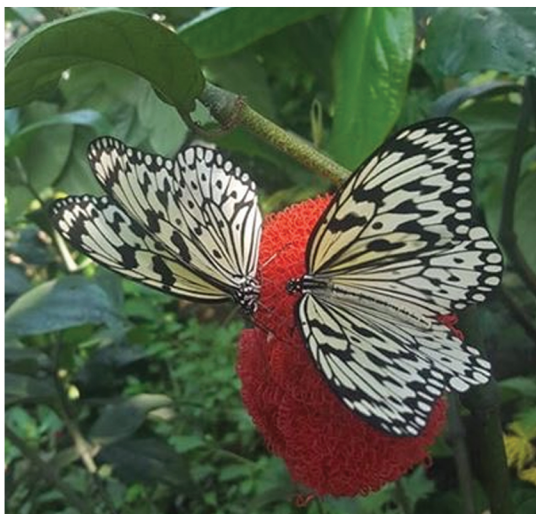


Texas Mathematics Teacher

Volume LXIV Issue 1

Spring/Summer 2018

Find the Mathematics...



...in Butterflies!

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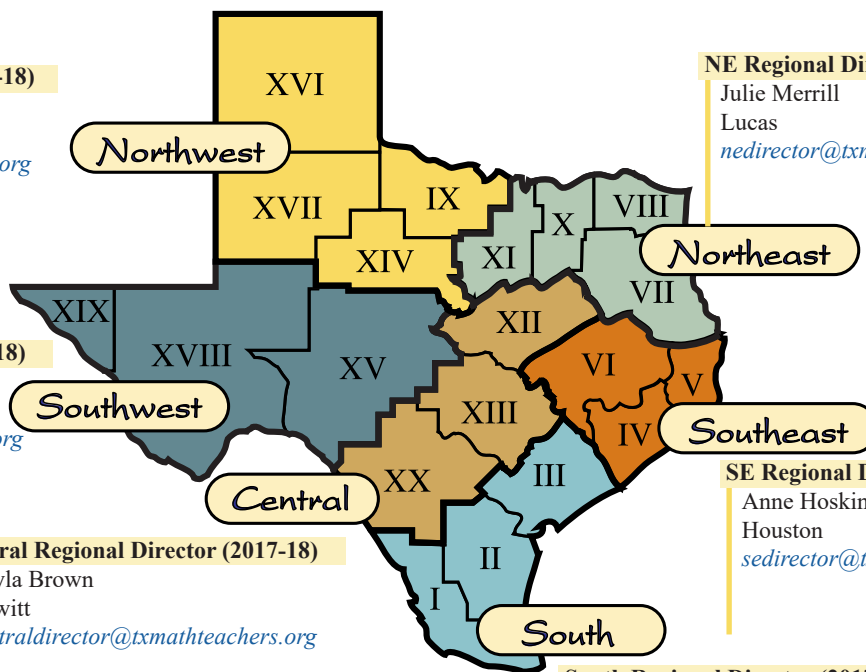
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Texas Mathematics Teacher

A PUBLICATION OF THE TEXAS COUNCIL OF TEACHERS OF MATHEMATICS

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All applications (including TCTM membership) are available online at:
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From the Editors

In this issue you will find the latest research-based practices, ready to use classroom activities, ideas for implementing technology, and support for implementing effective teaching strategies. Additionally, you will find legislative and local affiliate updates. We hope you will find the issue beneficial as you lead in mathematics education across Texas to support student learning. We thank the authors, reviewers, assistant editor, graphics designer, and editorial panel for their efforts to ensure a quality resource for your use. We invite you to send in your ideas to editor@txmathteachers.org for future issues! Check the website www.tctmonline.org for details on submission guidelines.

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Trena L. Wilkerson, Editor
Rachelle M. Rogers, Associate Editor



Letter from the President

Dear TCTM Colleagues,

TCTM leadership has worked hard this past year to elevate the level of communication with members. Each month, an eBlast newsletter is sent by the president via email to current members. If you have not received this email, be sure to check your spam/junk folder or you might verify that your membership is current. The email is sent from president@txmathteachers.org, so be sure to add this to your "safe senders" list. In addition, TCTM has a FaceBook page and Twitter account (@txmathteachers), so be sure to "Like" our page and "Follow" our tweets! This will provide the most up to date announcements, alert to special opportunities, and helpful resources. The Regional Directors will be contacting members in their region, so expect to receive an email from them as well. You can find your regional director information on the inside cover of this issue of TMT. They may host events in your area, so please contact them if you feel you are not receiving information.

In addition, current members have received login information to access the current issue and all archived issues, available on the TCTM web site. It will be important to keep your membership current so that you do not miss a single issue!

If you have colleagues who are not members, encourage them to join so that they can enjoy the wonderful resources included in each and every issue of TMT. If you are experiencing any issues with login information, please contact the membership director.

Be sure to make your plans now for participation in CAMT 2018, which will be held in Houston, and encourage a team from your school to attend. TCTM hosts a special reception just for members and we have a booth in the exhibit area. This is where you can interact directly with your TCTM leadership, so we hope to see you there! More details about events at CAMT will be sent via eBlasts, posted on Facebook, and tweeted on Twitter!

All my best for a great end of school year!

Sandi Cooper
President, 2016 - 2018

Website: www.txmathteachers.org
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TCTM Leader Spotlight

Each year since 1995, TCTM has accepted nominations for two awards for leaders in our professional community. The TCTM Leadership Award is presented to a TCTM member who is nominated by a TCTM affiliate. The second award, the E. Glenadine Gibb Achievement Award, is presented to someone nominated by a TCTM member. The following individuals have been honored and we wish to acknowledge their former and ongoing contributions this year in the leader spotlight. If you wish to nominate someone for 2019, please download the forms from our website. **Please submit your nomination by Dec. 31, 2018.** The award recipients for 2018 will be recognized at the TCTM business meeting at CAMT 2018. See the program for time and place of the meeting.

Our prior awardees are:

Year	Leadership(local/state)	Gibb (state/national)	Year	Leadership(local/state)	Gibb (state/national)
1995	Mary Alice Hatchett	Iris Carl	2007	Kathy Hale	Cynthia L. Schneider
1996	Betty Forte	Cathy Seeley	2008	Jim Wohlgeheagen	Juanita Copley
1997	Diane McGowan	Pam Chandler	2009	Jane Silvey	Jo Ann Wheeler
1998	----	----	2010	Elaine Young	Paula Steffen Moeller
1999	Linda Shaub	Eva Gates	2011	Beverly Burg Anderson	Jennie M. Bennett
2000	Lloy Lizcano	Bill Hopkins	2012	Paul Gray, Jr.	Linda Gann
2001	Susan Hull	Pam Alexander	2013	Vodene Schultz	Anne Papakonstantinou
2002	Janie Schielack	Judy Kelley	2014	Caren Sorrells	Noemi Rodriguez-Lopez
2003	Bonnie McNemar	Dinah Chancellor	2015	Jennifer Hylemon	Bea Luchin
2004	Dixie Ross	Jacqueline Weilmuenster	2016	Robb Wilson	Trena Wilkerson
2005	Barbara "Basia" Hall	Barrie Madison	2017	Anita Hopkins	James A. M. Alvarez
2006	Nancy Trapp	Lois Gordon Moseley			

Apply now for a MET Grant, Scholarship, or Award!

NCTM's Mathematics Education Trust (MET) channels the generosity of contributors through the creation and funding of grants, awards, honors, and other projects that support the improvement of mathematics teaching and learning.

MET provides funds to support classroom teachers in the areas of improving classroom practices and increasing mathematical knowledge. MET also sponsors activities for prospective teachers and NCTM Affiliates, as well as recognizing the lifetime achievement of leaders of mathematics education. Grant, scholarship, and award funding ranges from \$1,500 to \$24,000 and can be used for

conferences, workshops, seminars; research and in-service training in mathematics coursework; or professional development activities. There are various deadlines throughout the year, so be sure to check the NCTM website at www.nctm.org/Grants/ for specific dates. The next two upcoming deadlines are May 4, 2018 and November 2, 2018.

If you are a teacher, prospective teacher, or school administrator and would like more information about MET grants, scholarships, and awards, please visit their website at www.nctm.org/Grants/.

Affiliate Groups

These are local affiliated groups in Texas. If you are actively involved with them, please send future meeting and conference information to Trena Wilkerson at editor@txmathteachers.org so we may publicize your events. Contact information for each group is also available on the NCTM website, www.nctm.org. Contact information for regional directors is located on the inside front cover of this publication.

NORTHWEST REGION *Service Centers 9, 14, 16, 17*
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Texas South Plains CTM

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NORTHEAST REGION *Service Centers 7, 8, 10, 11*
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SOUTHWEST REGION *Service Centers 15, 18, 19*
Christopher Hiatt, Regional Director

Greater El Paso CTM

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SOUTHEAST REGION *Service Centers 4, 5, 6*
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Fort Bend CTM

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SOUTH TEXAS REGION *Service Centers 1, 2, 3*
Faye Bruun, Regional Director

The South Texas Region is on Project Share! The group is "Texas Council of Teachers of Mathematics: South Region."

Coastal CTM

Contact: Faye Bruun, faye.bruun@tamucc.edu
or visit sci.tamucc.edu/MATH/CTM/

ME by the SEa, a conference for k-12 math and science educators, is Friday, June 15, 2018 at Texas A&M Corpus Christi, sponsored by the Coastal Council of Teachers of Mathematics. You can register at me.tamucc.edu

CTM @ Texas A&M University at Corpus Christi (Student Affiliate)

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Kayla Brown, Regional Director

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Alamo District CTM

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Central Texas CTM

Contact: President of CTCTM Arash Abnoussi, aabnoussi@esc12.net, or visit www.ctctm.org
(CTCTM) 2018 Fall Meeting TBD.



*It is impossible to be a
mathematician without
being a poet in soul.*

~ Sofia Kovalevskaya (1850–1891)

STATEWIDE

Texas Association of Supervisors of Mathematics (TASM)

TxASMAOnline@gmail.com, or visit www.tasmonline.net

The Association of Mathematics Teacher Educators of Texas (AMTE-TX)

Contact Sarah Quebec-Fuentes at: s.quebec.fuentes@tcu.edu,
or visit www.amte-tx.org

NATIONAL

National Council of Teachers of Mathematics (NCTM)

visit: nctm.org

National Council of Supervisors of Mathematics (NCSM)

visit www.mathedleadership.org

Community-Building Discourse Practices in Math Workshop

“Implementing math workshop is complex and begins with the understanding that every student has the potential to understand mathematics conceptually.”

Have you ever wondered what a mathematical community would look like in an elementary classroom? What if the majority of interactions within this mathematical community encompassed rich, mathematical discourse practices that promote deep learning? According to Hattie, Fisher, and Frey (2017), mathematical discourse extends beyond discussion and encompasses the different ways that students agree, disagree, represent, talk, and think about mathematics. Specifically, mathematical discourse “includes not only ways of talking, acting, interacting, thinking, believing, reading, writing but also mathematical values, beliefs, and points of view” (Moschkovich, 2003, p. 326). The fundamental goal of mathematical discourse is to transcend traditional procedure- and rule-driven approaches to mathematics instruction and develop conceptual understandings about mathematics among all students (Griffin, League, Griffin, & Bae, 2013). Furthermore, mathematical discourse transforms classrooms into communities where “students hear one another’s ideas and have opportunities to articulate and refine or revise their own,” while also developing “confidence in themselves as mathematical knowers” (Ball, 1993, p. 394).

In 2014, the National Council of Teachers in Mathematics [NCTM] called for teaching practices that “facilitate discourse among students to build shared understanding of mathematical ideas” (p. 29). However, the concepts of ‘community’ and ‘discourse’ are not novel within the realm of mathematics instruction. For over 25 years, much literature has provided evidence supporting the notion that establishing a mathematical community to facilitate discourse practices develops mathematical understandings among elementary-aged students (e.g., Ball, 1993; Hiebert & Wearne, 1993; Prawat, 1989; Wood, Williams, McNeal, 2006). While there are many effective teaching practices that support this type of mathematical community, we propose the use of math workshop.

We believe that teachers could utilize math workshop as an effective strategy for supporting rich discourse to build community. In this article, we provide a description of math workshop and how its structure establishes a mathematical community to facilitate discourse practices among all students. Next, we present three classroom vignettes of elementary teachers who implement math workshop at the 2nd, 3rd, and 4th grade levels in varied ways. We conclude by offering recommendations and resources for teachers who are interested in learning more about math workshop.

What is Math Workshop?

Math workshop is a promising approach that transforms a class of students into “a community of mathematicians” (Hoffer, 2012, p. 5) and addresses all levels of learning (Hattie et al., 2017). According to Hoffer (2012), the structure of math workshop consists of four components:

1. Opening: Students engage with a problem connected to the lesson objective that unlocks prior knowledge.
2. Mini-lesson: The teacher develops student understandings with the lesson objective through the use of verbal explanations, or talk-alouds (Wright, 2014). During the mini-lesson, the teacher may introduce new information, revisit previous information, model thinking, or review strategies for work time (Hoffer, 2012).
3. Work Time: Students engage with a challenging, collaborative task in small groups that extends their thinking with the lesson objective. The teacher moves among small groups and confers with students.
4. Reflection: At the end of the lesson, students reflect upon their metacognitive understandings with the lesson objective.

The math workshop structure supports a paradigm shift in how mathematics is taught and learned (Hoffer, 2012). In math workshop, the teacher is no longer a presenter of information, but rather a facilitator of learning. Students are no longer passive recipients of information. Instead they “devote the majority of their time to thinking and talking about important mathematical ideas” (p. 3). As students engage with mathematical discourse practices individually and collaboratively, they utilize language, perspectives, and cognitive functions that are rooted in unique cultural and social identities (Moschkovich, 2007). In other words, understandings and contributions towards the co-construction of knowledge in math workshop are influenced and shaped by each student’s unique “ways of behaving, interacting, valuing, thinking, believing, speaking, and often reading and writing” (Gee, 2012, p. 3).

Implementing math workshop is complex and begins with the understanding that every student has the potential to understand mathematics conceptually (Hoffer, 2012). Therefore, the most critical aspect of designing a math workshop lesson is the third component, Work Time task. While planning for this task, the teacher must consider the learning levels of each student and incorporate differentiated tasks based upon learning needs. Work Time tasks must align with the lesson objective and promote exploration, sense-making, and the application of mathematical procedures (Hattie et al., 2017). The primary goal of the Work Time task is to present students with challenging and rigorous problems that promote engagement and persistence through the use of productive discourse practices in a supportive mathematical community (Boaler, 2016).

Classroom Vignettes

Tessa Delgado, Darla Crockett, and Monica Campbell (names were changed to pseudonyms to ensure anonymity) are seasoned teaching professionals affiliated with a school district in North West Texas. These teachers implement math workshop consistently and flexibly to meet the diverse learning needs of their students. We observed Ms. Delgado, Ms. Crockett, and Ms. Campbell implement a math workshop lesson and present the following classroom vignettes.

Ms. Delgado - 2nd Grade

Ms. Delgado has 10 years of teaching experience and employed math workshop for the past two years. The objective for the lesson we observed was: Students will solve problems by connecting repeated addition and subtraction to multiplication and division situations that involve equal groupings and shares. As shown in Table 1, this lesson aligned with grade-level state standards for the following strands: (a) mathematical process standards and (b) number and operations (Texas Education Agency, 2012).

Table 1

2nd Grade Math Workshop Lesson and Alignment with Targeted Grade-Level State Standards

Mathematical Process Standards	The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to: (E) create and use representations to organize, record, and communicate mathematical ideas. (G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.
Number and Operations	The student applies mathematical process standards to connect repeated addition and subtraction to multiplication and division situations that involve equal groupings and shares. The student is expected to: (A) model, create, and describe contextual multiplication situations in which equivalent sets of concrete objects are joined. (B) model, create, and describe contextual division situations in which a set of concrete objects is separated into equivalent sets.

Opening and Mini-lesson: (11 minutes)

Ms. Delgado called students to the carpet at the front of the classroom. She used guided practice to solve a division problem. During this time, Ms. Delgado explained her thinking with talk alouds, facilitated discourse by posing questions and calling on specific students to respond, and created an anchor chart to scaffold student understandings (see Figure 1). Next, Ms. Delgado handed each student a paper with the problem and facilitated a choral reading of the problem. Afterwards, she asked students a few questions to ascertain their understanding of the task (e.g., What's important? What's the question?).

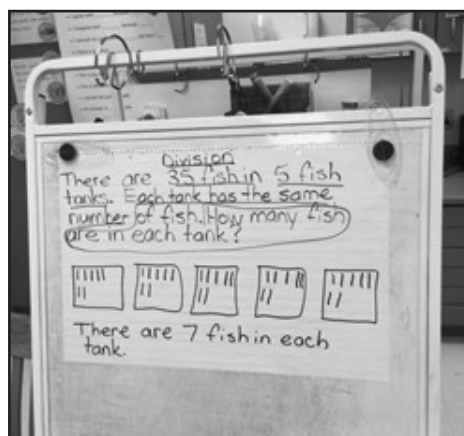


Figure 1. Anchor chart created during Ms. Delgado's math workshop lesson.

Work Time: (43 minutes)

- **Part I: Small Group Work Time (29 minutes)** - Ms. Delgado placed students into pairs, and each pair positioned themselves at various locations around the classroom to work. During this time, students engaged in a variety of discourse practices, such as talking with their partners, writing on their papers, and demonstrating problem-solving approaches on small dry erase boards. While students worked, Ms. Delgado moved among pairs and asked prompting questions (e.g., Okay – what's your plan? How many boxes are there? What do you know? Explain to me what you did and why.). As student pairs completed the task, they retrieved an iPad and created a brief video recording to explain their problem-solving process (see Figure 2 on the following page).

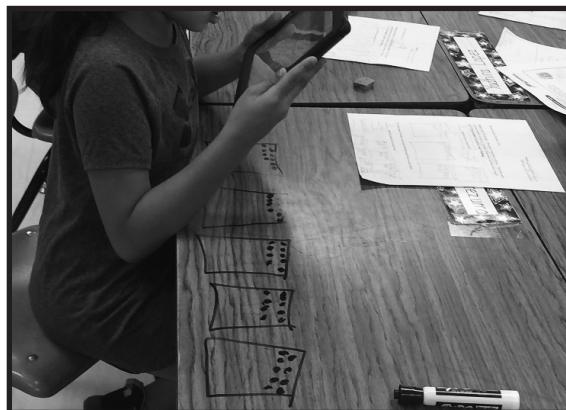


Figure 2. Students in Ms. Delgado's class used iPads to create brief video recordings that explained their problem-solving process during Work Time.

- Part II: Guided Work Time (14 minutes) - Ms. Delgado called students back to the carpet at the front of the classroom. Once students were seated, she played each video recording. Students watched each video and engaged in brief discourse after each viewing. Ms. Delgado led this discourse by asking students who created the video follow-up questions (e.g., Why did you ____?) or posing general questions to the whole group (e.g., Do you agree? Their video was a little different – what do you all think?). After viewing all of the videos, Ms. Delgado reinforced that the purpose of watching the videos was to agree, disagree, and discuss problem-solving strategies used among peers.

Reflection: (6 minutes)

Ms. Delgado asked students to go back to their desks and think about what they learned during the Work Time learning activities. She asked students to check their work and make any necessary revisions.

Ms. Crockett - 3rd Grade

Ms. Crockett has 25 years of teaching experience and employed math workshop for the past two years. The objective for the lesson we observed was: Students will solve problems by analyzing and using attributes of two-dimensional geometric figures. As shown in Table 2, this lesson aligned with grade-level Texas Essential Knowledge and Skills for the following strands: (a) mathematical process standards and (b) geometry and measurement (Texas Education Agency, 2012).

Table 2

3rd Grade Math Workshop Lesson and Alignment with Targeted Grade-Level State Standards

Mathematical Process Standards	<p>The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:</p> <p>(B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution and evaluating the problem-solving process and the reasonableness of the solution.</p> <p>(D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.</p> <p>(E) create and use representations to organize, record, and communicate mathematical ideas.</p>
Geometry and Measurement	<p>The student applies mathematical process standards to analyze attributes of two-dimensional geometric figures to develop generalizations about their properties. The student is expected to:</p> <p>(B) use attributes to recognize rhombuses, parallelograms, trapezoids, rectangles, and squares as examples of quadrilaterals and draw examples of quadrilaterals that do not belong to any of these subcategories.</p> <p>(C) determine the area of rectangles with whole number side lengths in problems using multiplication related to the number of rows times the number of unit squares in each row.</p> <p>(D) decompose composite figures formed by rectangles into non-overlapping rectangles to determine the area of the original figure using the additive property of area.</p>

Opening: (15 minutes)

Ms. Crockett used the document camera to project an image of a bird and potted flower that were formed with a variety of two-dimensional shapes. She called on several students and asked them to go to the projected image, point to a specific shape and identify the name of the shape. As students identified shapes, Ms. Crockett used talk alouds to provide information and simultaneously created an anchor chart that listed each shape and their respective number of sides.

Mini-lesson: (8 minutes)

Ms. Crockett explained to students that they were going to use some of the shapes on the anchor chart to calculate area. She handed each student an activity sheet that presented a word problem describing a scenario of tiling three rooms in a house. The activity sheet also contained a diagram of these rooms and included some measurements for the length and width of each room. Ms. Crockett asked students to read the problem to themselves for two minutes and think about what they know and what they need to know, to find the missing room dimensions. After two minutes, Ms. Crockett asked students to get into groups of three and solve the problem. As students moved to separate spaces throughout the classroom, she also provided each student with a paper that had five shapes drawn on it: an octagon, hexagon, pentagon, square, and triangle. She encouraged students to cut out the shapes and use them as models.

Work Time: (70 minutes)

- Part I: Small Group Work Time (28 minutes) - As students worked in their small groups, they engaged in a variety of discourse practices (see Figure 3). While students worked, Ms. Crockett circulated through the classroom and conferred with each group by asking prompting questions (e.g., Which room would it be easiest to find the area of first? What information do you have for each room? How do you calculate area?).

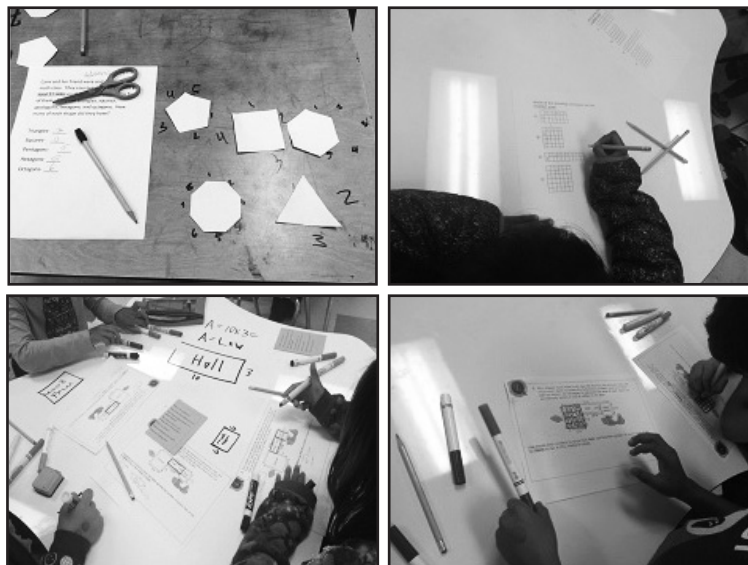


Figure 3. Small group work time in Ms. Crockett's 3rd grade classroom

- Part II: Guided Work Time (12 minutes) - Ms. Crockett noticed several groups were unable to recall how to find missing numbers, which prevented them from advancing forward in their thinking. Therefore, she asked students to come back together as a whole group and provided guided instruction regarding missing numbers. During this time, she asked prompting questions (e.g., How can we solve for missing numbers?), modeled how to create an equation, and asked each group to share strategies they were using.
- Part III: Small Group Work Time (7 minutes) - Ms. Crockett asked students to go back to their small group work areas. Once students were repositioned, she asked each small group to leave their work-in-progress in a visible location. Ms. Crockett then conducted a gallery walk. During this time, each small group rotated to the work area of other small groups to review their work for one minute. While reviewing the work of their peers, small group members exchanged verbal discourse related to their observations (e.g., Oh – this is their evidence. This is wrong – I think they're guessing; Look – $7+5$ is wrong.)
- Part IV: Small Group Work Time (23 minutes) – After the gallery walk, Ms. Crockett asked students to report back to their small group work area and resume solving the problem.

Reflection: (7 minutes)

Ms. Crockett got the attention of the whole class and asked, "Has anyone found a solution?" Ms. Crockett called on two students to explain their solution.

Ms. Campbell – 4th Grade

Ms. Campbell has 16 years of teaching experience and employed math workshop for the past three years. The objective for the lesson we observed was: Students will solve problems by organizing, displaying, and interpreting data in dot plots. As shown in Table 3, this lesson aligned with grade-level Texas Essential Knowledge and Skills for the following strands: (a) mathematical process standards and (b) data analysis (Texas Education Agency, 2012).

Table 3

4th Grade Math Workshop Lesson and Alignment with Targeted Grade-Level State Standards

Mathematical Process Standards	<p>The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:</p> <p>(B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution.</p> <p>(D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.</p> <p>(E) create and use representations to organize, record, and communicate mathematical ideas.</p>
Data Analysis	<p>The student applies mathematical process standards to solve problems by collecting, organizing, displaying, and interpreting data. The student is expected to:</p> <p>(A) represent data on a frequency table, dot plot, or stem-and-leaf plot marked with whole numbers and fractions.</p> <p>(B) solve one- and two-step problems using data in whole number, decimal, and fraction form in a frequency table, dot plot, or stem-and-leaf plot.</p>

Opening: (10 minutes)

Ms. Campbell used the document camera to display the opening problem, which was a dot plot with two questions. Students responded to the questions individually and then discussed their answers with their table groups, which consisted of three to four students.

Mini-lesson: (32 minutes)

Ms. Campbell handed each student daily notes that included important information related to the daily lesson, and asked them to read the first problem silently (see Figure 4). As students read independently, she reminded them to consider their schema and think about how they would solve the problem. After two minutes, Ms. Campbell asked students to talk within their table groups about what they know about dot plots. After a few minutes, Ms. Campbell asked several students to share their thinking with the whole group. Next, Ms. Campbell used the document camera to review the daily notes with students. During this time, she explained information using talk alouds and modeled mathematical processes with writing.

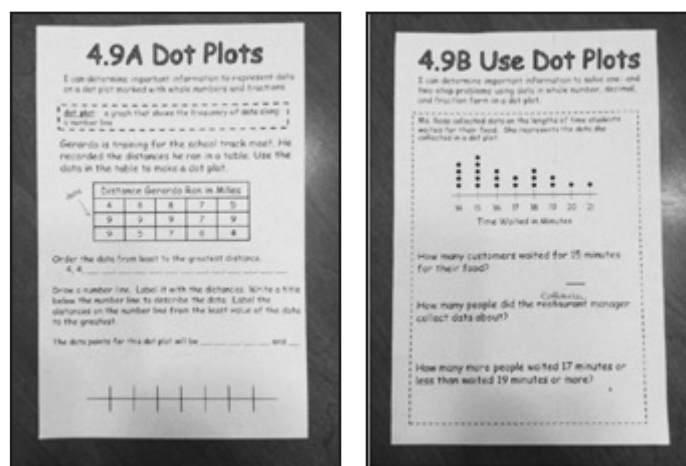


Figure 4. Daily notes that Ms. Campbell used during the mini-lesson in math workshop.

Work Time: (42 minutes)

Ms. Campbell handed each student a paper, then asked them to work in their table groups to solve the problems. As students worked in their table groups, they read the problem aloud, asked each other questions, talked about potential problem-solving strategies, and used small dry erase boards to demonstrate mathematical processes. During this time, Ms. Campbell circulated through the classroom and conferred with each group by asking prompting questions (e.g., What are you struggling with?), questioning their problem-solving strategy (e.g., You combined your data – do you need to combine it?), or commenting on information students shared (e.g., Oh – so you had trouble with counting. It's not that you don't understand – you just need to slow down.)

Reflection: (6 minutes)

Ms. Campbell facilitated a work time debriefing by posing the following question to students: Think about your ah-ha's. As you worked with dot plots today, what were ah-ha's that you had? Ms. Campbell then called on several students to share with the whole group.

Each classroom vignette we presented demonstrates different ways that these elementary teachers used math workshop to establish a mathematical community among their students. Within this community, the teacher and students utilized a variety of discourse practices to promote the development of individual and shared understandings. Moreover, these classroom vignettes illustrated how assessment fits into the math workshop approach, particularly during Work Time. As students engage with challenging and complex tasks with peers, conferring provides teachers with authentic formative assessment information that reveal the strengths, limitations, and struggles of each student (Hoffer, 2012). As a result, the teacher is equipped to plan subsequent lessons that support the different learning needs of each student.

We would also like to point out that the amount of time each teacher invested with the four math workshop components (i.e., Opening, Mini-lesson, Work Time, Reflection) varied by the amount of daily instructional time, as well as the learning needs of students. More importantly, we want to emphasize that the majority of math workshop time was dedicated to learning activities implemented during Work Time. Although these three classroom vignettes illustrated use of math workshop in elementary classrooms, the four math workshop components could easily be adapted and used in middle and high school classrooms.

Recommendations for Practice

Math workshop is not a quick-fix solution that enhances student performance with mathematics. Teachers who decide to implement math workshop do not receive pre-written lesson plans and reproducible learning activities that can be implemented in a turnkey manner. Rather, math workshop provides a structure for mathematics instruction that establishes a community of confident and competent students who engage in meaningful and purposeful discourse practices.

We visited with Ms. Delgado, Ms. Crockett, and Ms. Campbell and asked them to provide recommendations for teachers who were considering implementing math workshop. Based on their own experiences, they offered the following suggestions:

- Take it one step at a time. Focus on refining one component of math workshop each year.
- Implementing math workshop is a huge learning curve. It generally takes about three years to be comfortable with the structure of math workshop. However, the impact that math workshop has on students will be well worth the investment of time and energy.
- At the beginning of the school year, teach students thinking strategies first. These strategies include: (a) monitoring for meaning; (b) activating, using, and building schema; (c) asking questions; (d) drawing inferences; (e) determining importance; (f) creating sensory images; (g) synthesizing information; (h) and problem solving. By doing so, students will have a 'toolbox' for work time.
- Engage with a formal or informal professional network of other professionals who use math workshop. This is an excellent way to promote continuous professional learning, share resources, and find support among teachers of mathematics. Professional networks may be established among school campus and district colleagues, as well as among math professionals affiliated with professional organizations, such as NCTM (www.nctm.org) and Texas Council of Teachers of Mathematics (<http://tctmonline.org/>) or virtual platforms (e.g., Facebook, Twitter).

Learning More About Math Workshop

There are a few variations in methods of implementing math workshop that include ways of grouping students and different types of activities. For those interested in learning more about math workshop we recommend the following books:

- *Mathematical Mindsets: Unleashing Students' Potential through Creative Math, Inspiring Messages and Innovative Teaching* (2016) by Jo Boaler (ISBN: 978-0470894521)
- *Minds on Mathematics: Using Math Workshop to Develop Deep Understanding in Grades 4-8* (2012) by Wendy Ward Hoffer (ISBN: 978-0325044347)
- *Math Workshop in Action: Strategies for Grades K-5* (2016) by Nicki Newton (ISBN 978-1138785878)

Conclusion

In order to foster deep, conceptual understandings about mathematics, teachers could consider implementing teaching practices that facilitate mathematical discourse among students, such as math workshop. By doing so, math classrooms become communities where students are engaged in meaningful and productive interactions with each other. The classroom vignettes we presented exemplified how Ms. Delgado, Ms. Crockett, and Ms. Campbell used the math workshop framework to provide high-quality, challenging instruction that was aligned with state standards and addressed the diverse learning needs of students. Our hope is that the experiences of these seasoned professionals encourage readers to consult the resources we recommended and consider using math workshop in their classrooms.



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Publishing Opportunities for the Texas Mathematics Teacher

Call For Articles

Texas Mathematics Teacher seeks articles on issues of interest to mathematics educators, especially K-12 classroom teachers in Texas. All readers are encouraged to contribute articles. Manuscripts should adhere to the general publishing guidelines listed below. **Deadline for submissions: Fall, July 1; Spring, January 1.** Please contact our journal staff with your article and information at editor@txmathteachers.org

Call for Voices from the Classroom/Classroom Activities

Teachers are encouraged to submit articles for Voices From the Classroom, including inspirational stories, exemplary lessons, or management tools. Educators are encouraged to submit classroom activities or lessons for the journal. If submitting a lesson, it should include identification of the appropriate grade level and any prerequisites. **Deadline for submissions: Fall, July 1; Spring, January 1.** Please contact our journal staff with your story and information at editor@txmathteacher.org

Reviewers Needed

Apply or nominate a peer to become a reviewer by sending an email to our journal staff. Teachers are especially encouraged to apply.

Call for Lone Star News

Announcements for the Lone Star News include, but are not limited to, NCTM affiliated group announcements, announcements of upcoming professional meetings, and member updates. Please keep us and your members informed by contacting our journal and web staff with your affiliated group's updates.

Additional Calls

Texas Mathematics Teacher seeks submissions for the following departments. All readers are encouraged to contribute.

- Recommended Reading/Resources/Software/App
- On the Cover
- Puzzle Corner
- Quotes for Thought

Any images submitted for On the Cover must be original to the author and include 3-5 mathematical problems connected to the cover picture(s). Puzzle Corner submissions should include the solution.

Call for Advertisements:

For-Profit Organizations and businesses interested in placing an advertisement for mathematics materials should contact the journal staff. Advertisements do not imply endorsement by TCTM's board, editorial staff or members. **Deadline for submissions: Fall, September 1; Spring, February 1.** Price structure can be found on p. 33 of this issue or on the website, tctmonline.org

General Guidelines for Submission

As part of the submission process, authors are required to check their submission compliance with each of the following items. Submissions that do not adhere to these guidelines may be returned to authors.

General Preparation Checklist

- The manuscript has not been previously published and is not simultaneously being considered for publication elsewhere.
- The manuscript is in Microsoft Word format.
- All graphics, tables, and figures should be embedded in the manuscript. In addition, graphics should be saved as a jpg and attached as a separate document.
- The manuscript follows the most current APA guidelines with in-text citations as well as references at the end of the article. When possible, DOIs should be utilized in the references.
- The manuscript is double spaced, Times New Roman, size 12 font, with 1 inch margins on all sides.
- The manuscript's title page should include the title of the article/activity, author name, affiliation, mailing address, email address, phone number, and the intended target audience. No author names should appear on the manuscript after the title page.
- The author should indicate whether his or her email address can be published with the article.

Receipt of manuscripts will be acknowledged. If the manuscript is accepted for publication, the editors/reviewers may make suggestions or revisions in consultation with the principal author. Permission for all copyrighted materials must be received prior to publication.



TEA Updates

Spring 2018

Jo Ann Bilderback, Director of Mathematics, Curriculum Standards and Student Support Division

1. Instructional Resources on the Texas Gateway (www.texasgateway.org) – New instructional resources have been added to the Texas Gateway in the past year. These items include

- updates to the Supporting Information documents,
- Teacher2Teacher videos, and
- Open-Source Instructional Materials for high school courses.

2. Mathematics Achievement Academies – The 84th Texas Legislature (2015) established mathematics and reading academies for teachers. Funding for the academies was extended by the 85th Texas Legislature (2017) for the 2017-2018 and 2018-2019 school years. Teachers at eligible campuses who provide classroom instruction to students at the relevant grades will be eligible to attend an academy. This includes special education, bilingual, and ESL teachers. State law requires TEA to grant priority for academy participation to teachers at campuses with 50% or more educationally disadvantaged students. TEA plans to extend academy participation opportunities to campuses not currently on the eligible campus list in the near future.

3. ESTAR/MSTAR Universal Screeners and Diagnostic Assessments – The ESTAR/MSTAR Universal Screeners are a formative assessment system administered to students in grades 2-8 to support instructional decisions. The content of the screeners is based on algebra-readiness skills as identified in the Texas Response to Curriculum Focal Points. The results can help teachers identify if students are at-risk for algebra readiness. The spring window for the Universal Screeners is April 23, 2018 – May 18, 2018.

The ESTAR/MSTAR Diagnostic Assessments are designed to follow the ESTAR/MSTAR Universal Screener for students who have been identified as at-risk for algebra readiness. A Diagnostic Assessment is administered to identify why a student is struggling with algebra-related instruction and provides information that can be used to plan supplemental instruction. The spring window for the Diagnostic Assessments is April 30, 2018 – May 25, 2018.

ESTAR/MSTAR training modules are available at <http://texasmathsupportcenter.org/>. For technical assistance with the ESTAR/MSTAR assessment system, contact the help desk at mathtx@esc13.net or by phone at 1-855-462-8489 (toll free).

4. Interim Assessments – The Texas Education Agency (TEA) has created free, optional online interim assessments that align to the Texas Essential Knowledge and Skills (TEKS). The STAAR Interim Assessments are a TEA-provided tool to help educators tailor instructional practice to address students' needs. These assessments will be available at no cost to districts, and they are not tied to accountability. Additional information about interim assessments can be found at <https://tea.texas.gov/student.assessment/IA/>.

5. Presidential Awards for Excellence in Mathematics and Science Teaching (PAEMST) – The National Science Foundation, under the direction of the White House, approves the Texas candidates as finalists for the national PAEMST award. If chosen, the awardee will receive an all-expense paid trip to Washington, D. C. to attend recognition events, a certificate signed by the President of the United States, and a \$10,000 cash award from the National Science Foundation.

The 2018 competition is open to teachers who teach math or science in grades K-6. Teachers can sign up to apply or be nominated now through April 1, 2018. Applications are due by May 1, 2018. In Texas, eligible teachers who submit a completed application by the deadline will earn 25 hours of CPE credit. To sign up, apply, or find information about the awards, teachers can visit www.paemst.org. In Texas, questions can be sent to tx_paemst@tea.texas.gov.



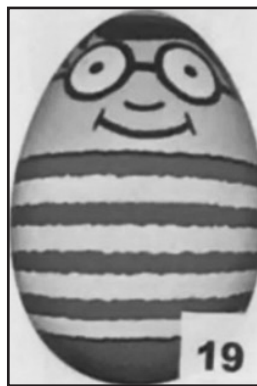
Using a Scavenger Hunt to Differentiate Mathematics Instruction

It can often be challenging to create ways to differentiate instruction in the mathematics classroom. Dimension 2.4 of T-TESS (Texas Teachers Evaluation and Support System) requires teachers to “provide differentiated instruction methods and content.” I have found that using a scavenger hunt along with purposeful grouping can create an engaging activity that also caters to the needs of a variety of students. The basic idea is that students will hunt for problems that are geared toward their algebraic readiness. I like to form groups of three by using data from a previous common or formative assessment and then group students who performed similarly on a concept or topic. Students enjoy working with peers who are at the same readiness level and collaborating on the various problems. In addition, students really enjoy this activity because it allows them to get up and move beyond the four walls of the classroom. Teachers can also make the scavenger hunt into a race for the finish if students enjoy competitions.

Activity:

The last time I used this activity it was in the spring and my students were solving algebraic equations. The activity was called “egg equations” and students were sent on a scavenger hunt. I taped pictures of eggs randomly throughout the hallway (see Figure 1) that were numbered from one to thirty-two. I tiered the problems with four levels of complexity.

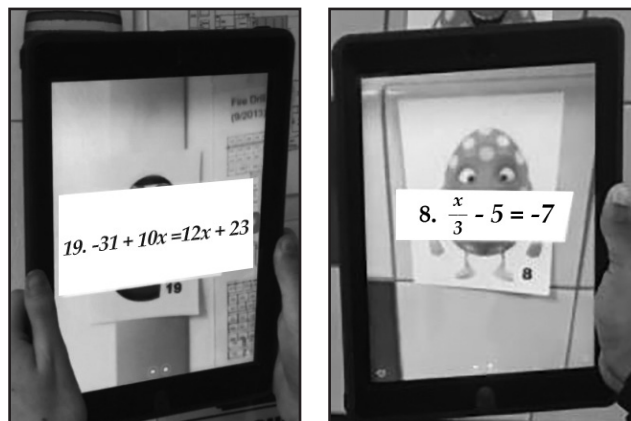
Figure 1. Picture of “egg equations” taped throughout the hallways.



1. Level 1 problems were basic two-step equations (Eggs 1-8) Ex: $2x + 5 = 13$
2. Level 2 problems included distribution (Eggs 9-16) Ex: $2(x + 5) = 18$
3. Level 3 problems had variables on both sides (Eggs 17-24) Ex: $5x + 8 = 2x + 26$
4. Level 4 problems included distribution and variables on both sides (Eggs 25-32) Ex: $3(2x + 1) = 4(x - 6)$

I also “hid” the problems in the eggs by using the app, *Aurasma* (an augmented reality app that is free and user friendly) so that students would not see the problem until they got to the egg and scanned it using their smart device (see Figure 2). The app can be found in the Apple App Store and additional information can be found at www.aurasma.com (will be redirected to new site www.hpreveal.com). Alternatively, this could be done with QR codes or by simply putting the problems on the back of the eggs.

Figure 2. Pictures of sample problems using the *Aurasma* app.



As previously noted, students were placed into groups of three based on similar algebraic readiness and were given a hunt card that told them what eggs (equations) to go find. Each hunt card (recording sheet) was numbered differently and students were to complete only those problems. All students were required to find the same amount of eggs (I had them find twelve), but hunts A & B contained mostly two-step equations (level one problems). Hunts C & D had fewer level one problems and had more level two and level three problems. Hunts E & F only had one or two problems from level one and contained several from level four. Finally, hunts G & H did not contain level one problems and were primarily comprised of level three and level four problems. The illustrations (see Figure 3) on the following page represent examples of problems assigned to level one and level four problems based on students’ algebraic readiness. Hunt A contained problems: 1, 5, 6, 8, 10, 13, 14, 15, 18, 21, 24, & 27. While hunt H contained problems: 9, 11, 12, 13, 17, 19, 20, 23, 25, 28, 30, 32. A few of the problems were used in multiple hunts.

Figure 3. Examples of hunt cards for level one and level four equations.

Egg Equations Hunt A

Directions: Find the eggs with the numbers that match the numbers in the boxes below. Write the equation and solve for x. You must show your work. Write your answer to each problem in the answer box.

1	3
Answer:	Answer:
10	14
Answer:	Answer:
8	6
Answer:	Answer:

Egg Equations Hunt F

Directions: Find the eggs with the numbers that match the numbers in the boxes below. Write the equation and solve for x. You must show your work. Write your answer to each problem in the answer box.

12	32
Answer:	Answer:
25	6
Answer:	Answer:
27	18
Answer:	Answer:

Once students completed a hunt, they could check their answers with the teacher and receive feedback if they missed some of the problems. This provided a great opportunity for small group instruction while other students were still out hunting for problems. If a group completed all their problems successfully, then they were sent out on another hunt that allowed them to move to the next level of problems. For instance, if a group completed hunt A easily, then I would send them out on hunt C. In regards to assessing students' understanding, students had to show how they would solve each problem by writing all necessary steps to solving each equation. The goal of the lesson was not for students to get the answers to the problems, but rather for students to focus on the procedures and to learn the intended mathematics by using the problems as a basis for thinking (Seeley, 2017).

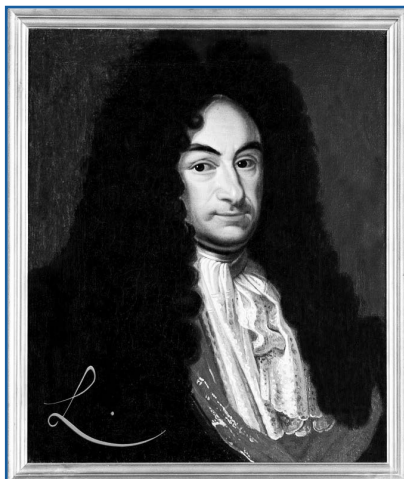
This activity could be used with a variety of topics. For example, students could engage in "hunting" problems to practice mathematical skills, explore concepts and encourage problem solving. When teachers create activities that are customized for each student, there is greater impact on student learning. In addition, by having more interactive activities students' engagement levels dramatically increase and students, regardless of their age, still enjoy a good egg hunt.



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The pleasure we obtain from music comes from counting, but counting unconsciously. Music is nothing but unconscious arithmetic.

~ Gottfried Leibniz



TCTM Government Relations Update

January 2018

Dr. Paul Gray, TCTM Governmental Relations Representative

Legislative Update

2018 is an election year. The primary elections for the Democratic and Republican parties were in March and the general election will be in November. There are a variety of candidates in both parties with varying degrees of support for public education. We strongly recommend that all educators and their eligible family members register to vote, do their homework, and vote according to their interests in the November general election.

State Board of Education

Each even-numbered year, half of the SBOE offices are up for election. For 2018, the following districts are on the November ballot. There are three open seats.

- District 2 (South Texas) - incumbent Ruben Cortez, Jr. (D-Brownsville) will be running against Charles Hasse (R).
- District 3 (San Antonio) - incumbent Marisa Perez (D-San Antonio) will be running unopposed
- District 4 (Houston) - incumbent Lawrence Allen (D-Houston) will be running unopposed.
- District 7 (SE Texas) - incumbent David Bradley (R-Beaumont) is retiring. Elizabeth Markowitz (D) and Matt Robinson (R) are running for this open seat.
- District 11 (Fort Worth/N Texas) - incumbent Pat Hardy (R-Fort Worth) will be running against Carla Morton (D).
- District 12 (Dallas/N Texas) - incumbent Geraldine "Tincy" Miller (R-Dallas) is retiring. Pam Little (R) will be running against either Laura Malone-Miller (D) or Suzanne Smith (D), pending the results of the May 22 primary runoff election.
- District 13 (Dallas and Fort Worth) - incumbent Erika Beltran (D-Fort Worth) is not running. Aicha Davis (D) and A. Denise Russell (R) are running for this open seat.

Texas Education Agency Updates

Interim Assessments: TEA has unveiled the pilot for the STAAR Interim Assessments. They need schools to use the assessments and provide feedback. Assessments are online. An informational webinar is available.

- Information from TEA: https://tea.texas.gov/About_TEA/News_and_Multimedia/Correspondence/TAA_Letters/Interim_Assessment_Pilot_Available_for_Educators/
- More Information: <https://tea.texas.gov/student.assessment/IA/>

Accountability Watch: As a part of the new federal Every Student Succeeds Act (ESSA), the federal law succeeding No Child Left Behind (NCLB), each state must submit a plan to the U.S. Department of Education explaining how they intend to measure student success at Grades 3-8 and once in high school for both mathematics and reading. As well, states must develop a school accountability plan that includes both student achievement and student progress.

Texas's initial plan was rejected in December and several revisions were requested. TEA filed a revised plan in mid-January and as of press time for this publication, the U.S. Department of Education has not responded.

One of the requested revisions was to clarify how Texas measures student success. Originally, TEA proposed using the "Approaches Grade Level" standard, based on the STAAR phase-in passing standard of about 1 standard deviation below the final panel recommendation, as a metric of student success. The US Department of Education requested that TEA use "Meets Grade Level," which is the final panel recommendation.

Additionally, the U.S. Department of Education requested that TEA clarify their methods of measuring student progress. Thus, TEA has proposed a graduated plan for all student populations to increase their 2016-17 "meets grade level" rates by 30 percentage points in the 2031-32 school year.

Read more about TEA's revised ESSA plan on the TEA website, https://tea.texas.gov/About_TEA/Laws_and_Rules/ESSA/Every_Student_Succeeds_Act/

ESSA compliance is an evolving issue as of press time for this publication. We will keep TCTM members updated through our Facebook page (@TexasCTM) and Twitter feed (@txmathteachers).

Resources for Texas Teachers

- TEA's Curriculum Division maintains a resource list for teachers, parents, and community members. https://tea.texas.gov/Academics/Subject_Areas/Mathematics/Resources_for_the_Revised_Mathematics_TEKS/.
- The Texas Gateway contains a new Teacher2Teacher video series with hints on instructional strategies <https://www.texasgateway.org/node/227936>.
- Region 11 maintains the math4texas website (www.math4texas.org) with short videos and other resources elaborating on content found in the revised mathematics TEKS.

What Can Elementary Teachers Do with Technology? There's an App for That

The National Council of Teachers of Mathematics (NCTM, 2015) indicated that strategic use of technology should enhance the learning process. Many teachers have access to current technologies but do not always know how to use the technology in ways that enhance both their teaching and students' learning. Research has indicated that student engagement can positively impact both teaching and learning (Christenson, Reschly, & Wylie, 2012). In order for the integration of technology to enhance the classroom, the technology should be engaging and interactive. Engaging and interactive use of technology is not simply watching a video or using a calculator to perform procedural algorithms where students plug in numbers and chug through the operations, often referred to as "plug and chug." Engaging and interactive technology grabs students' interest, solicits analytical thought, and facilitates student discussion of mathematical concepts. The iPad can do all of these things.

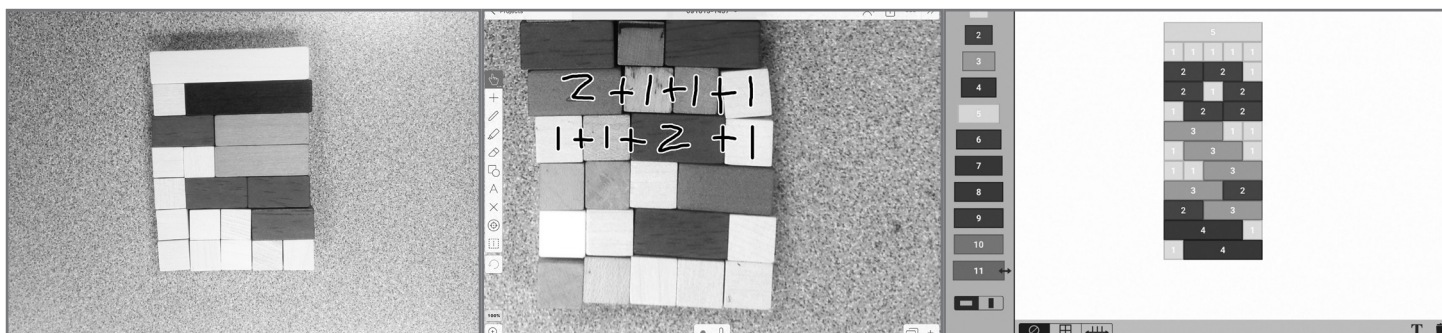
The purpose of this article is to help teachers engage students in mathematical explorations using iPads and independent apps. While this article focuses on the use of iPads, comparable applications may be available on other platforms and devices. Manipulatives have been integral tools in teaching mathematics for many years. In 1960, Jerome Bruner, at a joint conference between the Mathematics Association of America and NCTM, stressed the importance of using manipulatives when teaching mathematics (Klein, 2003). In the late 1980s, Marilyn Burns advocated for the use of manipulatives and created a professional development video series entitled *Mathematics with Manipulatives* (Burns, 1996). In the late 1990's Utah State University created a library of virtual manipulatives in order to support students' conceptual and procedural knowledge in mathematics



(Reimer & Moyer, 2005). More recently, iPad apps, related to manipulatives, have shown positive results in early childhood performance and efficiency in mathematics (Moyer-Packenham, et al., 2015). In addition, Burns and Hamm (2011) found no difference in using either virtual or concrete manipulatives when attempting to engage students and improve their understanding of mathematics. There are iPad apps available for numerous mathematical manipulatives. A few examples of how a teacher can use these apps to enhance both teaching and learning are described here.

First, imagine a first grade teacher who has a class set of concrete *Cuisenaire® Rods* and an iPad connected to a projector. The teacher wants to have students explore different addends of five using rod trains in order to address elements of the first grade TEKS 3(B): use objects and pictorial models to solve word problems involving joining, separating, and comparing sets within 20 and unknowns as any one of the terms in the problem such as $2 + 4 = []$; $3 + [] = 7$; and $5 = [] - 3$. Figure 1 describes one way to use the iPad, in conjunction with the concrete manipulatives, to enhance student learning within this standard.

Figure 1: The iPad engages students and dynamically models content.



The teacher begins by handing pairs of students a set of *Cuisenaire® Rods* and asking them to find all the different combinations of rods that equal the same length as the yellow rod. As the students work to find all of the rod trains, the teacher takes photographs of complete and incomplete sets of trains that can be used for discussion related to student thinking.

The teacher uses the Explain Everything App¹ to capture photographs of students' constructions of the rod trains to expand the lesson towards recording number sentences. The Apple Pencil can be used with the iPad Pro to allow students to record their number sentences on top of the photograph of their constructions.

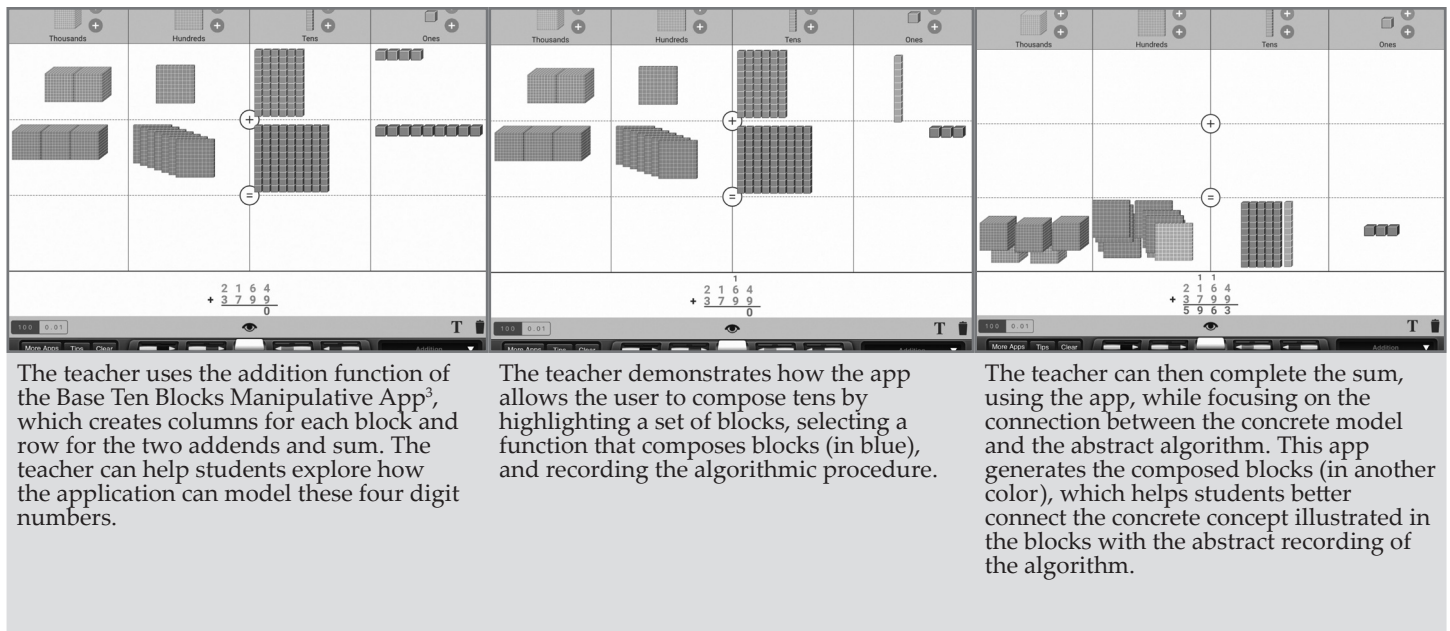
The teacher then uses a manipulative app, like Brainiaccamp's *Cuisenaire® Rods App*², that has a function to label the rods. The teacher can then proceed to construct all of the possible combinations of rods that equal the yellow, five rod. If the teacher has the available technology to wirelessly mirror their iPad screen, the iPad could be given to students who can complete the above constructions.

Secondly, imagine a third or fourth grade teacher exploring the sum of 2,164 and 3,799. Students are required to compose numbers in concrete ways that connect to the abstract vertical algorithm in these grade levels. Third grade teachers can extend TEKS 4(A): solve with fluency one-step and two-step problems involving addition and subtraction within 1,000 using strategies based on place value, properties of operations, and the relationship between addition and subtraction; and fourth grade teachers can address concepts associated with TEKS 4(A): add and subtract whole numbers and decimals to the hundredths place using the standard algorithm.

Suppose these third and fourth grade teachers have access to a classroom set of iPads, each equipped with Brainiaccamp's Base Ten Blocks App³. Figure 2 demonstrates how a teacher could use this application to help students gain a better understanding of composing tens when adding large numbers.

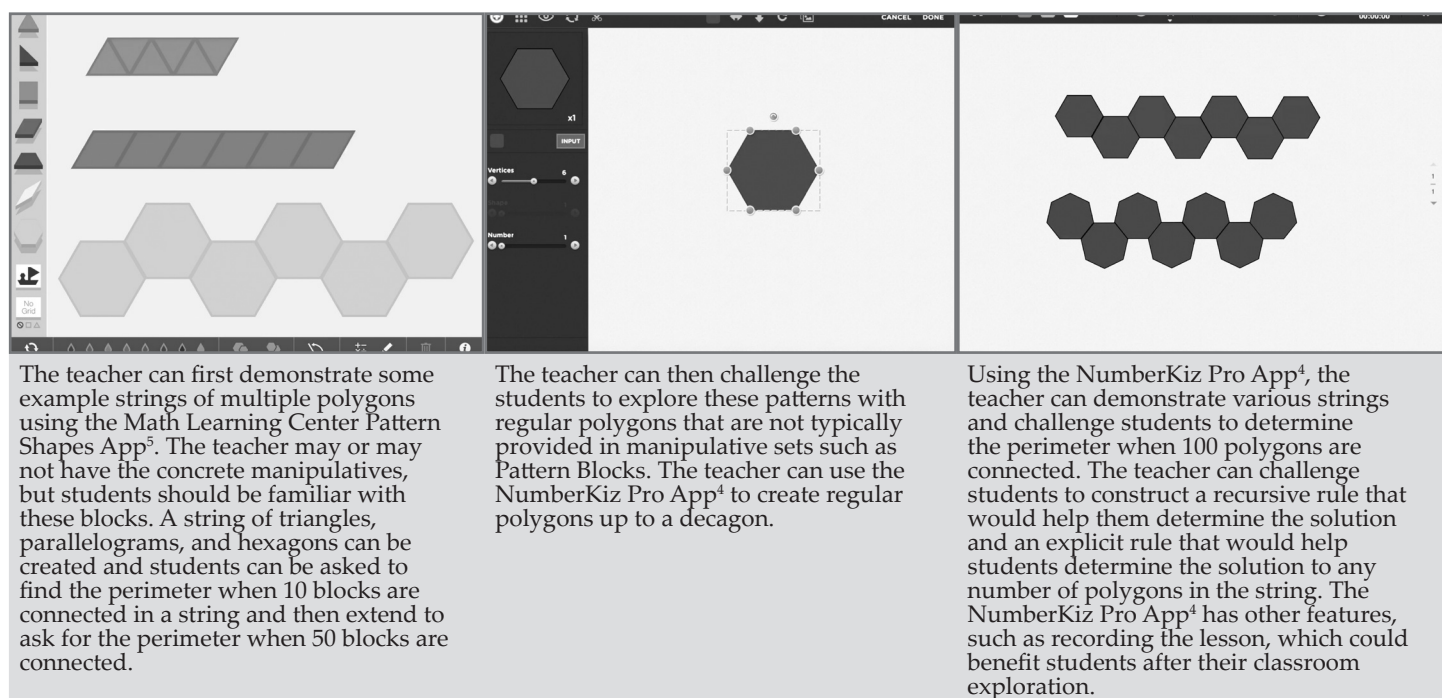


Figure 2: iPad applications can help students connect concrete concepts with procedural algorithms.



Thirdly, imagine a fifth or sixth grade teacher exploring algebraic patterns involving perimeter and strings of regular polygons. The student objective is to determine a rule for the perimeter when any number of regular polygons are connected in a string. For the following examples, the teacher does not have any manipulatives of polygons with five or seven sides, but has access to a single iPad that is equipped with the NumberKiz Pro App⁴. Figure 3 demonstrates how a teacher could use the iPad to help students explore patterns and rules related to polygons and perimeter. A fifth grade teacher could extend TEKS 4(H): represent and solve problems related to perimeter and/or area and related to volume, while a sixth grade teacher could address the TEKS 6(B): write an equation that represents the relationship between independent and dependent quantities from a table.

Figure 3: Teachers can create multiple polygons and explore patterns using a single iPad.



Conclusion

Teachers of mathematics are called to enhance the learning of their students through the use of technology (NCTM, 2015). The iPad has become a very robust device that can fit the technological needs of most teachers. Elementary school teachers can supplement their concrete manipulatives by using dynamic applications. Teachers can challenge students to explore patterns as well as make connections to algorithms through the use of technology and hands-on materials. Technology can help engage students in the learning process and create classrooms that solicit interactions between teachers and students and among students, as well. When it comes to teaching mathematics with an iPad, there's an app for that!



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Apps from Article

1. Explain Everything: <https://itunes.apple.com/us/app/explain-everything/id1020339980?mt=8>
2. Cuisenaire© Rods: <https://itunes.apple.com/us/app/cuisenaire-rods/id536204074?mt=8>
3. Base Ten Blocks Manipulative: <https://itunes.apple.com/us/app/base-ten-blocks-manipulative/id966324173?mt=8>
4. NumberKiz Pro: <https://itunes.apple.com/us/app/numberkiz-math-interactive-whiteboard/id594702283?mt=8>
5. Pattern Shapes: <https://itunes.apple.com/us/app/pattern-shapes-by-the-math-learning-center/id908511013?mt=8>

Other Suggested Apps for Elementary School

- Manipulative Apps by Brainiaccamp: <https://www.brainiaccamp.com/product/mobile.html>
- Manipulative Apps by The Math Learning Center: <https://www.mathlearningcenter.org/resources/apps>

Butterfly Art

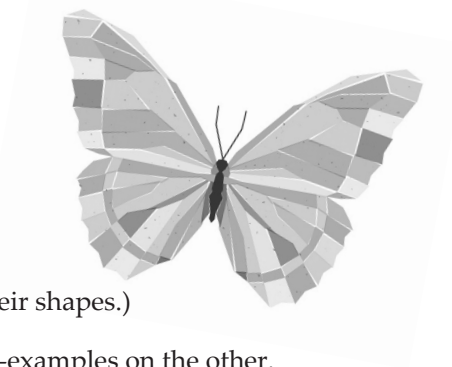
No garden truly blooms until butterflies have danced upon it. - K. D'Angelo

Spring is here! Our students, like a garden, can blossom when butterflies are included in math instruction. Young and old are mesmerized by their beauty and grace. These creatures lend themselves well to many geometry and measurement topics as well as science. Here are some ways to incorporate butterflies into your lessons this spring.

Kinder – 6th Grade Ideas

Draw a butterfly that has wings decorated with:

- 2-dimensional figures
 - Polygons
 - Quadrilaterals
 - Parallelograms
 - Figures with parallel lines
 - Figures with perpendicular lines
 - Figures with right angles, acute angles, and/or obtuse angles.
- (Students could use a protractor and label the angle measurements of their shapes.)



Another option would be to have students draw examples on one wing and non-examples on the other.

○ Example:

- On one wing draw examples of polygons. On the other wing draw examples of figures that are not polygons.

Follow up questions:

- How many of each figure did you use to decorate your butterfly?
- What fraction of the total figures does each 2-dimensional shape represent?
- Can these fractions be simplified? If so, simplify them.
- Compare the fractions of two shapes.
 - Examples:
 - Compare the fraction of rectangles to the fraction of trapezoids.
 - Compare the fractions of figures with right angles to figures with only acute angles.
- What fraction of the figures are _____ and _____?
 - Examples:
 - What fraction of the figures are squares and triangles?
 - What fraction of the figures are quadrilaterals and octagons?

7th – 12th Grade Ideas

Stage of Monarch Butterfly Life Cycle	Size
Egg	1.2 mm long x 0.9 mm wide
Larvae when first born	4.5 mm long x 1 mm wide
Larvae before forming a chrysalis	35 mm long x 6.5 mm wide
Chrysalis	25.4 mm long
Butterfly wingspan	96.5 mm

The students at your school are studying about the life cycle of Monarch Butterflies. The table shows the estimated sizes of a Monarch Butterfly during each stage of its life cycle. You decide to create a model of each stage. If your models' scale is 1 inch represents 5 mm, what will be the dimensions for each model you create? Create your models based on your results.



..... On the Cover: Find Mathematics in Butterflies

Using construction paper colors of their own choice, have students cut out and combine triangles, parallelograms, trapezoids, rhombi, regular polygons, and/or sectors of circles to create a butterfly.

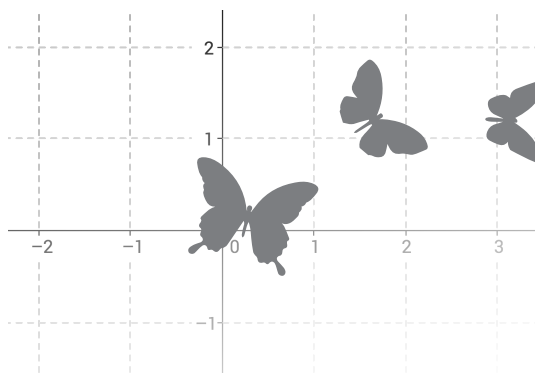
Follow up questions:

- What is the area of the butterfly (the composite figure)?
- What proportion of the area of the butterfly is _____?
 - o Examples:
 - What proportion of the area of the butterfly is parallelograms?
 - What proportion of the area of the butterfly is kites or sectors of circles?
 - What proportion of the area of the butterfly is red?

Have students draw a butterfly on their coordinate plane. This can be done on graph paper or with graphing programs such as *Desmos* (www.desmos.com).

You might require:

- Linear functions
- Quadratic functions
- Restrictions by domain
- Restrictions by range
- Conic sections
- Parametric equations



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1966

We will always have STEM with us. Some things will drop out of the public eye and will go away, but there will always be science, engineering and technology. And there will always, always be mathematics. Everything is physics and math.

~ Katherine Johnson, born August 26, 1918



2008

Reflections: Planning for Guided Math in the Kindergarten Classroom

When it comes time to preparing lessons for the week, I find that planning gives me a sense of calm and preparedness; however, when it comes to guided math I have had a personal struggle with being able to find that sense of calmness. I used to sit down on Fridays and try to make a plan for the upcoming week: the groups I would meet with, the skills we would work on, and the exact activities and games we would play together. I went through several templates trying to find the right one to keep track of my lessons and groups. I finally settled on creating my own template after realizing that pre-made pages just weren't designed for my classroom.

When sitting down to create my template, I thought about how many rotations of groups I would be able to meet with and what meeting days would be realistic with my schedule each week. I created a planning template in order to successfully track my students and their planned activities (see Figure 1).



Figure 1. Weekly planning template created for guided math groups.

Monday would arrive, and I would start out strong, referencing my binder, and following my well-prepared plan. This worked really well, until Tuesday arrived. I would reference my planning sheet, pull the groups, do the activities, and then realize that this was not the best way I could utilize my small group time. The curriculum was changing skills and introducing new concepts often, and my small group plans did not reflect my students' needs for more instruction on skills they had not yet mastered. I decided to take inventory of what my students were doing during independent work stations along with really listening to their answers during whole group discussions. I quickly realized that being in tune with what my students understood about the various mathematics concepts was what actually made me feel prepared and organized, not having a pre-made plan designed well in advance.

After deciding that planning the way I had been for small groups was not effective for my students, I started fresh with a different approach. I built a repertoire of simple games and activities to reinforce certain skills that I could pull from as needed. Having a good collection of activities that required minimal materials and copies made it simple to pull an un-planned group to practice a skill (see Figure 2). Below are pictures of students working on various skills during guided time.



Figure 2. Students working on various skills during guided math time.

I designated Mondays as the day to reinforce skills my students learned from the previous week, or skills my students still needed to practice. Usually these groups were more intentional, as most of what the students were working on was "catch up" from previous concepts. Tuesdays became the day that my small groups were focused on a new concept, and I could gauge the depth of my students' understanding.

Tuesdays really gave direction to my small group instruction for the rest of the week because I could then start grouping according to what my students knew. These groups were flexible, and for the rest of the week I would meet with different students in different combinations. My most successful way to keep track of this was using a clipboard with a template I made to fit my classroom (see Figure 3). I included only the critical information needed to best address my students' needs. For example, which students did I need to meet with, what skill(s) should be addressed, and what materials were needed.

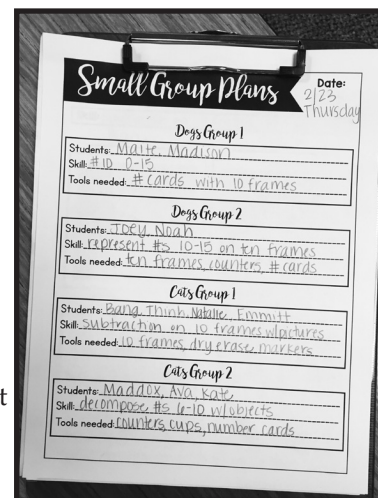


Figure 3. Daily planning template for guided math groups.

On Fridays, I made a point to move around the room during work station time and meet with students in their work stations as needed (see Figure 4). This gave me a good idea of which students I needed to meet with on the upcoming Monday and what skills we should review.

My new planning system consisted of observation and note taking, no weekly template needed as I had originally thought. Sometimes it was as simple as writing on the table with a dry erase marker and using a check mark for “understood and worked independently,” an “x” for “does not understand,” and a squiggle type shape for “partially understood.” (see Figure 5).

When I gave up the idea of planning so far in advance, it was a little terrifying. I love the idea of having a planning template that checks all the necessary boxes and is completely filled out for the upcoming week. Teaching kindergarten is all about being flexible, though, and meeting students where they are, especially when they each have different prior knowledge and learn at such different paces. When your students are five years old, they may be at a completely different place on Wednesday than they were on Monday. Having an idea of the direction my small groups needed to go turned out to be a more effective system for me and my students than having a regimented plan.



Figure 4. Working in a guided math group exploring with manipulatives.

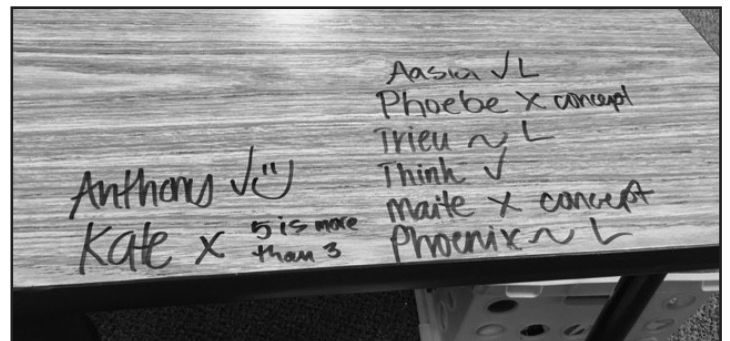
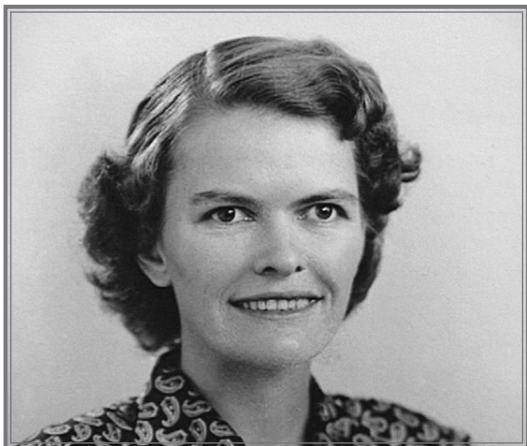


Figure 5. A picture of informal observation notes.



Elizabeth Leimbach • eleimbach@pasadenaisd.org
Kindergarten Teacher, Richard Moore Elementary, Pasadena ISD



I like to think of mathematicians as forming a nation of our own without distinctions of geographical origin, race, creed, sex, age or even time...all dedicated to the most beautiful of the arts and sciences.

~ Julia Robinson (1919-1985)

CAMT Board Report

January 2018

Nancy Trapp, Sherita Wilson-Rodgers, Paul Gray, CAMT Board Representatives

Upcoming CAMT Dates

- CAMT 2018 will be in Houston at the George R. Brown Convention Center on July 16-18, 2018 (Monday, Tuesday, Wednesday).
- CAMT 2019 will be in San Antonio at the Henry B. Gonzales Convention Center on July 8-10, 2019 (Monday, Tuesday, Wednesday).
- CAMT 2020 will be in Fort Worth at the Fort Worth Convention Center on July 8-10, 2020 (Wednesday, Thursday, Friday).

CAMT Personnel Changes

Anita Hopkins, longtime CAMT Executive Director, has announced her retirement after CAMT 2019 in San Antonio. The CAMT Board is pleased to announce that Dr. Paula Moeller will become the CAMT Executive Director upon Anita's retirement. Paula is now the CAMT Executive Director-Designate and will serve alongside Anita for CAMT 2018 and 2019. At the conclusion of CAMT 2019, Paula will assume the leadership role for CAMT.

CAMT 2018 Registration

Registration for CAMT 2018 is open right now on the CAMT website, <http://camtonline.org/conference-information-2-2/>. You may pay when you register online with a credit card or you may print the paper registration form and mail it in with a check. Your registration will not be confirmed until payment is received. Please note that CAMT does not accept Purchase Orders.

- **On-Line Registration:** Online registration may be done at any time. The fee for registering by May 1 is \$185. After May 1, the fee is increased to \$280. Payment may be made with a credit card or offline with a check. You will not have a finalized registration until your payment has been received. If your payment is received after May 1, you will be invoiced for the full registration amount of \$280.00.
- **On-Site Registration:** The fee for registration on-site is \$280. On-site registration will be held from 3:00 p.m.-7:00 p.m. on Sunday (July 15); 7:00 a.m.-3:00 p.m. on Monday (July 16) and Tuesday (July 17); and 7:00-9:00 a.m. on Wednesday (July 18).

Name badge distribution will be done onsite this year.

Program Updates

Beginning with 2018, the CAMT program book will be available electronically only. Attendees can retrieve the electronic program book through a CAMT app on their mobile device. Once downloaded, the program book can be viewed online or offline. Moving into an app format affords CAMT the flexibility to instantly announce cancellations, new sessions, or sessions that are repeated by popular demand. Watch TCTM's Facebook page (@TexasCTM) and Twitter feed (@txmathteachers) for details on how to obtain the CAMT app prior to the conference.

Have you ever attended CAMT, went to hear your favorite speaker, and found out that the session was full? This year, we are using our new registration system to help solve this problem. When you register online this year, you'll be able to indicate your interest in particular sessions when you register. Please, please, please take advantage of this new and exciting opportunity! If we know that a certain number of people want to attend that session, we can make sure that the room size matches the demand.

Anne Hoskin, of Alief ISD, is the 2018 CAMT Program Chair. She and her committee are following a similar approach as previous years with opening sessions each day followed by 1-hour breakout sessions and 2-hour extended sessions.

8:00 - 9:30 – OPENING

1-HOUR SESSIONS

10:00-11:00
11:30-12:30
1:00-2:00
2:30-3:30
4:00-5:00 (not on Wed)

2 HOUR SESSIONS

10:00-12:00
12:30-2:30
3:00-5:00 (not on Wed)

Featured speakers include:

- Karim Ani, Mathalicious
- Jennifer Bay-Williams, University of Louisville
- Sharon Benson, Region 4 ESC
- Jules Bonin-Ducharme, CFORP
- Susan Brookheart, Brookheart Enterprises
- Edward Burger, Southwestern University
- James Burnett, ORIGO
- Ramona Davis, Cypress-Fairbanks ISD
- Juli Dixon, University of Central Florida
- Brianna Donaldson, American Institute of Mathematics
- Barbara Dougherty
- Annie Fetter, The Math Forum
- Graham Fletcher, McDonough, Georgia
- Pam Weber Harris, Texas State University
- Tim Kanold, NCSM
- Steve Leinwand, American Institutes for Research
- Sue O'Connell, Quality Teacher Development
- Janet Pittock, McGraw-Hill Education
- Cathy Seeley, Author/Speaker/Consultant
- Robyn Silbey, Montgomery County (MD) Schools (retired)
- Debbie Silver, Debbie Silver Presents
- Marian Small, University of New Brunswick
- Caren Sorrells, Independent Consultant
- Kim Sutton, Creative Mathematics
- Dinah Zike, Dinah Zike Academy

Pythagoras Unchained

This article describes work done at a summer institute for secondary mathematics teachers. Participants in the institute were presented with problems about Pythagorean triples. The problems were sequenced in a way that allowed for a wide variety of discoveries to develop. The following observations were made by one of the participants, which the authors thought would be of interest to secondary mathematics teachers.

Introduction

The Rice University School Math Project (RUSMP), established in 1987 through a National Science Foundation grant, has grown into a leading mathematics education center in the state of Texas, providing support in mathematics education for districts, schools, teachers, and students. In 2016, Rice University (through RUSMP) was awarded a grant through the Robert Noyce Teacher Scholarship program. The purpose of the grant was to identify and develop 16 Houston Independent School District middle and high school teachers as Master Teacher Fellows (MTFs). As part of their professional growth, MTFs participated in the first of two 3-week summer institutes during the summer of 2017. During this summer, the MTFs investigated the problem sets in the book *Applications of Algebra and Geometry to the Work of Teaching* (Kerins, Sinwell, Young, Cuoco, & Stevens, 2015).

Many of the problems in the book centered on an in-depth examination of the properties of Pythagorean triples. Pythagorean triples are whole numbers a , b , and c such that $a^2 + b^2 = c^2$. Participants were introduced to the topic by exploring the following introductory problem:

Picture a piece of graph paper. Now picture a dot at each intersection. We'll call this square dot paper. A 5-by-5 piece of square dot paper would have 5 dots in each direction – also known as a "geoboard." But the dot paper can be any size, really. We'll say the distance from a dot to its nearest neighbor is 1. Segments drawn on square dot paper must start and end at dots, but can be horizontal, vertical, or diagonal at any angle.

Question: On a 6-by-6 piece of square dot paper (see Figure 1), what lengths of segments are possible?

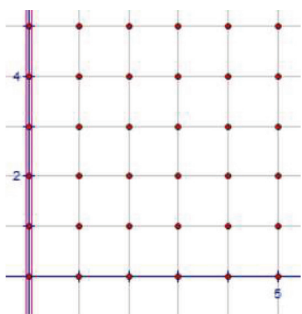
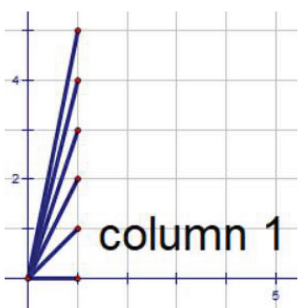


Figure 1. Dot paper provided to participants for their exploration.

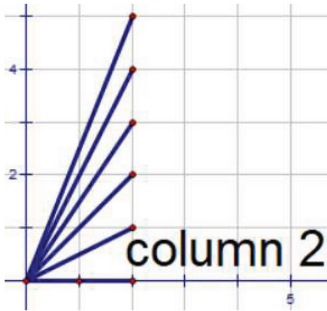


To begin, participants labeled one point as the origin $(0,0)$ and started drawing segments in the first column, $(0,0)$ to $(1,0)$, $(0,0)$ to $(1,1)$, $(0,0)$ to $(1,2)$, $(0,0)$ to $(1,3)$, $(0,0)$ to $(1,4)$, and $(0,0)$ to $(1,5)$ (see Figure 2).

Figure 2. Drawn segments with an x-coordinate of 1.

Using the Pythagorean theorem, starting at the bottom and working upwards, participants calculated the lengths of the drawn segments. These lengths were $\sqrt{1}$, $\sqrt{2}$, $\sqrt{5}$, $\sqrt{10}$, $\sqrt{17}$, $\sqrt{26}$.

“The challenge for teachers is to create similar experiences and opportunities for their students that lead naturally to important concepts.”



Likewise, participants drew the segments in the second column (those with an x-coordinate of 2) (see Figure 3).

Figure 3. Drawn segments with an x-coordinate of 2.

Again, using the Pythagorean theorem, participants started at the bottom and calculated the lengths of these segments. They were: $\sqrt{4}$, $\sqrt{5}$, $\sqrt{8}$, $\sqrt{13}$, $\sqrt{20}$, $\sqrt{29}$.

Continuing for all five columns participants produced a table of segment lengths (see Figure 4).

Figure 4. Participants' exploration results for segment lengths using the Pythagorean theorem.

Column	Lengths					
1	$\sqrt{1}$	$\sqrt{2}$	$\sqrt{5}$	$\sqrt{10}$	$\sqrt{17}$	$\sqrt{26}$
2	$\sqrt{4}$	$\sqrt{5}$	$\sqrt{8}$	$\sqrt{13}$	$\sqrt{20}$	$\sqrt{29}$
3	$\sqrt{9}$	$\sqrt{10}$	$\sqrt{13}$	$\sqrt{18}$	$\sqrt{25}$	$\sqrt{34}$
4	$\sqrt{16}$	$\sqrt{17}$	$\sqrt{20}$	$\sqrt{25}$	$\sqrt{32}$	$\sqrt{41}$
5	$\sqrt{25}$	$\sqrt{26}$	$\sqrt{29}$	$\sqrt{34}$	$\sqrt{41}$	$\sqrt{50}$

While Figure 4 reflects an accurate solution, the authors saw some additional patterns. Because this presentation of the problem has essentially “unchained” Pythagoras from the usual context of solving sides of a specific triangle, we thought there might be a more compact and general description, perhaps even a function which would generate *all* the possible segment lengths. Therefore, we generalized the problem to include all possible segment lengths.

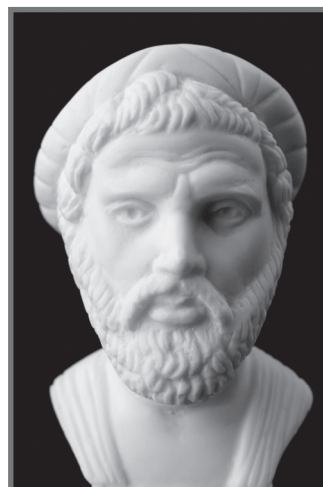
The Problem and Investigation

Given an infinitely large piece of paper on which dots have been placed in a square grid pattern 1 unit apart, what segment lengths can be produced?

To begin, we looked for a pattern in the table represented in Figure 4. For simplicity, we focused on the radicands (see Figure 5). We will call the list of radicands from column 1, sequence 1. We will call the list of radicands from column 2, sequence 2, and so on.

Figure 5. The radicands from the participants' exploration using the Pythagorean theorem.

Column	Radicands					
1	1	2	5	10	17	26
2	4	5	8	13	20	29
3	9	10	13	18	25	34
4	16	17	20	25	32	41
5	25	26	29	34	41	50



Pythagoras of Samos
c. 570 – c. 495 BC

To identify patterns, we looked at the first and second order differences in the sequences. Below is sequence 1.

Sequence 1:	1	2	5	10	17	26
First Order Difference:	1	3	5	7	9	
Second Order Difference:		2	2	2	2	

In sequence 2, the first and second order differences are identical to those for sequence 1:

Sequence 2:	4	5	8	13	20	29
First Order Difference:	1	3	5	7	9	
Second Order Difference:		2	2	2	2	

Likewise, in sequence 3:

Sequence 3:	9	10	13	18	25	34
First Order Difference:	1	3	5	7	9	
Second Order Difference:		2	2	2	2	

Also, sequence 4:

Sequence 4:	16	17	20	25	32	41
First Order Difference:	1	3	5	7	9	
Second Order Difference:		2	2	2	2	

When the second order difference of a sequence is constant, one knows the sequence can be produced by a quadratic function. That is, there exist a , b , and c such that, in sequence 1, $a \cdot 1^2 + b \cdot 1 + c = 1$, $a \cdot 2^2 + b \cdot 2 + c = 2$, $a \cdot 3^2 + b \cdot 3 + c = 5$, and so on. Because there are 3 unknowns, 3 equations are sufficient to find the values of a , b , and c .

This leads to the following system of equations for sequence 1:

$$\begin{array}{rclcl} 1a & + & 1b & + & c & = & 1 & & 1 & 1 & 1 & 1 \\ 4a & + & 2b & + & c & = & 2 & \text{which, written as an augmented matrix, is} & 4 & 2 & 1 & 2 \\ 9a & + & 3b & + & c & = & 5 & & 9 & 3 & 1 & 5 \end{array}$$

$$\begin{array}{cccc} & 1 & 0 & 0 & 1 \\ \text{The row reduced echelon form of this matrix is} & 0 & 1 & 0 & -2 \\ & 0 & 0 & 1 & 2 \end{array}$$

That is, $a = 1$, $b = -2$, and $c = 2$. Therefore, the function $f_1(n) = n^2 - 2n + 2$ will produce sequence 1. As n takes on the values 1, 2, 3, 4, ..., $f_1(n)$ takes on the values 1, 2, 5, 10, ... which are the radicands of the lengths of segments in column 1.

Repeating the process results in the following equations for the remaining sequences:

Sequence 1 is produced by the function $f_1(n) = n^2 - 2n + 2$

Sequence 2 is produced by the function $f_2(n) = n^2 - 2n + 5$;

Sequence 3 is produced by the function $f_3(n) = n^2 - 2n + 10$; and

Sequence 4 is produced by the function $f_4(n) = n^2 - 2n + 17$

Notice the constant term c of $f_1(n)$ is 2, the constant term of $f_2(n)$ is 5, the constant term of $f_3(n)$ is 10, and the constant term of $f_4(n)$ is 17. We have seen this sequence 2, 5, 10, 17, ... before. In general, it appears the constant terms of the functions are the elements of sequence 1. However, the constant term of $f_1(n)$ is the second term of sequence 1, the constant term of $f_2(n)$ is the third term of sequence 1 and so on. We see the value of c in any given f_m is the $f(m+1)$ term from sequence 1. To obtain a formula for the c value of equation f_m we must translate the function of f_1 by 1 unit. The resulting equation is:

$$c_{f_m} = f_1(m+1) = (m+1)^2 - 2(m+1) + 2 \text{ or } m^2 + 1.$$

Recalling that the lengths of the segments that can be produced are the square roots of the elements of the sequences, we see all the lengths which can be formed on an infinitely large piece of paper on which dots have been placed in a square grid pattern 1 unit apart can be produced by the function:

$$F_m(n) = \sqrt{n^2 - 2n + c_m}, \text{ where } c_m = m^2 + 1, \text{ where } m \text{ and } n \text{ are positive integers.}$$

Although this has been explored through patterns, it is important to connect this formula to the Pythagorean theorem. Given a right triangle with legs of lengths a and b and hypotenuse of length c , we can see that $F_a(b+1) = F_b(a+1) = c$.

For example, given a right triangle with legs of lengths $a=3$ and $b=4$ and hypotenuse of $c=5$.

$$F_a(b+1) = F_3(4+1) = F_3(5) = \sqrt{5^2 - 2(5) + c_3}$$

$$\text{Since } c_3 = 3^2 + 1 = 10, F_3(5) = \sqrt{5^2 - 2(5) + 10} = \sqrt{25} = 5$$

$$\text{Similarly, } F_b(a+1) = F_4(3+1) = F_4(4) = \sqrt{4^2 - 2(4) + c_4} = \sqrt{4^2 - 2(4) + 17} = \sqrt{25} = 5$$

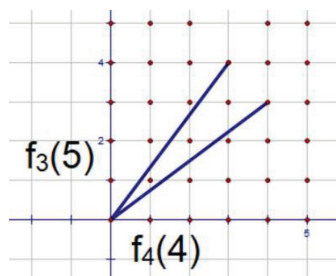


Figure 6. Example when solutions are not unique.

It directly follows then that although this formula gives all possible lengths, solutions are not necessarily unique. For example, not only $F_3(5)$ and $F_4(4)$ give a result of 5, but $F_5(1)$ does as well (see left Figure 6).

Applications to Teaching

The authors of the previously mentioned book aimed at creating problems structured to provide opportunities, primarily for teachers, for mathematical discovery through exploration of patterns and through making connections across different content domains within mathematics. Explorations such as this one are very much in this spirit. The challenge for teachers is to create similar experiences and opportunities for their students that lead naturally to important concepts.

For instance, younger students might draw a square with side lengths of 1 unit, color half of it, and label the colored part as part 1. The area of part 1 is $\frac{1}{2}$ square unit. Students can then be asked to color half of the remainder with a different color and label that part 2. The area of part 2 is $\frac{1}{4}$ square unit. If the sequence of exercises is carefully structured even very young students might be led to conjecture $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} \dots$ in some sense gets closer and closer to 1 but never gets larger than 1. Students could then discuss their own intuitive ideas of what they observe. Such exposure in early math courses could be invaluable later when the students must confront rigorous concepts of limit and infinity. It also gives students a chance to do math rather than just learn it.

Algebra 1 students who have recently learned to calculate slope might be given this challenge: On a piece of dot paper like that described above, approximate a line through the origin with a slope of $\sqrt{2}$. For each dot near the line, have them determine if the dot is above or below the line, correcting their approximate line placement as necessary. Then ask the students “If line and the paper are extended forever, what dots will lie on the line?” The students will not realize it at first but, if there is a point (x, y) that falls on a line through the origin, the slope of that line will be y/x ; however, the slope of this line is $\sqrt{2}$. We are sending students on a hunt for a rational representation of an irrational number. The discovery that the line will not ever hit any dot can lead in a natural way to student conversations about the subtleties of irrationality.

Conclusion

The English mathematician G. H. Hardy (2015) in his book *A Mathematician’s Apology*, wrote: “A mathematician, like a painter or a poet, is a maker of patterns. If his patterns are more permanent than theirs, it is because they are made with ideas” (p. 84).

The purpose of the original problem, in the authors’ opinion, was to help mathematics teachers remember what it is like to be a mathematician and to challenge them to think of ways to help students think like mathematicians as well. The applications offered here are only possibilities. The authors are confident that there are many creative teachers who can construct their own mathematical explorations for their students. We would be grateful to anyone who does so and shares the ideas and the classroom experiences with us.



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References:

Kerins, B., Sinwell, B., Yong, D., Cuoco, A., & Stevens, G. (2015). *Applications of algebra and geometry to the work of teaching*. Providence: American Mathematical Society.

Hardy, G. (2015). *A mathematician’s apology*. Cambridge [England]: Cambridge University Press.

Application Information

2018-19 Mathematics Preservice Teacher Scholarship

There are five \$2000 scholarships available for 2018-19. Any student attending a Texas college or university - public or private - and who plans on student teaching during the 2018-19 school year in order to pursue teacher certification at the elementary, middle or secondary level with a specialization or teaching field in mathematics is eligible to apply. A GPA of

3.0 overall and 3.25 in all courses that apply to the degree (or certification) is required. Look for the scholarship application online at www.txmathteachers.org. The application deadline is May 1, 2018. Winners will be announced in July 2018.



2019 TCTM Grant

This grant is for K-12 educators, university faculty and NCTM affiliate groups in Texas. Please note, pre-service teachers are not included as they can apply for the Mathematics Preservice Teacher Scholarship. The grant can be awarded to an individual, a group of teachers or to another NCTM or NCSM affiliate organization, if they are in Texas. Grant requests up to \$800 will be accepted.

Uses include (1) improving mathematics classroom(s), or (2)

helping your school achieve its goals related to mathematics, or (3) promoting mathematics teaching and learning, or (4) improving your ability to teach mathematics.

The online application may be found at www.txmathteachers.org. The application deadline is November 30, 2018. Awardees will be notified by January 31, 2019.



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TCTM 2018 Mission, Focus, and Goal Statements

Mission of the Texas Council of Teachers of Mathematics:

To promote mathematics education in Texas

To support this mission, TCTM has five **focus areas**:

Recruit and Retain
Mathematics Teachers

Curriculum and Instruction
Support

Advocacy

Promote Communication
among Teachers

Serve as Partner Affiliate
for NCTM

TCTM activities will align to the five strategic goals. Goals of the organization include six strands:

Administration

- Streamline online membership registration through CAMT

Publications

- Survey membership to identify what they want in the Texas Mathematics Teacher (TMT)
- Review and refine the TMT journal and the TCTM website
- Improve the review protocol, establish criteria for reviewers
- Provide tips for new teachers in the TMT and on the website

Service

- Increase the donations toward Mathematics Specialist College Scholarships
- Staff CAMT with volunteers as necessary
- Advertise affiliated group conferences on the TCTM website, in the TMT and at CAMT

Communication

- Maintain an e-mail list of members for timely announcements
- Communicate with affiliated groups in a timely manner

Membership

- Encourage affiliated groups to include TCTM registration on their membership forms

Public Relations

- Sponsor and staff the TCTM booth at CAMT
- Follow NCTM Advocacy Toolkit (2004) for increased voice of TCTM membership on issues relevant to our mission

TCTM Past-Presidents

1970-1972	James E. Carson	1988-1990	Otto Bielss	2008-2010	Paul Gray
1972-1974	Shirley Ray	1990-1992	Karen Hall	2010-2012	Nancy Trapp
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1982-1984	Betty Travis	2000-2002	Kathy Mittag		
1984-1986	Ralph Cain	2002-2006	Cynthia L. Schneider		
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About This Publication

Since 1971, the Texas Council of Teachers of Mathematics (TCTM) has produced the journal *Texas Mathematics Teacher* for our members. Our mission is to promote mathematics education in Texas. In the journal we accomplish this by publishing peer-reviewed articles by leading authors and local news from around the state. TCTM is committed to improving mathematics instruction at all levels. We place an emphasis on classroom activities that are aligned to the Texas Essential Knowledge and Skills and the NCTM Principles and Standards for School Mathematics. The *Texas Mathematics Teacher* seeks articles on issues of interest to mathematics educators, especially K-12 classroom teachers in Texas. All readers are encouraged to contribute articles and opinions for any section of the journal. Teachers are encouraged to submit articles for Voices from the Classroom, including inspirational stories, exemplary lessons, or management tools. More specific guidelines for submissions may be found below. Original artwork on the cover is another way teachers may contribute. We publish the journal twice each school year, in the fall and spring semesters. Our website archives the journals in PDF format. If you wish to view past issues, please see www.txmathteachers.org.

Advertising Guidelines for Texas Mathematics Teacher

All advertising is subject to the approval of the publisher. The journal staff shall be responsible for ascertaining the acceptability of advertisements. All advertisements should be sent “copy-ready” by the closing dates of September 1 for the fall issue and February 1 for the spring issue. Position preference, such as right-hand pages or first half of issue will be honored on a first-come basis. All advertisements must be pre-paid by the closing date with a check made payable to TCTM, and mailed to our current treasurer, Sherita Wilson-Rodgers, treasurer@txmathteachers.org.

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TCTM Communications



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The journals are posted on the TCTM website and are accessible through your member log in. E-blasts from the TCTM President are sent to the e-mail address you indicated on your membership form or the e-mail address that was used when you registered for CAMT. Please update your e-mailing address if it is not correct. If you do not have an email address, please let us know. You may update your information online at www.txmathteachers.org.

Send Us Your Cover Ideas!



Can you find mathematics in the real world? If so, capture real world mathematics problems by taking pictures and sending them to editor@txmathteachers.org. Submit 3-5 mathematics problems connected to your pictures and you will have the opportunity for your pictures to appear on the cover of the *Texas Mathematics Teacher* journal. All pictures should be saved as a jpg.

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Coming Soon!

New digital layout for *Texas Mathematics Teacher* journal! Announcement will be made in an upcoming President e-blast.

Texas Mathematics Teacher (ISSN# 0277-030X), the official journal of the Texas Council of Teachers of Mathematics (TCTM), is published in the fall and spring. Editorial correspondence should be mailed or e-mailed to the editor.

This journal is funded by the Texas Council of Teachers of Mathematics.

Call For Articles

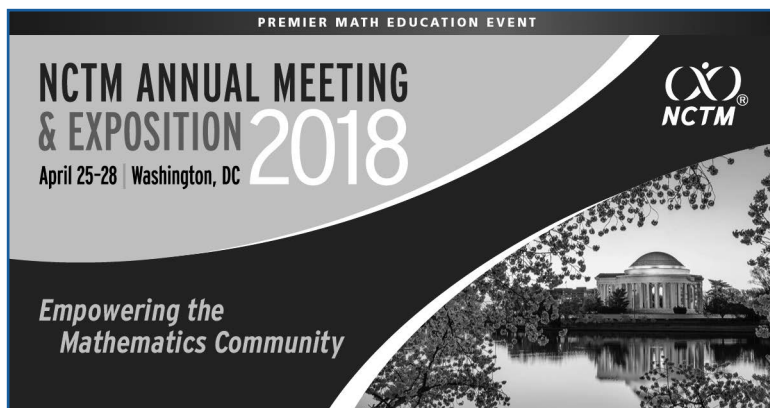
See page 13 for the updated Call for Articles and other components of the journal.

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Erratum

In Volume LXIII Issue 2 Fall/Winter 2017/2018: Rodriquez, R., C., Bingham, T., & Juarez, L. M. (2017). Putting literacy in financial literacy. *Texas Mathematics Teacher*, 63(2), 36-41.

At the bottom of page 36 the last sentence was cut off and should read "equal to the amount they spend."



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