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# A Conversation about Language in the Mathematics Classroom

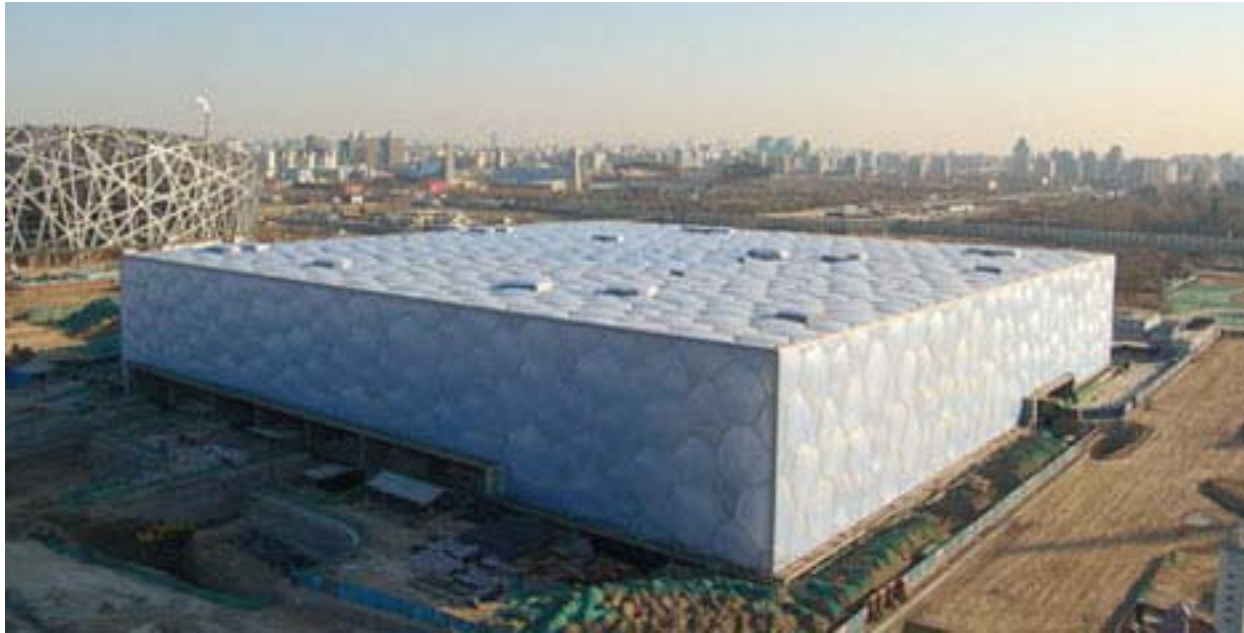
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CAMT 2009

# The Water Cube located north of Beijing City



Is this a cube?



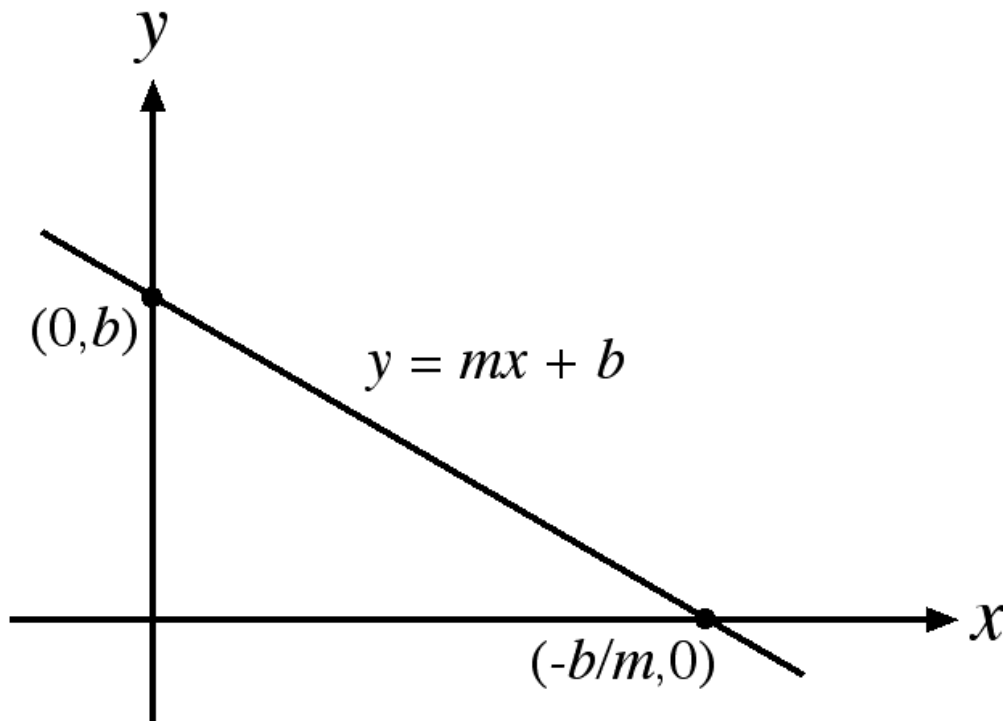
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## A Money Cube

Is this a cube?



What is the  $y$ -intercept of  
 $y = m x + b$ ?



$b$  or  $(0, b)$ ?

Is the  $y$ -intercept  
the starting  
point?



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The word “inverse” is a loaded term.  
It confuses many students.

Why?



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Is there such a thing as  
an inverse function?

Is  $f(x) = \frac{1}{x}$  the inverse function?

## The Syntax of Mathematics

What does  $f^{-1}(x)$  mean?

What does  $[f(x)]^{-1}$  mean?

Are they equal?

## The Syntax of Mathematics

What does  $\sin^{-1}(x)$  mean?

What does  $[\sin(x)]^{-1}$  mean?

Are they equal?



## The Syntax of Mathematics

What does  $\sin^2 x$  mean?

What does  $[\sin(x)]^2$  mean?

What does  $\sin x^2$  mean?

Which two are equal?



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# The Semantics of Mathematics

5 less 3

5 less than 3



## The Semantics of Mathematics

Write an equation using the variables  $S$  and  $P$  to represent the following statement: “There are six times as many students as professors. Use  $S$  for the number of students and  $P$  for the number of professors.”



## Words and Phrases to Avoid?

- Cancel or cancel out
- Flip
- Plug in
- Reduce
- Top and bottom
- Cross cancel



## Words and Phrases to Avoid?

Cancel or Cancel out

$$4 - 4$$

$$x - x$$

$$\frac{4}{4}$$

$$\frac{x}{x}$$

$$\frac{\sin x}{x}$$

$$\frac{\ln 2x}{x}$$



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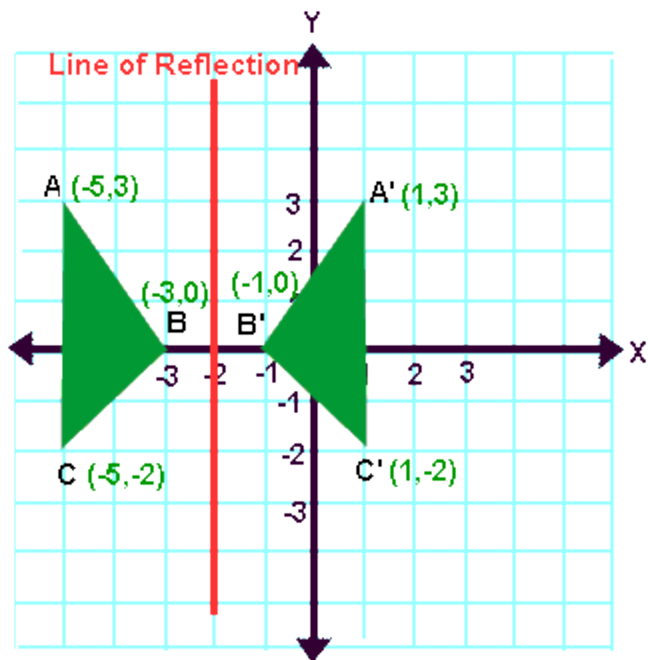
## Words and Phrases to Avoid? Flip

$$\frac{4}{1} \rightarrow \frac{1}{4}$$

What could you say?



## Words and Phrases to Avoid? Flip



What could you say?



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## Words and Phrases to Avoid? Flip

$$\frac{2}{7} \div \frac{8}{21}$$

What could you say?





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## Words and Phrases to Avoid? Reduce

$$\frac{8}{16} = \frac{1}{2}$$

What could you say?



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## Words and Phrases to Avoid? Cross Cancel

$$\frac{2}{7} \times \frac{21}{8}$$

What could you say?



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Simplify or solve?

Cross multiply or invert and multiply?

$$\frac{12}{5} = \frac{2x}{8}$$

$$\frac{5}{12} \div \frac{15}{8}$$



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Words and Phrases to Avoid?  
Top and Bottom

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

## English vs. Mathematics

Sequence – the following of one thing after another; a succession; a series

Series – a group or a number of related or similar things, events, etc. arranged or occurring in temporal, spatial, or other order or succession; a sequence

## English vs. Mathematics

Sequence – A sequence is a function whose domain is the set of positive integers.

e.g., 1, 4, 7, 10,...

$$\{a_n\} = \{a_1, a_2, a_3, \dots, a_n, \dots\}$$

Series – If  $\{a_n\}$  is an infinite sequence, then

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \dots + a_n + \dots$$

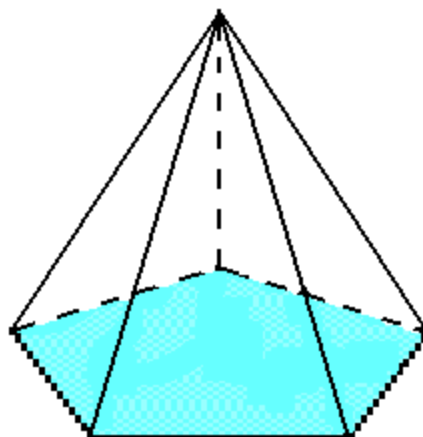
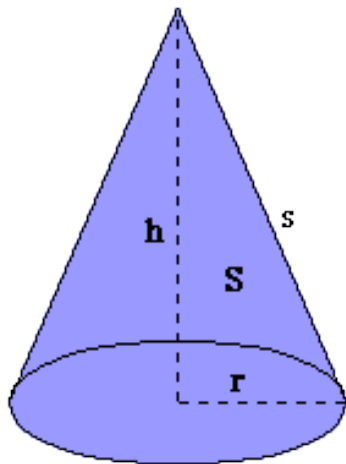
is an infinite series (or simply a series).



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# Mathematics vs. Mathematics

Does a cone have a face?

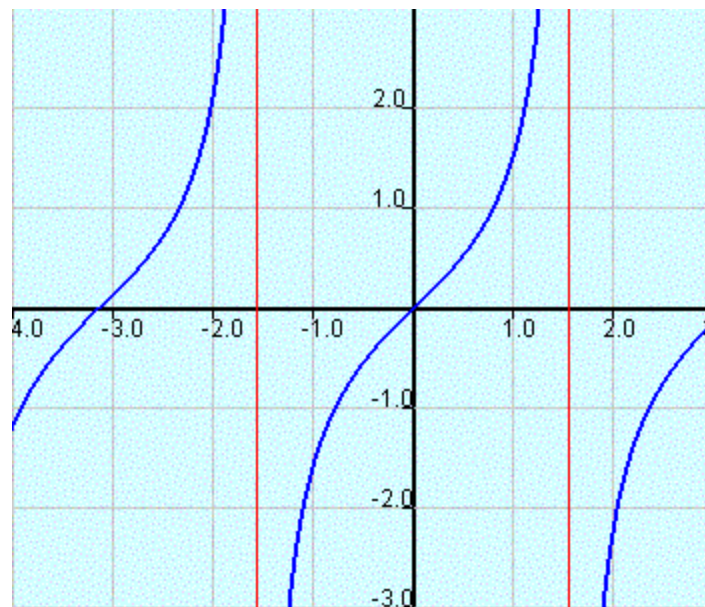
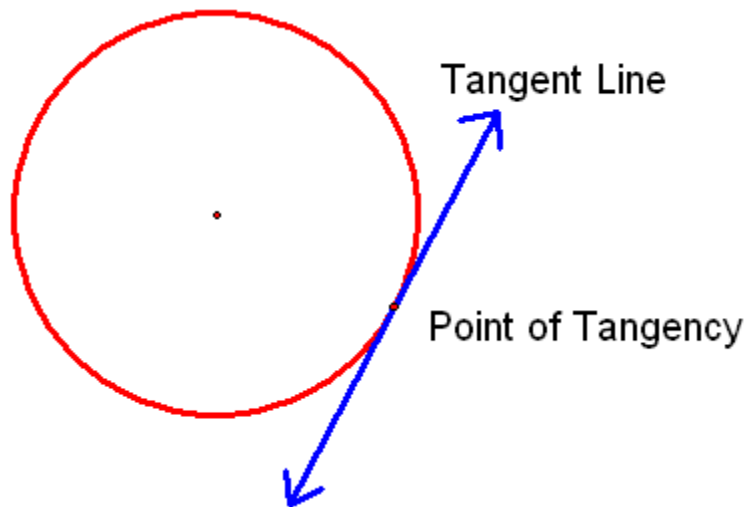
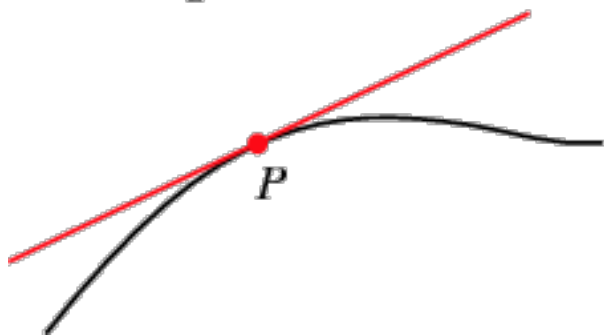




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# Mathematics vs. Mathematics

*tangent line*



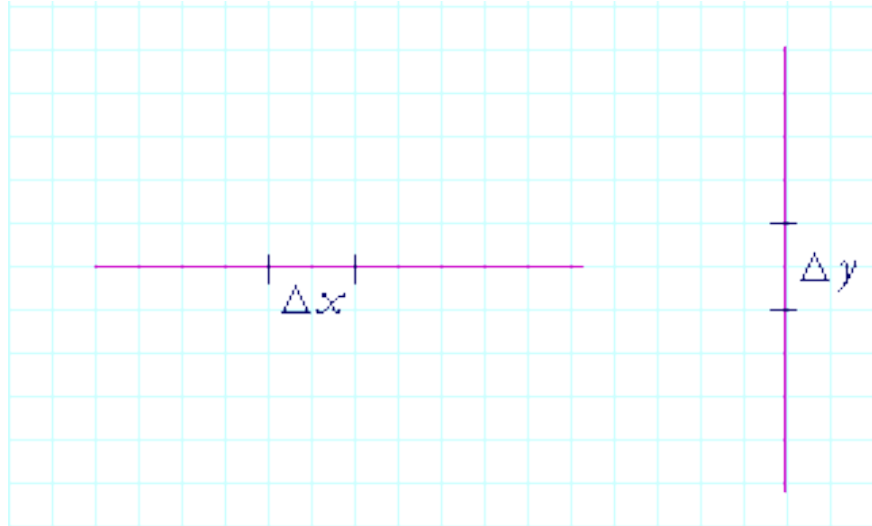
*tangent function*





## Slope

- Zero slope
- No slope
- Infinite slope
- Undefined



$$\text{Slope} = \frac{\Delta y}{\Delta x}$$

What's the difference?

- Inductive reasoning
- Proof by mathematical induction

What's the difference?

Inductive reasoning:

$4 + 6 = 10$  and 10 is an even number.

$24 + 40 = 64$  and 64 is an even number.

Then the sum of two even numbers is an even number.

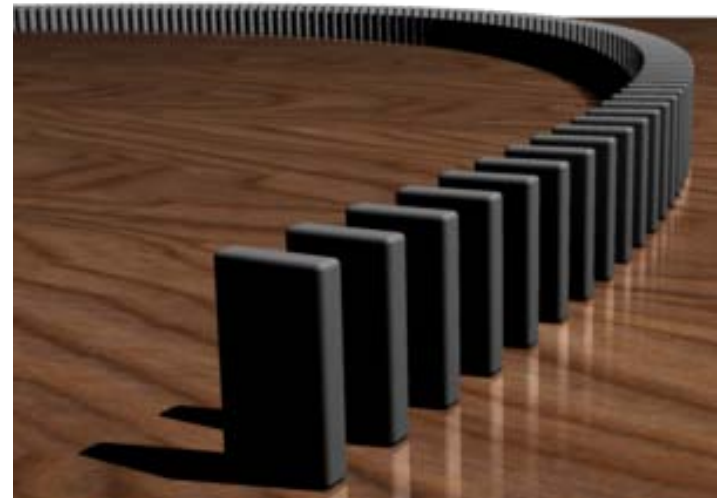
## What's the difference?

Mathematical Induction:

Let  $S(1), S(2), \dots, S(n), \dots$  be a list of statements, one for each positive integer. If the following two conditions hold:

- (i)  $S(1)$  is a true statement
- (ii) For each positive integer  $k$ , if  $S(k)$  is true, then  $S(k + 1)$  is true

then every statement on the list is true.



## Mathematical Induction

Show that

$$\sum_{k=1}^n k = \frac{n(n+1)}{2} \quad \text{for } n \geq 1.$$

$$n! > 2^n \quad \text{for } n \geq 4.$$



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Find the solution set of the system of equations

$$\begin{cases} y = 2(x - 2) + 4(x + 1) \\ y = 6x \end{cases}$$



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This presentation  
may be found at  
<http://rusmp.rice.edu>