Developing Mathematical Problem Solving, Communication, and Reflection through Online Learning Experiences



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Spring Networking Conference RICE UNIVERSITY SCHOOL MATHEMATICS PROJECT



Problem Solving

- Schoenfeld calls problems "starting points for serious explorations, rather than tasks to be completed"
- As opposed to most students' approach—find the answer and move to the next problem.

Explore, Develop, Extend, Wonder

Schoenfeld, A.H. "Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics." *Handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics.* Ed. D.A. Grouws. New York, NY: Macmillan, 1992. 334–370.

Principles and Standards

"A problem-centered approach to teaching mathematics uses interesting and well-selected problems to launch mathematical lessons and engage students. In this way, new ideas, techniques, and mathematical relationships emerge and become the focus of discussion"

Principles and Standards for School Mathematics. Reston, VA: The National Council of Teachers of Mathematics, Inc., 2000.



math.rice.edu./~lanius/Lessons





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Problems of the Week

The Math Forum's Problems of the Week service (the PoWs), is one of the most successful examples of challenging mathematical learning on the web, having mentored approximately 120,000 students. Students access non-routine problems online. They submit solutions and explanations, then receive feedback from volunteer math professionals/enthusiasts with encouragement to reflect more on their solutions, refine, and re-submit.





Answer Check

After submitting a solution, students are encouraged to view the Answer Check. If their answer doesn't match ours, they might:

- See that they've made a calculation error.
- Notice that they've misinterpreted the problem.
- Realize they've completed only part of the problem or not quite answered the question asked.
- -Tell us where they think they are stuck.

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Mentoring Student Submissions

- The Math Forum provides limited free mentoring with the help of volunteers and groups of education students.
- Mentoring involves careful reading of submissions, scoring, and replying to students about their solutions.
- Priority Mentoring is available by subscription.

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Evaluation

Evaluation has shown that both weak and strong elementary, middle, and high school students who worked with PoWs over a 10-month period were found to make more connections to, generate more effective strategies for, and work more independently with the problems than they had before.



PoW Scoring **Rubric**

Reflection: check the answer, reflect on its reasonableness, summarize the process, and connect it to prior knowledge and experience.

apprentice novi ce **Problem Solving** \mathbf{C} \cap interpretation () C^{*} strategy C accuracy Communication C C completeness C C clarity. C reflection mathforum.org

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These seven congruent rectangles form a larger rectangle. If the area of the larger rectangle is 756 units², what is its perimeter?





These seven congruent rectangles form a larger rectangle. If the area of the larger rectangle is 756 units², what is its perimeter?

What does the answer show that Jamesunderstands?

James:

The perimeter is 166 units. I knew that area was length times width. So I knew that 27 times 28 gave me 756. So I added 28 plus 28 plus 27 plus 27 to get my answer of 166.



PoW Scoring **Rubric**

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These seven congruent rectangles form a larger rectangle. If the area of the larger rectangle is 756 units², what is its perimeter?

What I would tell James.

James:

The perimeter is 166 units. I knew that area was length times width. So I knew that 27 times 28 gave me 756. James, it is good that you understood that you needed to find two numbers whose product was 756, but are there other numbers that also give 756? Is there other information given in the problem that you need to consider to decide which



Bob:

If 756 units² is the area, and area is calculated by multiplying width times length, then the width and height most both be 756 units. Then if perimeter is calculated by 2L + 2 W the perimeter must be 3024 units.

The Math Forum @ Drexel

Congruent Rectangles

Lisa: What does Lisa understand?

Let the dimensions of the small rectangles be a, b, then the area of the large rectangle is: 4b(a+b)=756From the graph, we can see that 4b=3a, so b=3a/4, then we have 3a(a + 3a/4) = 7567/4a^2=252 b a^2=144 a=12 (-12 rejected) a b=3/4a=(3/4)*12=9 The perimeter of the larger rectangle is 4b+a+b= a+5b=12+5*9=12+45=57



These seven congruent rectangles form a larger rectangle. If the area of the larger rectangle is 756 units², what is its perimeter?



3*I*=4*w I*=4*w*/3

(l+w)(4w)=756(4w/3+w)(4w)=756 $28w^2/3=756$ $w^2=81$ w=9, l=12P=10w+2l=114



These seven congruent rectangles form a larger rectangle. If the area of the larger rectangle is 756 units², what is its perimeter?







algebra problem of the week





Encouragement

This problem may seem different from the types of problems that you're used to, but don't assume that that means you can't do it. Think about how the shape of the glass changes the rate at which the water rises. If you're stuck, no problem. Tell us what you are thinking and make sure to leave a comment about the help you want.





algebra problem of the week













algebra problem of the week



















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Basics

How do I read and write Roman numerals? What are the rules for subtracting letters? Can I write MIM? What about IIII? How do I write large numbers? Web resources: introductions, charts, calculators, and converters <u>Calculations</u> Did the Romans use fractions? How can I use Roman numerals to do arithmetic problems? <u>Uses Past and Present</u> How did the Romans use math? How are Roman numerals used today?

Resources for other numeral systems





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<u>Research Area</u> Developers Area A community library of technology tools, lessons, activities, and support materials for teaching and learning mathematics.

New to Math Tools? Welcome!

There's a lot of stuff in here. Join our community! You can get a nice tour in the <u>Getting Started</u> section.

What's Hot?

[January 26] <u>Upper middle schoolers that haven't yet mastered</u> <u>multiplication facts</u>: Bethy writes, "We have a number of students who, at 7th and 8th grade, still don't know 75% or more of the multiplication facts. What does the research indicate? Does anyone know of an effective method for equipping these students with a procedure so that they can move ahead?"

[January 24] <u>Fractions, concept and calculations</u>: Cynthia writes, "How do you teach conceptual understanding of fractions? What tools have you



William Cullen Bryant Elementary Sch



Tool: <u>Bobbie Bear: Counting Strategi</u> [Java applet]

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Searching For: pattern, blocks (24 results)

Search discussions for: pattern, blocks

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Home About	Attribute Trains	Patterns and sequences (Kindergarten) +	Java Applet	Tool	***
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Web Site Help	Pattern Blocks	General (Math 3)+	Java Applet	Tool	****
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Research	Pattern Blocks: Describing a P	General (Pre-Kindergarten)+	Java Applet	Activity	****
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Developers Area	Pattern Blocks: Finding Simila	Tessellations (Geometry)	Java Applet	Activity	****

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Search discussions for: fraction

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Home	Adding and Subtracting Fractio	Adding fractions (Math 4)+	JavaScript	Tool		****
About	Addition and Subtraction	Operations with numbers (Math 4)+	TI Calculator	Activity		****
Contact Us	Advanced Join Fractions	Adding fractions (Math 7)+	JavaScript	Tool		****
Math 1 opics	Advanced Partial Fractions	Partial Fractions (PreCalculus)+	JavaScript	Tool		****
Recent Activity	Ask Dr. Math: FAQ: Dividing Fr	Dividing fractions (Math 5)+	Java Applet	Support Material		
New Discussions	Basic Algebra: Adding and Subt	Adding fractions (Math 4)+	Flash	Tool	\$	****
Help	Basic Algebra: Fractions, Deci	Fractions <> decimals (Math 4)+	Flash	Tool	<u>\$</u>	
Getting Started	Basic Algebra: Multiplying and	Multiplying fractions (Math 5)+	Flash	Tool	\$	****
<u>Software Heip</u> Web Site Help	Basic Algebra: Rational Expone	Rational (Algebra)	Flash	Tool	<u>\$</u>	
Math Help	Basic Join Fractions	Adding fractions (Math 7)+	JavaScript	Tool		****
Discussions	Basic Partial Fractions	Partial Fractions (PreCalculus)+	JavaScript	Tool		
<u>Highlighted</u> Developers	BasketMath Interactive	General (Math 4)+	JavaScript	Tool		****
Research	Bounded Fraction Finder	Comparing & ordering (Math 3)+	Java Applet	Tool		
Research Area	Bounded Fraction Pointer	Comparing & ordering (Math 3)+	Java Applet	Tool		****
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	Basic Partial Fractions	Partial Fractions (PreCalculus)+	JavaScript	Tool		
	BasketMath Interactive	General (Math 4)+	JavaScript	Tool		****
	Bounded Fraction Finder	Comparing & ordering (Math 3)+	Java Applet	Tool		
ea	Bounded Fraction Pointer	Comparing & ordering (Math 3)+	Java Applet	Tool		****
Area	Cancel Fractions To Their Simp	Simplifying fractions (Math 3)+	Computer	Tool	<u>\$</u>	

