A Conversation about Academic Language in the Mathematics Classroom in Light of the ELPS

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"Understanding mathematics requires language capacity on the part of the learner."

Heidi Hayes Jacobs (2010)

The Texas English Language Proficiency Standards (ELPS)

19 Texas Administrative Code §74.4

Chapter 74. Curriculum Requirements

Subchapter A. Required Curriculum

§74.4 English Language Proficiency Standards

Adopted December, 2007

The ELPS

- Required curriculum grades K-12
- Social and academic language
- Integrated within content areas (mathematics, science, social studies, etc.) for all language skills

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Explaining the English Language Learner Achievement Gap

by Richard Fry

Senior Researcher

Pew Hispanic Center

June 26, 2008

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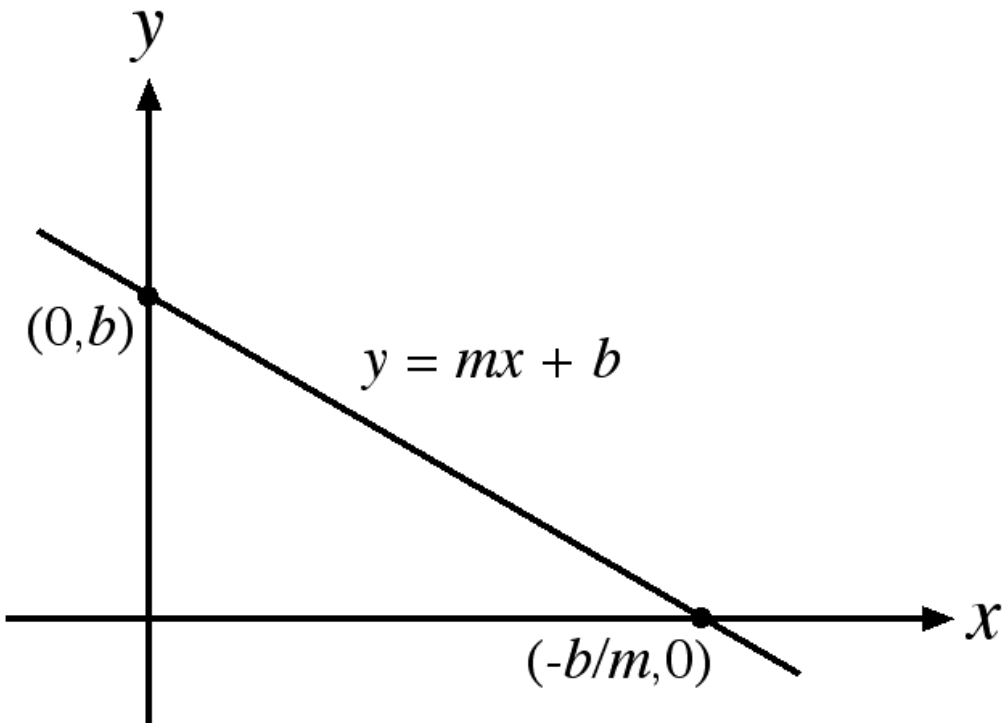
The Water Cube located north of Beijing City

Is this a cube?

A Money Cube

Is this a cube?

What is the *y*-intercept of
$y = m\,x + b$?



b or (0, *b*)?

Is the *y*-intercept the "starting point"?

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The word “inverse” is a loaded term.

It confuses many students.

Why?

Is there such a thing as
an inverse function?

Is $f(x) = \frac{1}{x}$ the inverse function?

The Syntax of Mathematics

What does $f^{-1}(x)$ mean?

What does $[f(x)]^{-1}$ mean?

Are they equal?

The Syntax of Mathematics

What does $\sin^{-1}(x)$ mean?

What does $[\sin(x)]^{-1}$ mean?

Are they equal?

The Syntax of Mathematics

What does $\sin^2 x$ mean?

What does $[\sin(x)]^2$ mean?

What does $\sin x^2$ mean?

Which two are equal?

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The Semantics of Mathematics

3 less 5

3 less than 5

3 is less than 5

The Semantics of Mathematics

Write an equation using the variables S and P to represent the following statement: “There are six times as many students as professors. Use S for the number of students and P for the number of professors.”

Clement, Lochhead, & Soloway, 1979

Words and Phrases to Avoid?

- Cancel or cancel out
- Flip
- Plug in
- Reduce
- Top and bottom

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Words and Phrases to Avoid?

Cancel or Cancel out

$$4 - 4 \qquad x - x$$

$$\frac{4}{4} \qquad \frac{x}{x}$$

$$\frac{\sin x}{x} \qquad \frac{\ln 2x}{x}$$

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Words and Phrases to Avoid?

Flip

$$\frac{4}{1} \rightarrow \frac{1}{4}$$

What could you say?

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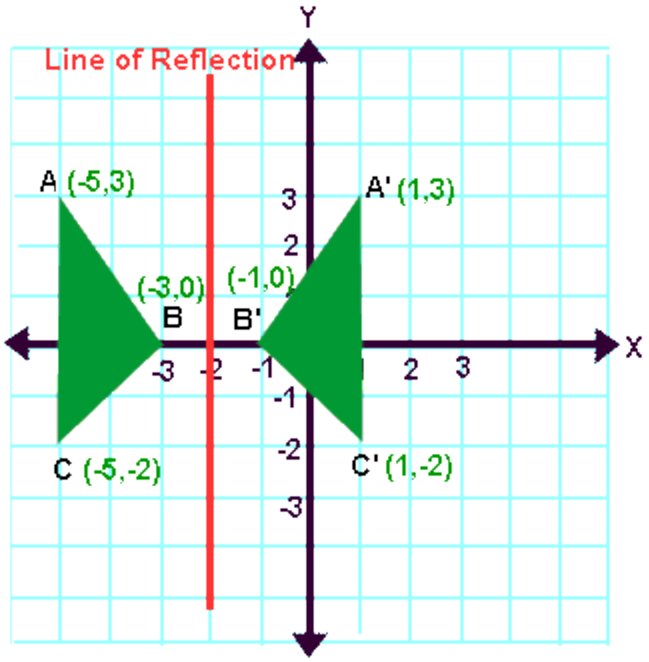
Words and Phrases to Avoid?
Flip



What could you say?

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Words and Phrases to Avoid?
Flip

$$\frac{2}{7} \div \frac{8}{21}$$

What could you say?

Flip a Coin

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Words and Phrases to Avoid?
Reduce

$$\frac{8}{16} = \frac{1}{2}$$

What could you say?

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Simplify or solve?
Cross multiply or invert and multiply?

$$\frac{12}{5} = \frac{2x}{8} \qquad \frac{5}{12} \div \frac{15}{8}$$

Words and Phrases to Avoid?
Top and Bottom

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

English vs. Mathematics

Sequence – the following of one thing after another; a succession; a series

Series – a group or a number of related or similar things, events, etc. arranged or occurring in temporal, spatial, or other order or succession; a sequence

English vs. Mathematics

Sequence – A sequence is a function whose domain is the set of positive integers.

e.g., 1, 4, 7, 10,…

$$\{a_n\} = \{a_1, a_2, a_3, \ldots a_n, \ldots\}$$

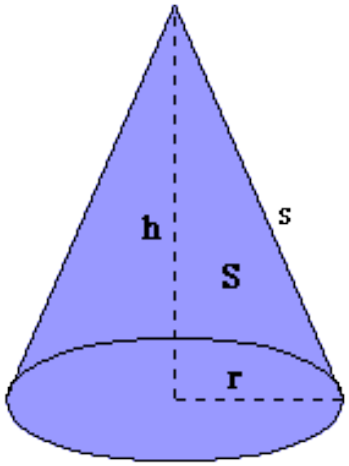
Series – If $\{a_n\}$ is an infinite sequence, then

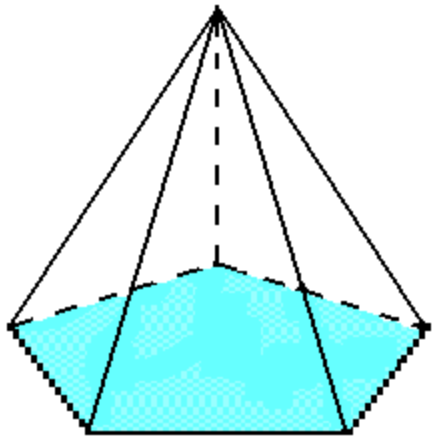
$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \cdots + a_n + \cdots$$

is an infinite series (or simply a series).

Mathematics vs. Mathematics

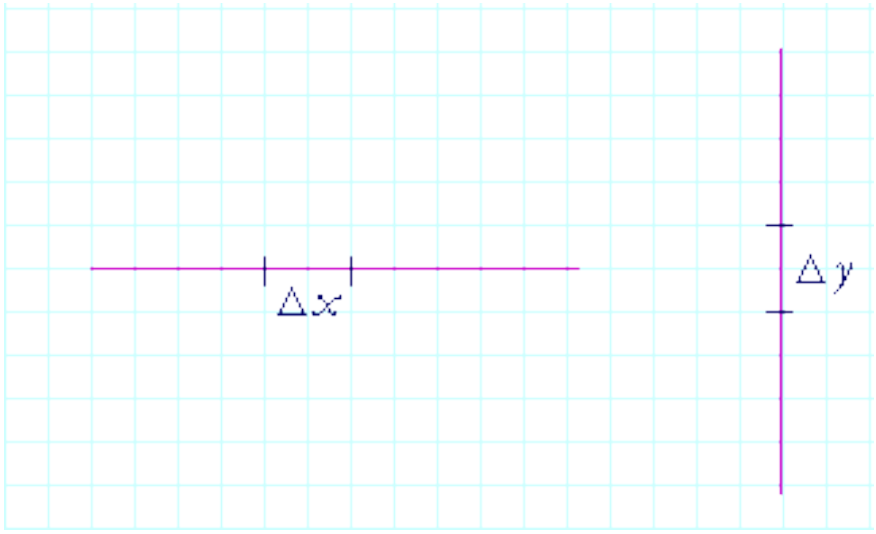
Does a cone have a face?





Slope

- Zero slope
- No slope
- Infinite slope
- Undefined



$$\text{Slope} = \frac{\Delta y}{\Delta x}$$

What's the difference?

- Inductive reasoning
- Proof by mathematical induction

What's the difference?

Inductive reasoning:

4 + 6 = 10 and 10 is an even number.

24 + 40 = 64 and 64 is an even number.

Then the sum of two even numbers is an even number.

What's the difference?

Mathematical Induction:

Let S(1), S(2), ..., S(n), ... be a list of statements, one for each positive integer. If the following two conditions hold:

(i) S(1) is a true statement

(ii) For each positive integer k,
if S(k) is true, then S(k + 1) is true

then every statement on the list is true.

Mathematical Induction

Show that

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2} \quad \text{for } n \geq 1.$$

$$n! > 2^n \quad \text{for } n \geq 4.$$