RUSM/Robert Noyce Master Fellowship Meeting
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Is it Scope and Sequence versus Higher Level Thinking and Deep Knowledge?

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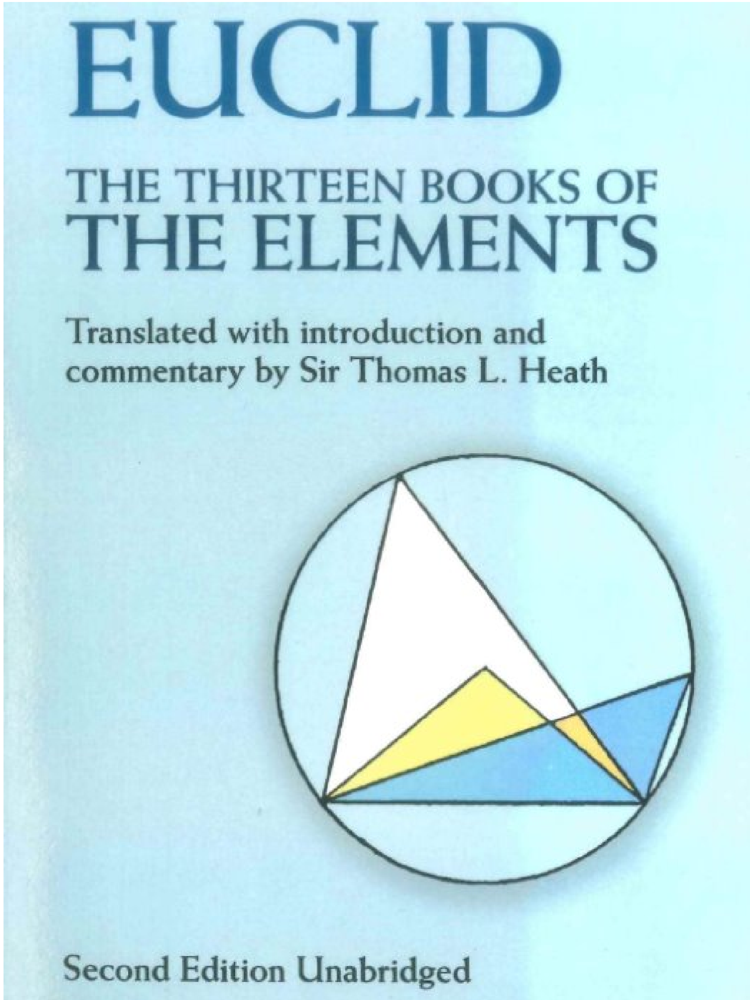
School year

- 180 days or 40 weeks
- - 6 weeks (2 review weeks, 2 weeks of tests, STAAR/AP tests)
- 180 theorems (properties to learn)

GEOMETRY course

- USA - 1 year
- Mexico – 2 years
- Russia – 4 years

Proofs in Geometry: SAS theorem

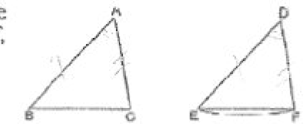


PROPOSITION 4.

If two triangles have the two sides equal to two sides respectively, and have the angles contained by the equal straight lines equal, they will also have the base equal to the base, the triangle will be equal to the triangle, and the remaining angles will be equal to the remaining angles respectively, namely those which the equal sides subtend.

Let ABC, DEF be two triangles having the two sides AB, AC equal to the two sides DE, DF respectively, namely AB to DE and AC to DF, and the angle BAC equal to the angle EDF.

I say that the base BC is also equal to the base EF, the triangle ABC will be equal to the triangle DEF, and the remaining angles will be equal to the remaining angles respectively, namely those which the equal sides subtend, that is, the angle ABC to the angle DEF, and the angle ACB to the angle DFE.



For, if the triangle ABC be applied to the triangle DEF, and if the point A be placed on the point D and the straight line AB on DE, then the point B will also coincide with E, because AB is equal to DE.

Again, AB coinciding with DE, the straight line AC will also coincide with DF, because the angle BAC is equal to the angle EDF;

hence the point C will also coincide with the point F, because AC is again equal to DF.

But B also coincided with E;

hence the base BC will coincide with the base EF.

[For if, when B coincides with E and C with F, the base BC does not coincide with the base EF, two straight lines will enclose a space: which is impossible.

Therefore the base BC will coincide with EF] and will be equal to it. [C.N. 4]

Thus the whole triangle ABC will coincide with the whole triangle DEF,

and will be equal to it.

GEOMETRY text books:

- **McDougal Littell**: Ron Larsen, Laurie Boswell, Timothy D. Kanold, Lee Stiff,
- **Pearson**: Randall I. Charles, Allan E. Bellman, Basia Hall, William G. Handlin, Dan Kennedy, Stuart G. Murphy, Grant Wiggins
- **Pensacola Christian College**: F. Eugine Seymour
- **Publicaciones Cultural**: Mario Baldor , Mexico
- **Prosveshenie: Pogorelov (Погорелов), Russia**

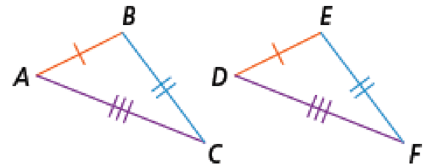
	SAS	ASA	SSS	
McDougal Littell	postulate	postulate	postulate	
Pearson	postulate	postulate	postulate	
Pensacola	theorem	theorem	theorem	
Baldor	theorem	theorem	theorem	
Pogorelov	theorem	theorem	theorem	

Postulate 4-1 Side-Side-Side (SSS) Postulate

take note

Postulate
If the three sides of one triangle are congruent to the three sides of another triangle, then the two triangles are congruent.

If . . .
$\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, $\overline{AC} \cong \overline{DF}$

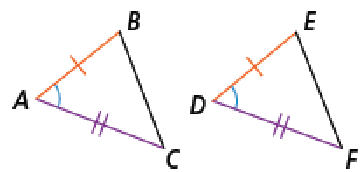


Then . . .
$\triangle ABC \cong \triangle DEF$

Postulate 4-2 Side-Angle-Side (SAS) Postulate

Postulate
If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the two triangles are congruent.

If . . .
$\overline{AB} \cong \overline{DE}$, $\angle A \cong \angle D$, $\overline{AC} \cong \overline{DF}$



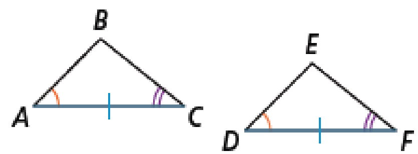
Then . . .
$\triangle ABC \cong \triangle DEF$

Postulate 4-3 Angle-Side-Angle (ASA) Postulate

take note

Postulate
If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the two triangles are congruent.

If . . .
$\angle A \cong \angle D$, $\overline{AC} \cong \overline{DF}$, $\angle C \cong \angle F$



Then . . .
$\triangle ABC \cong \triangle DEF$

take note

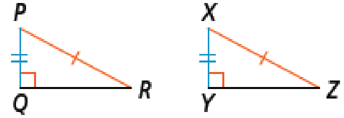
Theorem 4-6 Hypotenuse-Leg (HL) Theorem

Theorem

If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and a leg of another right triangle, then the triangles are congruent.

If . . .

$\triangle PQR$ and $\triangle XYZ$ are right △s, $\overline{PR} \cong \overline{XZ}$, and $\overline{PQ} \cong \overline{XY}$



Then . . .

$\triangle PQR \cong \triangle XYZ$

For a proof of Theorem 4-6, see the Reference section on page 683.

p.685

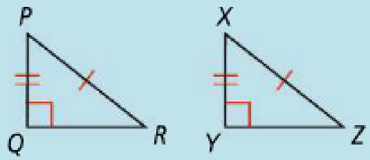
Theorem 4-6

Hypotenuse-Leg (HL) Theorem

If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and a leg of another right triangle, then the triangles are congruent. (p. 174)

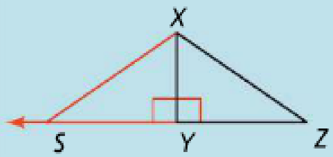
Proof

Given: $\triangle PQR$ and $\triangle XYZ$ are right triangles, with right angles Q and Y. $\overline{PR} \cong \overline{XZ}$ and $\overline{PQ} \cong \overline{XY}$.



Prove: $\triangle PQR \cong \triangle XYZ$

Proof: On $\triangle XYZ$, draw $\overrightarrow{ZY}$.



Mark point S so that $YS = QR$. Then, $\triangle PQR \cong \triangle XYS$ by SAS.

Since corresponding parts of congruent triangles are congruent, $\overline{PR} \cong \overline{XS}$. It is given that $\overline{PR} \cong \overline{XZ}$, so $\overline{XS} \cong \overline{XZ}$ by the Transitive Property of Congruence. By the Isosceles Triangle Theorem, $\angle S \cong \angle Z$, so $\triangle XYS \cong \triangle XYZ$ by AAS. Therefore, $\triangle PQR \cong \triangle XYZ$ by the Transitive Property of Congruence.

take note

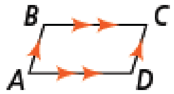
Theorem 6-3

Theorem

If a quadrilateral is a parallelogram, then its opposite sides are congruent.

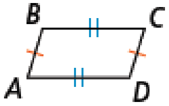
If . . .

ABCD is a ▱



Then . . .

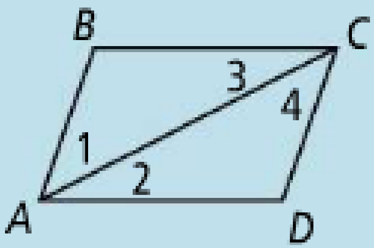
$\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{DA}$



For a proof of Theorem 6-3, see the Reference section on page 683.

p.688

Proof



Given: ▱*ABCD*

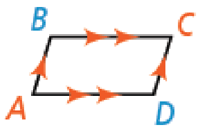
Prove: $\overline{AB} \cong \overline{CD}$, $\overline{BC} \cong \overline{DA}$

Proof: $\angle 1 \cong \angle 4$ because $\angle 1$ and $\angle 4$ are alternate interior angles and $\overline{AB} \parallel \overline{CD}$. Likewise, $\angle 2 \cong \angle 3$ because $\angle 2$ and $\angle 3$ are alternate interior angles and $\overline{BC} \parallel \overline{DA}$. $\overline{AC} \cong \overline{AC}$ by the Reflexive Property of Congruence. $\triangle ABC \cong \triangle CDA$ by ASA. Thus $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{DA}$ because corresponding parts of congruent triangles are congruent.

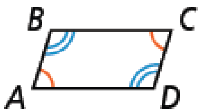
Theorem 6-5

Theorem
If a quadrilateral is a parallelogram, then its opposite angles are congruent.

If . . .
$ABCD$ is a ▱.



Then . . .
$\angle A \cong \angle C$ and $\angle B \cong \angle D$



For a proof of Theorem 6-5, see Problem 2.

p.256

p.688

Theorem 6-5

If a quadrilateral is a parallelogram, then its opposite angles are congruent. (p. 256)

- Proof on p. 257, Problem 2

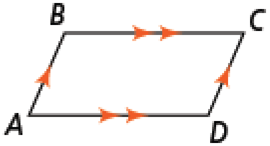
Problem 2

TEKS Process Standard (1)(G)

Proof — Using Properties of Parallelograms in a Proof

Given: $\square ABCD$

Prove: $\angle A \cong \angle C$ and $\angle B \cong \angle D$



ABCD is a $\square$.
Given

$\angle A$ and $\angle B$ are consecutive $\angle$s.
Def. of consecutive $\angle$s

$\angle B$ and $\angle C$ are consecutive $\angle$s.
Def. of consecutive $\angle$s

$\angle C$ and $\angle D$ are consecutive $\angle$s.
Def. of consecutive $\angle$s

$\angle A$ and $\angle B$ are supplementary.
Consecutive $\angle$s are supplementary.

$\angle B$ and $\angle C$ are supplementary.
Consecutive $\angle$s are supplementary.

$\angle C$ and $\angle D$ are supplementary.
Consecutive $\angle$s are supplementary.

$\angle A \cong \angle C$
Supplements of the same $\angle$ are $\cong$.

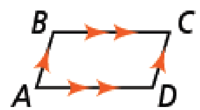
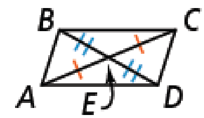
$\angle B \cong \angle D$
Supplements of the same $\angle$ are $\cong$.

Think

Why is a flow proof useful here?
A flow proof allows you to see how the pairing of two statements leads to a conclusion.

p.256

Theorem 6-6

Theorem	If . . .	Then . . .
If a quadrilateral is a parallelogram, then its diagonals bisect each other.	*ABCD* is a ▱ 	$\overline{AE} \cong \overline{CE}$ and $\overline{BE} \cong \overline{DE}$ 

You will prove Theorem 6-6 in Exercise 11.

P.259

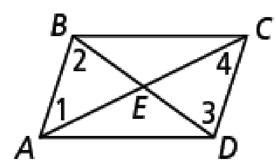
11. Justify Mathematical Arguments (1)(G)

Proof

Complete this two-column proof of Theorem 6-6.

Given: ▱*ABCD*

Prove: $\overline{AC}$ and $\overline{BD}$ bisect each other at *E*.



Statements	Reasons
1) *ABCD* is a parallelogram.	**1)** Given
2) $\overline{AB} \parallel \overline{DC}$	**2) a.** ?
3) $\angle 1 \cong \angle 4$; $\angle 2 \cong \angle 3$	**3) b.** ?
4) $\overline{AB} \cong \overline{DC}$	**4) c.** ?
5) d. ?	**5)** ASA
6) $\overline{AE} \cong \overline{CE}$; $\overline{BE} \cong \overline{DE}$	**6) e.** ?
7) f. ?	**7)** Definition of bisector

What is the area as a Geometry concept/term?

Topic 13 | Area

TOPIC OVERVIEW

13-1 Areas of Parallelograms and Triangles
13-2 Areas of Trapezoids, Rhombuses, and Kites
13-3 Areas of Regular Polygons
13-4 Perimeters and Areas of Similar Figures
13-5 Trigonometry and Area

VOCABULARY

English/Spanish Vocabulary Audio Online:

English	Spanish
altitude of a parallelogram, *p. 520*	altura de un paralelogramo
apothem, *p. 532*	apotema
base of a parallelogram, *p. 520*	base de un paralelogramo
base of a triangle, *p. 520*	base du un triángulo
center of a regular polygon, *p. 532*	centro de un polígono regular
composite figure, *p. 520*	figura compuesta
height of a parallelogram, *p. 520*	altura de un paralelogramo
height of a trapezoid, *p. 526*	altura de un trapecio
height of a triangle, *p. 520*	altura de un triángulo
radius of a regular polygon, *p. 532*	radio de un polígono regular

Area is not a formula!

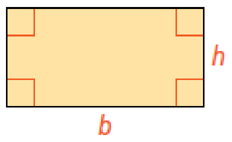
p.520

Key Concept Area of a Rectangle

take note

The area of a rectangle is the product of its base and height.

$A = bh$



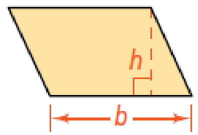
p.521

Key Concept Area of a Parallelogram

take note

The area of a parallelogram is the product of a base and the corresponding height.

$A = bh$

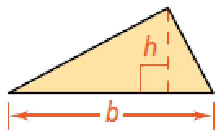


Key Concept Area of a Triangle

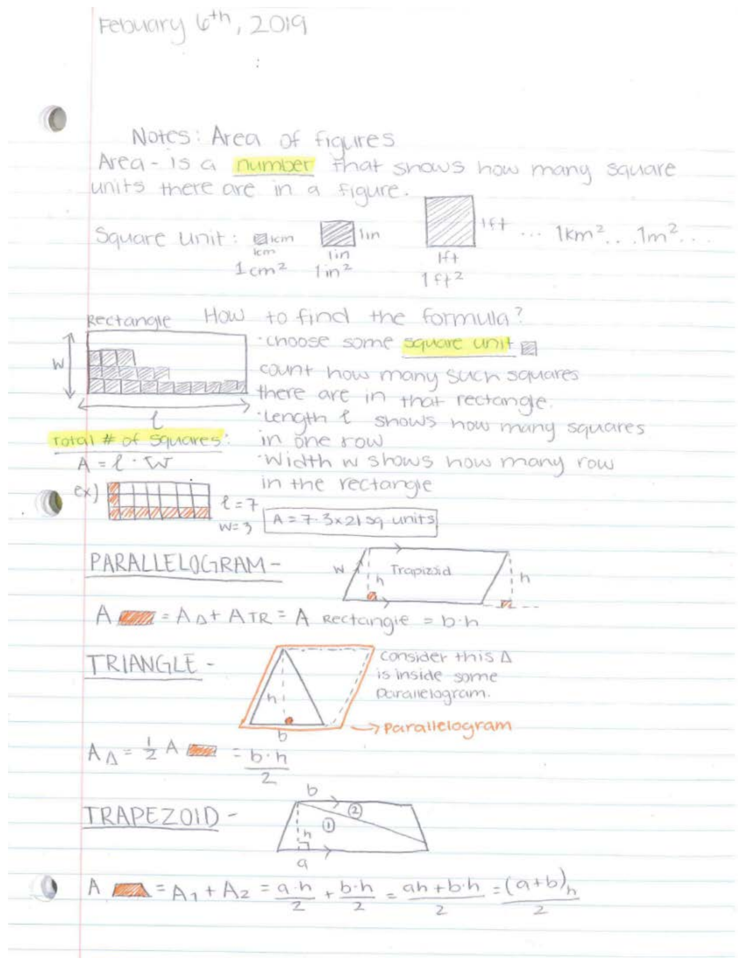
take note

The area of a triangle is half the product of a base and the corresponding height.

$A = \frac{1}{2}bh$



Student notes



Febuary 6^{th}, 2019

Notes: Area of figures

Area - is a number that shows how many square units there are in a figure.

Square Unit: 1km 1km $1 cm^2$ 1in 1in $1 in^2$ 1ft 1ft $1 ft^2$ ... $1km^2$ .. $1m^2$ ..

Rectangle How to find the formula?

· choose some square unit

count how many such squares there are in that rectangle.

· Length ℓ shows how many squares in one row

· Width w shows how many row in the rectangle

Total # of squares:

$A = \ell \cdot w$

ex) $\ell = 7$, $w = 3$ $A = 7 \cdot 3 = 21$ sq units

PARALLELOGRAM -

w, Trapizoid, h, h

$A_{▱} = A_{\Delta} + A_{TR} = A$ Rectangle $= b \cdot h$

TRIANGLE -

consider this Δ is inside some parallelogram.

h, b, → parallelogram

$A_{\Delta} = \frac{1}{2} A_{▱} = \frac{b \cdot h}{2}$

TRAPEZOID -

b, ②, ①, h, a

$A_{⏢} = A_1 + A_2 = \frac{a \cdot h}{2} + \frac{b \cdot h}{2} = \frac{ah + b \cdot h}{2} = \frac{(a+b)h}{2}$

Scope and Sequence 2018-2019

Sep	Oct	Nov	Dec	Jan	Feb	Mar	Apr	May
Main Terms; Cond. Stateme nts	Parallel Lines; Triangles	Transf- tions; Triangl es, Pypha goras	Similari ty	Right Triangl es; Circles	Quadril aterals	Trigono metry	Area, 3D Figures	3D Figures

PROPOSALS FOR FUTURE GEOMETRY COURSES

- Geometry 1 for Middle School (academic course with proofs rather than a set of activities)
- Geometry 2 for High Schools
- Proofs of theorems and derivation of main folmulae (s) have to be an intrinsic part of the course; they have be explicitly represented and explained in textbooks
- Quadrilaterals have to follow after Triangles
- Similarity should go after Quadrilateral and even after Areas
- Missed theorems:
    - Diagonal in Parallelogram makes two congruent triangles (formula for the area of triangles)
    - Angles with parallel sides are congruent (centripetal force in Physics)
    - Angles with perpendicular sides are congruent (forces and motion of the inclined plane)

Thank you!