for precalculus for me to catch up and multiple links for me to practice pre-calculus, calculus, and some geometry for my future IB class."

Some students offered insights into other aspects of the course such as productive struggle (Hiebert & Grouws, 2007), wide range of applications and deep understanding rather than a focus on rote memorization, and motivation. The following quotes attest to these aspects:

"The calculus course helped my need of branching out, and really pushing my limits and knowledge when it comes to calculus."

"This course exceeded my needs as a high school student, because the class isn't just forcing kids to memorize fact but understand and apply to our lives."

"This course helped me by getting a deeper understanding of what calculus really means and how to really explain mathematics in the correct way. Helped me get a head start and feeling prepared to take cal[culus] in the fall."

"It helps me realize that this (calculus) is what I truly live for. I get to learn lots of things in a short [amount] of time is actually impressive."

Students were also asked about their perceptions of the positive aspects of the course. In their responses, most students noted the positive learning environment that provided "collaboration," formation of a "community" of learners, ownership in their learning, and "hands-on learning." A student eloquently summarizes the perceptions of the learning environment and norms set in this course "...*the lovely atmosphere made learning calculus such a delight.*" Another student's perception is also a representative of the overall perceptions of the learning environment:

"I enjoyed that the course was very collaborative and the students were given the time to work out problems and proofs on our own so we could have the "A-ha" moment."

When responding to positive aspects of the course, several students pointed out the instructional quality of the course. The following verbatim phrases of students to describe the instruction verify the high quality of instruction and the effective constructivist approach to teaching (Windschitl, 2002) in this course: "learning mathematical language," "deep understanding," "real-world application of calculus," "critical thinking," and "interactive lessons." One student appreciated "get[ting] to know the meaning behind all those rules."

Another common theme in students' perceptions of the positive aspects of the calculus course was the "teacher" factor and teacher characteristics. They pointed out the instructor's enthusiasm and caring for every student. They also indicated the importance of the "*teacher's and teaching assistants' knowledge*" and "*willingness*" to help students and in their willingness to make learning calculus effective and enjoyable. The following two quotes clearly indicate the importance of the "teacher" in student learning:

"... our professor Dr. Papa, she made sure we always felt smart and is a wonderful teacher to have."

"I've also loved Dr. Papakonstantinou's determination to help us fully understand this material."

Lastly, students also commented about the aspects that can be improved in the course. First of all, the majority of the students stated their desire for a longer course in terms of the days/weeks (e.g., 4 fours weeks instead of three weeks). The rest of the comments were related to the daily structure of the course except for one comment that indicated a need for "*[m]ore fun activities/games that are related to calculus*..." Regarding the daily schedule one student's response—"*shorten[ing] the duration of the math class*" in the day—is in contrary to another student's: "*I would prefer if this class took more of the day*." One student recommended hourly breaks to make learning more effectively for students. Lastly, another student stated a need for "*more assistance with [their] work outside of class*."

### References

- Eccles, J. S. (2009). Who am I and what am I going to do with my life? Personal and collective identities as motivators of action. *Educational Psychologist*, 44, 78–89.
- Hiebert, J., & Grouws, D. A. (2007). The effects of classroom mathematics teaching on students' learning. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 371–404). Charlotte, NC: Information Age.
- Lim, S. Y., & Chapman, E. (2013). Development of a short form of the attitudes toward mathematics inventory. *Educational Studies in Mathematics*, 82(1), 145–164.
- Windschitl, M. (2002). Framing constructivism in practice as the negotiation of dilemmas: An analysis of the conceptual, pedagogical, cultural, and political challenges facing teachers. *Review of Educational Research*, 72(2), 131–175.

# Appendix A

**Preparing for Success in Calculus** 

July, 2023 (9:00 a.m. – noon) at Rice University (Humanities 117)

Office hours (8:15 a.m. - 8:45 a.m.) in Humanities 117 before class each day beginning July 11

Dr. Anne Papakonstantinou Email: apapa@rice.edu RUSMP Office: 713-348-6076 Instructor:

| lay       |      |                                                                                        |    |                                                                                                                                                                                                                                             |    |                                                                                                                                |
|-----------|------|----------------------------------------------------------------------------------------|----|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----|--------------------------------------------------------------------------------------------------------------------------------|
| Saturday  |      |                                                                                        |    |                                                                                                                                                                                                                                             |    |                                                                                                                                |
| Friday    | 14   | One-sided limits<br>Infinite limits<br>Limits at infinity                              | 21 | Chain Rule<br>Derivatives of the 6 trig<br>functions<br>L'Hôpital's Rule<br>for $0/0$ and $\infty/\infty$                                                                                                                                   | 28 | Analysis of Functions and<br>their Graphs<br>Guess My Graph<br>Multiple Choice A.P. items                                      |
| Thursday  | 13   | Evaluating limits algebraically,<br>pictorially, & for piece-wise<br>defined functions | 20 | Differentiation Rules (power,<br>sum, difference, product,<br>quotient)                                                                                                                                                                     | 27 | Analysis of Functions<br>and their Graphs                                                                                      |
| Wednesday | 12   | NASA Field Trip                                                                        | 19 | Equivalent forms of the<br>derivative<br>Find <i>dy/dx</i> using definition<br>Find the derivative from tables<br>Find the derivative from graphs<br>(A.P. item on limits, continuity,<br>rates of change, definition of the<br>derivative) | 26 | Analysis of Functions<br>and their Graphs                                                                                      |
| Tuesday   | 11   | Definition of limit of a<br>function<br>ε-δ definition<br>Properties of Limits         | 18 | Limits of trig functions Continuity and Limits<br>Squeeze Theorem Intermediate Value Theorem Definition of Continuity Graphic representation of the derivative of a function Average vs instantaneous rates of change                       | 25 | Analysis of Functions<br>and their Graphs                                                                                      |
| Monday    | 10   | Parent Functions<br>Transformations of<br>Functions<br>Trig Facts                      | 17 | Limits of trig functions<br>Squeeze Theorem<br>Definition of Continuity<br>Examples for Continuity                                                                                                                                          | 24 | Relationship between<br>differentiability &<br>continuity<br>(including proof that<br>differentiability implies<br>continuity) |
| Sunday    | July |                                                                                        |    |                                                                                                                                                                                                                                             |    |                                                                                                                                |

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## Appendix B

## Tapia Camp Calculus Course Evaluation Pre-and Post Survey Items

ID:

## How much do you agree with the following statements?

|                                                                                               | Strongly disagree | Somewhat disagree | Neither<br>agree or<br>disagree | Somewhat<br>agree | Strongly<br>agree |
|-----------------------------------------------------------------------------------------------|-------------------|-------------------|---------------------------------|-------------------|-------------------|
| I have usually enjoyed studying mathematics in school.                                        |                   |                   |                                 |                   |                   |
| I am confident that I could learn advanced mathematics.                                       |                   |                   |                                 |                   |                   |
| Studying mathematics makes me feel nervous.                                                   |                   |                   |                                 |                   |                   |
| Mathematics is a very worthwhile and necessary subject.                                       |                   |                   |                                 |                   |                   |
| I like to solve new problems in mathematics.                                                  |                   |                   |                                 |                   |                   |
| I am willing to take more than the required amount of mathematics.                            |                   |                   |                                 |                   |                   |
| I am always under a terrible strain in a mathematics class.                                   |                   |                   |                                 |                   |                   |
| Mathematics is important in everyday life.                                                    |                   |                   |                                 |                   |                   |
| I really like mathematics.                                                                    |                   |                   |                                 |                   |                   |
| I plan to take as much mathematics as I can during my education.                              |                   |                   |                                 |                   |                   |
| It makes me nervous to even think about having to do a mathematics problem.                   |                   |                   |                                 |                   |                   |
| Mathematics is one of the most important subjects for people to study.                        |                   |                   |                                 |                   |                   |
| I am happier in a mathematics class than in any other class.                                  |                   |                   |                                 |                   |                   |
| The challenge of mathematics appeals to me.                                                   |                   |                   |                                 |                   |                   |
| I am always confused in my mathematics class.                                                 |                   |                   |                                 |                   |                   |
| College mathematics lessons would be very helpful no matter what I decide to study in future. |                   |                   |                                 |                   |                   |
| Mathematics is a very interesting subject.                                                    |                   |                   |                                 |                   |                   |
| I feel a sense of insecurity when attempting mathematics.                                     |                   |                   |                                 |                   |                   |
| A strong mathematics background could help me in my professional life.                        |                   |                   |                                 |                   |                   |

Date:

## Tapia Camp Calculus Course Evaluation Post-Survey Poen-Ended Questions

How did this calculus course meet your needs?

What are three positive aspects of this calculus course?

What could be done to improve this calculus course?