## Developing Mathematical Problem Solving, Communication, and Reflection through Online Learning Experiences



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Spring Networking Conference RICE UNIVERSITY SCHOOL MATHEMATICS PROJECT

## Problem Solving

## Schoenfeld calls problems "starting points for serious explorations, rather than tasks to be completed"

## As opposed to most students' approach-find the answer and move to the next problem.

## Explore, Develop, Extend, Wonder

Schoenfeld, A.H. "Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics." Handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics. Ed. D.A. Grouws. New York, NY: Macmillan, 1992. 334-370.

## Principles and Standards

## "A problem-centered approach to

 teaching mathematics uses interesting and well-selected problems to launch mathematical lessons and engage students. In this way, new ideas, techniques, and mathematical relationships emerge and become the focus of discussion"Principles and Standards for School Mathematics. Reston, VA: The National Council of Teachers of Mathematics, Inc., 2000.

$\square$

## Mathematics Lessons


math.rice.edu./~lanius/Lessons

Author: Cynthia Lanius


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## Problems of the Week

The Math Forum's Problems of the Week service (the PoWs), is one of the most successful examples of challenging mathematical learning on the web, having mentored approximately 120,000 students. Students access non-routine problems online. They submit solutions and explanations, then receive feedback from volunteer math professionals/enthusiasts with encouragement to reflect more on their solutions, refine, and re-submit.


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C．The Math Forum＠Drexel


## Problems of the Week

| Curent Problems： | Math Fundamentals Algebra |  |
| :---: | :---: | :---: |
| （using and submitting is free for all） | Pre－Algebra | Geometry |
| Mentor Groups at Work Now： <br> （more information） | none |  |

The Math Forum＇s Problems of the Week（PoWs）are designed to provide creative， non－routine challenges for students in grades three through twelve．Problem－solving and mathematical communication are key elements of every problem．Program Update， August 16， 2004.

## Memberships and Subscriptions

Info：Mentoring｜｜Teacher Accounts｜｜Pricing Guide｜｜Purchase Online
Free browsing．fill access to members：Active Problem Library｜｜Lesson Support Pages
$\square$ TASB2
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## Answer Check

After submitting a solution, students are encouraged to view the Answer Check. If their answer doesn't match ours, they might:

See that they've made a calculation error. Notice that they've misinterpreted the problem.

Realize they've completed only part of the problem or not quite answered the question asked.

Tell us where they think they are stuck.

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## Mentoring Student Submissions

The Math Forum provides limited free mentoring with the help of volunteers and groups of education students.

Mentoring involves careful reading of submissions, scoring, and replying to students about their solutions.

Priority Mentoring is available by subscription.

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## Evaluation

Evaluation has shown that both weak and strong elementary, middle, and high school students who worked with PoWs over a 10-month period were found to make more connections to, generate more effective strategies for, and work more independently with the problems than they had before.

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## PoW Scoring Rubric

Reflection: check the answer, reflect on its reasonableness, summarize the process, and connect it to prior knowledge and experience.

## RUBRIC:

Problem Solving

| interpretation | $C$ | $C$ | $C$ | $C$ |
| :--- | :--- | :--- | :--- | :--- |
| strateg | $C$ | $C$ | $C$ | $C$ |
| accuracy | $C$ | $C$ | $C$ | $C$ |
| Communication |  |  |  |  |
| contipleteness | $C$ | $C$ | $C$ | $C$ |
| Clarity | $C$ | $C$ | $C$ | $C$ |
| teflection | $C$ | $C$ | $C$ | $C$ |

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## Congruent Rectangles

These seven congruent rectangles form a larger rectangle. If the area of the larger rectangle is 756 units $^{2}$, what is its perimeter?


## Congruent Rectangles

These seven congruent rectangles form a larger rectangle. If the area of the larger rectangle is 756 units $^{2}$, what is its perimeter? What does the answer show that James understands?

James:
The perimeter is 166 units. I knew that area was length times width. So I knew that 27 times 28 gave me 756. So I added 28 plus 28 plus 27 plus 27 to get my answer of 166 .

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## PoW Scoring Rubric

Reflection: check the answer, reflect on its reasonableness, summarize the process, and connect it to prior knowledge and experience.

## RUBRIC:

Problem Solving

| interpretation | $C$ | $C$ | $C$ | $C$ |
| :--- | :--- | :--- | :--- | :--- |
| strateg | $C$ | $C$ | $C$ | $C$ |
| accuracy | $C$ | $C$ | $C$ | $C$ |
| Communication |  |  |  |  |
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| Clarity | $C$ | $C$ | $C$ | $C$ |
| teflection | $C$ | $C$ | $C$ | $C$ |

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## Congruent Rectangles

These seven congruent rectangles form a larger rectangle. If the area of the larger rectangle is 756 units $^{2}$, what is its perimeter?

## What I would tell James.

 James:The perimeter is 166 units. I knew that area was length times width. So I knew that 27 times 28 gave me 756. James, it is good that you understood that you needed to find two numbers whose product was 756 , but are there other numbers that also give 756? Is there other information given in the problem that you need to consider to decide which

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## Congruent Rectangles



Bob:
If 756 units $^{2}$ is the area, and area is calculated by multiplying width times length, then the width and height most both be 756 units. Then if perimeter is calculated by $2 \mathrm{~L}+2 \mathrm{~W}$ the perimeter must be 3024 units.

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## Congruent Rectangles

Lisa:
What does Lisa understand?
Let the dimensions of the small rectangles be $a, b$, then the area of the large rectangle is:
$4 b(a+b)=756$
From the graph, we can see that $4 b=3 a$, so $b=3 a / 4$, then we have
$3 a(a+3 a / 4)=756$
$7 / 4 a^{\wedge} 2=252$
$\mathrm{a}^{\wedge} 2=144$
a=12 (-12 rejected)
b=3/4a=(3/4)*12=9


The perimeter of the larger rectangle is $4 b+a+b=$ $a+5 b=12+5 * 9=12+45=57$

## Congruent Rectangles

These seven congruent rectangles form a larger rectangle. If the area of the larger rectangle is 756 units $^{2}$, what is its perimeter?


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## Congruent Rectangles

These seven congruent rectangles form a larger rectangle. If the area of the larger rectangle is 756 units $^{2}$, what is its perimeter?


756/7=108
Iw=108
Look for factors of 108 in the ratio of 4:3. Trial and error gave 12 and 9.



Graph 1


Graph 2


Graph 3


Graph 4

Note to Cynthia: not yet

## Encouragement

This problem may seem different from the types of problems that you're used to, but don't assume that that means you can't do it. Think about how the shape of the glass changes the rate at which the water rises. If you're stuck, no problem. Tell us what you are thinking and make sure to leave a comment about the help you want.



Graph 1


Graph 2


Graph 3


Graph 4

Note to Cynthia: not yet





Graph 1
Graph 2
Graph 3


## algebra problen of the week



Graph 1
Phone a friend


Graph 2


Graph 3


Graph 4

Poll the audience






Graph 2

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and
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Stars indicate particularly interesting answers or good places to begin browsing
$\qquad$

21 Nickels，Dimes，and Pennies in a Dollar［11／8／1994］澊
How can you have 21 coins of nickels，dimes，pennies that equal one dollar？No quarters．My son and I need an explanation．

8 dimes， 5 quarters， 4 half dollars［1／13／1995］湦：
Jacob has 8 dimes，Laura has 5 quarters，and Mellisa has 4 half dollars．How much money is this all together？

## ELEMENTARY

## Arithmetic

 addition subtractionBlending Seed［09／27／1998］澡
How much of $\$ 0.65 / \mathrm{lb}$ seed does a dealer need to combine with 200 lbs of $\$ 0.45 / \mathrm{lb}$ seed to make a $\$ 0.55 / \mathrm{lb}$ blend？

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What are the rules for subtracting letters? Can I write MIM? What about DII?
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Calculations
Did the Romans use fractions?
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## New to Math Tools? Welcome!

There's a lot of stuff in here. Join our community! You can get a nice tour in the Getting Started section.

## What's Hot?

[January 26] Upper middle schoolers that haven't yet mastered multiplication facts: Bethy writes, "We have a number of students who, at 7 th and 8 th grade, still don't know $75 \%$ or more of the multiplication facts. What does the research indicate? Does anyone know of an effective method for equipping these students with a procedure so that they can move ahead?"
[January 24] Fractions, concept and calculations: Cynthia writes, "How do you teach conceptual understanding of fractions? What tools have vou


William Cullen Bryant Elementary Sch


Tool: Bobbie Bear:Counting Strategi [Java applet]

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| Advanced Partial Fractions | Partial Fractions（PreCalculus）＋ | JavaScript | Tool |  | 大 |
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