



RICE UNIVERSITY  
SCHOOL MATHEMATICS PROJECT

# Developing Student's Algebraic Thinking Through Problem Solving



**Rice University School Mathematics Project  
Houston, Texas**

**Carolyn L White  
clwhite@rice.edu  
Director of Elementary Programs**



# RICE UNIVERSITY SCHOOL MATHEMATICS PROJECT

**Carolyn L. White**  
**clwhite@rice.edu**  
**Director of Elementary Programs**

Handouts available electronically at  
**<https://rusmp.rice.edu/>**



## **Why Algebraic Thinking?**



## **What is Algebraic Thinking?**

“Algebraic thinking or algebraic reasoning involves forming generalizations from experiences with number and computation, formalizing these ideas with the use of a meaningful symbol system, and exploring the concepts pattern and function.”

(Van De Walle, 2010, p. 254)



**Where is number in algebraic thinking?**



## **Algebraic Thinking**

- ❖ Generalization from arithmetic
- ❖ Meaningful use of symbols
- ❖ Study of patterns and functions



## **Generalization from Arithmetic**



## **Developing Arithmetic in the Elementary Grades**

- ❖ The separation of arithmetic and algebra deprives students of powerful ways of thinking about mathematics.
- ❖ Fundamental properties that scholars use in calculating are the basis for most of symbolic manipulation in algebra.





## Using Playing Cards

**Let's play the game 'Salute'**

- ❖ Three players on each team
- ❖ Deck of cards
- ❖ Paper to record (optional)



# Using Playing Cards to form Equations

$$\boxed{\phantom{000}} + \boxed{\phantom{000}} = \boxed{\phantom{000}}$$



## Using Playing Cards to form Equations

Four problems involving playing cards

$$\square + \boxed{7} = \boxed{10} \quad (x + 7 = 10)$$

$$\square + \square = \boxed{8} + \boxed{6} \quad (2x = 8 + 6)$$

↑ ↑  
The first two cards are the same.

$$\square + \square + \square = \square + \square \quad (3x = 2y)$$

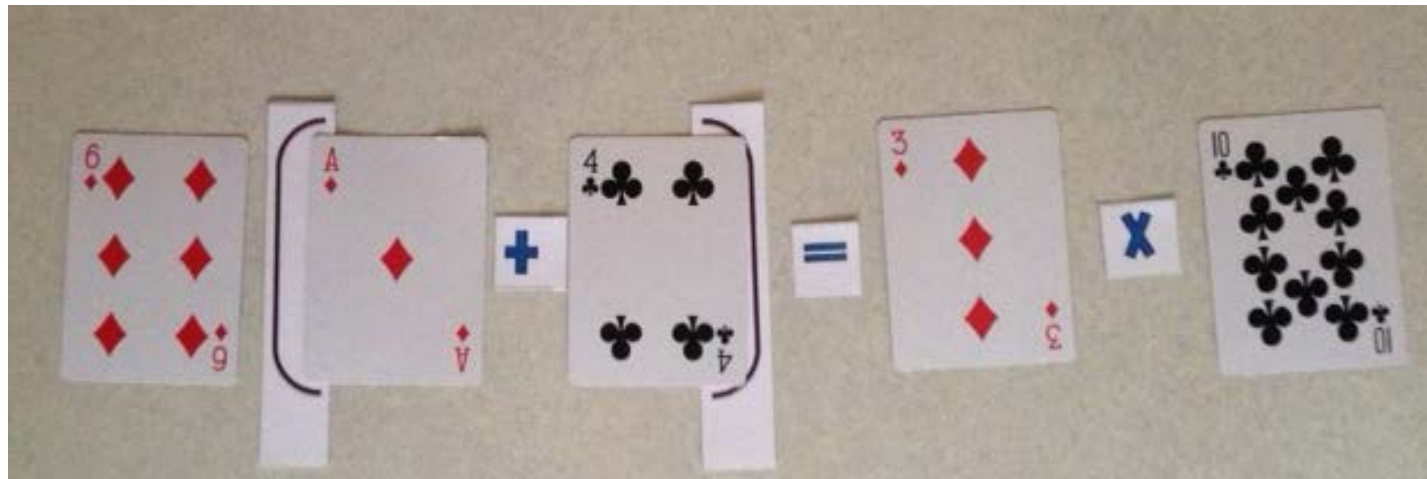
↑ ↑ ↑      ↑ ↑  
The first three cards are the same, and the last two cards are the same.

$$\square + \square + \square + \square = \square + \square + \square + \square$$

All eight cards must be different.



# Using order of operations to evaluate expressions and solve equations





## **Meaningful Use of Symbols**


















# RICE UNIVERSITY

## SCHOOL MATHEMATICS PROJECT

The students were introduced to a system of equations.

HOW MUCH IS EACH SYMBOL WORTH?
















			32
			29
			25
			24
33	40	37	
			



# RICE UNIVERSITY

## SCHOOL MATHEMATICS PROJECT

HOW MUCH DOES EACH FISH COST?
















			SUM
			\$6
			\$12
			\$19
			\$16
SUM	\$13	\$19	\$21
			
Goldfish	Beta	Clown Fish	



# RICE UNIVERSITY

## SCHOOL MATHEMATICS PROJECT

HOW MUCH DOES EACH WHALE WEIGH IN TONS?

			SUM
			330
			170
			200
			185
SUM	215	375	295
			
Right Whale	Gray Whale	Blue Whale	





## Questions to ask students

- ❖ Can you tell me what you were thinking?
- ❖ Did you solve this in a different way?
- ❖ How do you know this is true?
- ❖ Does this always work?



# RICE UNIVERSITY

## SCHOOL MATHEMATICS PROJECT



**Figure This!**  
Math Challenges for Families

Which is worth more, a  
**SMILE** or a **FROWN?**

	Sum					
	+		+		=	\$40
	+		+		=	\$32
	+		+		=	\$35
	+		+		=	\$37
=	=	=				
Sum	\$52	\$50	\$42			

**Figure This!** The costs of combinations of frowns, smiles, and neutral faces are shown. How much is a smile worth?

**Hint:** Find a way to combine two of the rows or columns that have something in common.

Reasoning about unknowns is essential in studying equations. Economists, nurses, chemists, and engineers all use equations in their work.



# **Systems of Equations**

## **Objectives of the investigation**

### **Students will:**

- ❖ Develop their ability to reason with and represent with variables;
- ❖ Move away from random guess-and-check to a more logical approach for finding values for variables in a system of equations; and
- ❖ Understand various approaches to solving the same problem.



## Make up your own chart

			_____
			_____
			_____

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_



# **Study of Patterns and Functions**



# Two of Everything

By Lily Toy Hong





## Two Of Everything

- ❖ Read the book.
- ❖ Act out the story using a magical pot.
- ❖ Develop a table of values using Input and Output.
- ❖ Utilize pattern found from the table to generalize a rule verbally and using symbols.



## **What would you choose ?**

- ❖ Choice A: 100 coins each day for 10 days
- ❖ Choice B: 5 coins and a magical pot that doubled the coins each day for 10 days

**Justify your reasoning**





RICE UNIVERSITY  
SCHOOL MATHEMATICS PROJECT

# **Applying Algebraic Thinking to Mathematics Lessons from Better Test Scores**



# Using Better Test Scores to develop Webb's DOK Level 3 Strategic Thinking

Grade	Pages
3.5E	50,51
4.5B	36,37
5.4C	38,39

**Verbs for Level 3 are:**

**justify, revise, compare, construct, use concepts to solve problems**

<http://facstaff.wcer.wisc.edu/normw/All%20content%20areas%20%20DOK%20levels%2032802.pd>



## Better Test Scores Grade 3

Figure Number	Number of Squares
1	3
2	6
3	9
4	12

- ❖ What is the relationship between the figure number and the number of squares?
- ❖ How do you know this is true?
- ❖ Does this always work?
- ❖ What is the rule that describes the relationship between figure number and the number of squares?
- ❖ Create a picture for four figures then devise a table where the relationship between the figure number and the number of squares is plus 2.



## STAAR Grade 3 2018 #7

- 7 The table shows the relationship between the number of toy airplanes made in a factory and the number of batteries needed for the airplanes.

Batteries for Toy Airplanes

Number of Toy Airplanes	5	7	9	11	13	15
Number of Batteries	15	21	27	33	39	45

Based on the relationship shown in the table, which statement is true?

- A The number of batteries is equal to the number of toy airplanes times 3.
- B The number of batteries is equal to the number of toy airplanes times 2.
- C The number of batteries is equal to the number of toy airplanes times 6.
- D The number of batteries is equal to the number of toy airplanes times 5.



## Better Test Scores Grade 4

BJs Number	Process Column	Tori's Number
15	BJ's number - 7 = Tori's number $15 - 7 = 8$	8
8	$8 - 7 = 1$	1
10	$10 - 7 = 3$	3
20	$20 - 7 = 13$	



## STAAR Grade 4 2018 # 8

8 A number pattern begins with these values.

6, 12, 18, 24, . . .

Which table correctly represents the relationship between the position of a number in the pattern and the value of that number?

F

Position	Numerical Expression	Value
6	$6 \times 1$	6
12	$12 \times 1$	12
18	$18 \times 1$	18
24	$24 \times 1$	24

H

Position	Numerical Expression	Value
6	$6 \div 6$	1
12	$12 \div 6$	2
18	$18 \div 6$	3
24	$24 \div 6$	4

G

Position	Numerical Expression	Value
1	$1 + 6$	7
2	$2 + 6$	8
3	$3 + 6$	9
4	$4 + 6$	10

J

Position	Numerical Expression	Value
1	$1 \times 6$	6
2	$2 \times 6$	12
3	$3 \times 6$	18
4	$4 \times 6$	24



# Better Test Scores Grade 5

Randel	Process Column	Mellony
18	Randel's number + 15=Mellony's number $18 + 15 = 33$	33
42	$42 + 15 = 57$	57
99	$99 + 15 = 114$	114
7	$7 + 15 = 22$	22



### STAAR Grade 5 2018 # 21

- 21** The equation  $y = 1.5x$  can be used to determine  $y$ , the number of cups of water needed to cook  $x$  cups of rice. Which table shows the relationship between  $x$  and  $y$ ?

**A**

Cooking Rice

Number of Cups of Rice, $x$	9	11	13	15
Number of Cups of Water, $y$	13.5	16.5	19.5	22.5

**B**

Cooking Rice

Number of Cups of Rice, $x$	10	12	14	16
Number of Cups of Water, $y$	11.5	13.5	15.5	17.5

**C**

Cooking Rice

Number of Cups of Rice, $x$	13	15	17	19
Number of Cups of Water, $y$	19.5	21	22.5	24

**D**

Cooking Rice

Number of Cups of Rice, $x$	14	16	18	20
Number of Cups of Water, $y$	14.5	16.5	18.5	20.5





## Internet Resources

- ❖ Pan Balance Shapes  
<http://illuminations.nctm.org/Activity.aspx?id=3531>
- ❖ Function Machine  
<http://www.shodor.org/interactivate/activities/FunctionMachine/>
- ❖ Function Machine Math Playground  
<http://www.mathplayground.com/functionmachine.html>



# RICE UNIVERSITY

## SCHOOL MATHEMATICS PROJECT

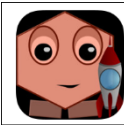
- **Visual Algebra Puzzles**



Create your own algebra puzzles then try to solve them! This easy to use, educational tool was designed to work together with Shuttle Mission Math, an algebraic reasoning game in the app store. Puzzles can be solved with at least one of the following visual strategies: Scale Up, Scale Down (multiply or divide)

<https://itunes.apple.com/us/app/visual-algebra-puzzles/id662990649?mt=8>

- **Shuttle Mission Math**



Shuttle Mission Math is a mathematical puzzle game that makes algebraic thinking both visual and interactive. The goal is to find the weight of each space creature and assemble a team for the next shuttle mission.

<https://itunes.apple.com/us/app/shuttle-mission-math/id498617241?mt=8>



# RICE UNIVERSITY SCHOOL MATHEMATICS PROJECT

**Rice University School Mathematics Project**  
**Houston, Texas**

**Website: [www.rusmp.rice.edu](http://www.rusmp.rice.edu)**



**Carolyn L. White**

clwhite@rice.edu

Director of Elementary Programs