A Framework for Geometry K – 12

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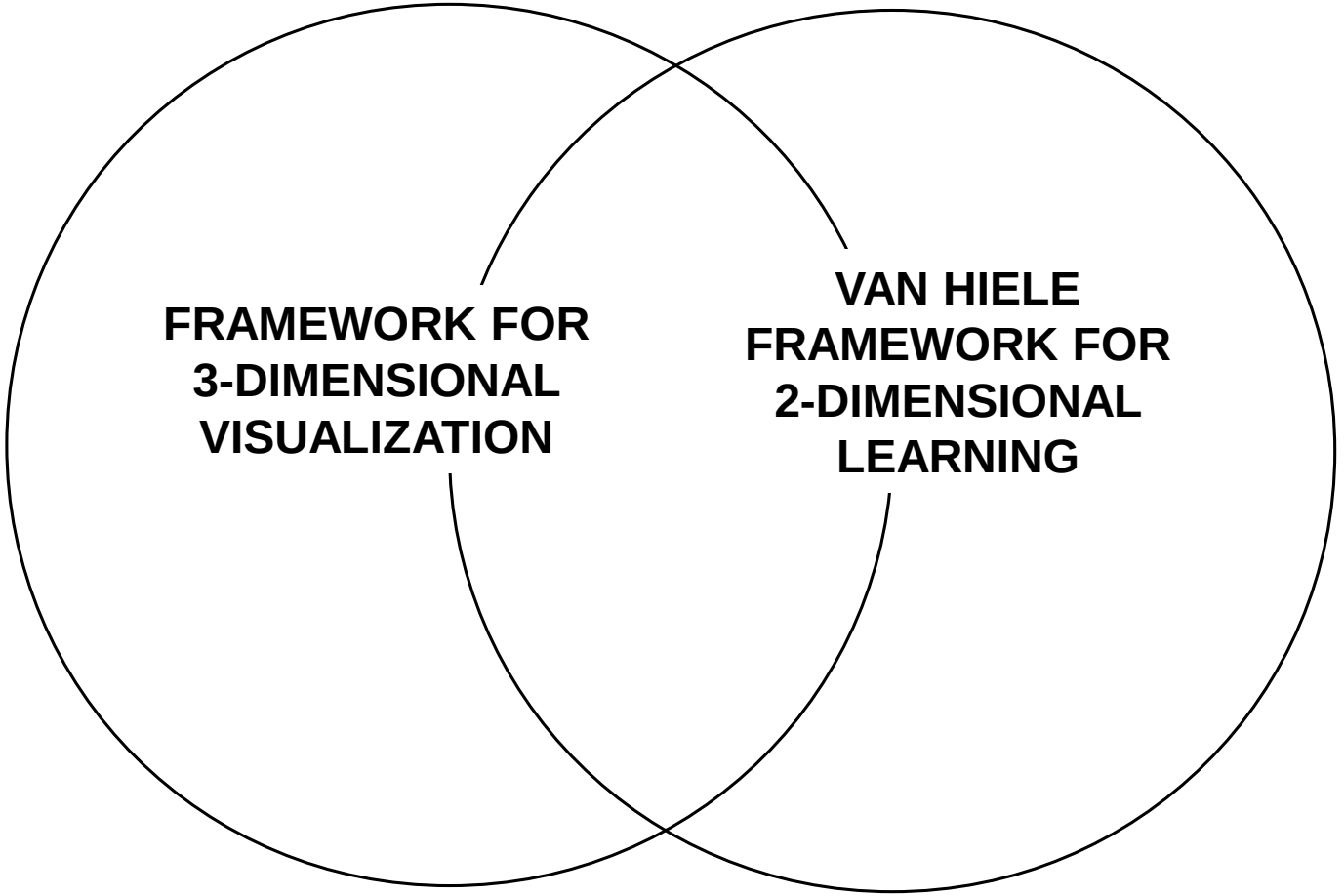
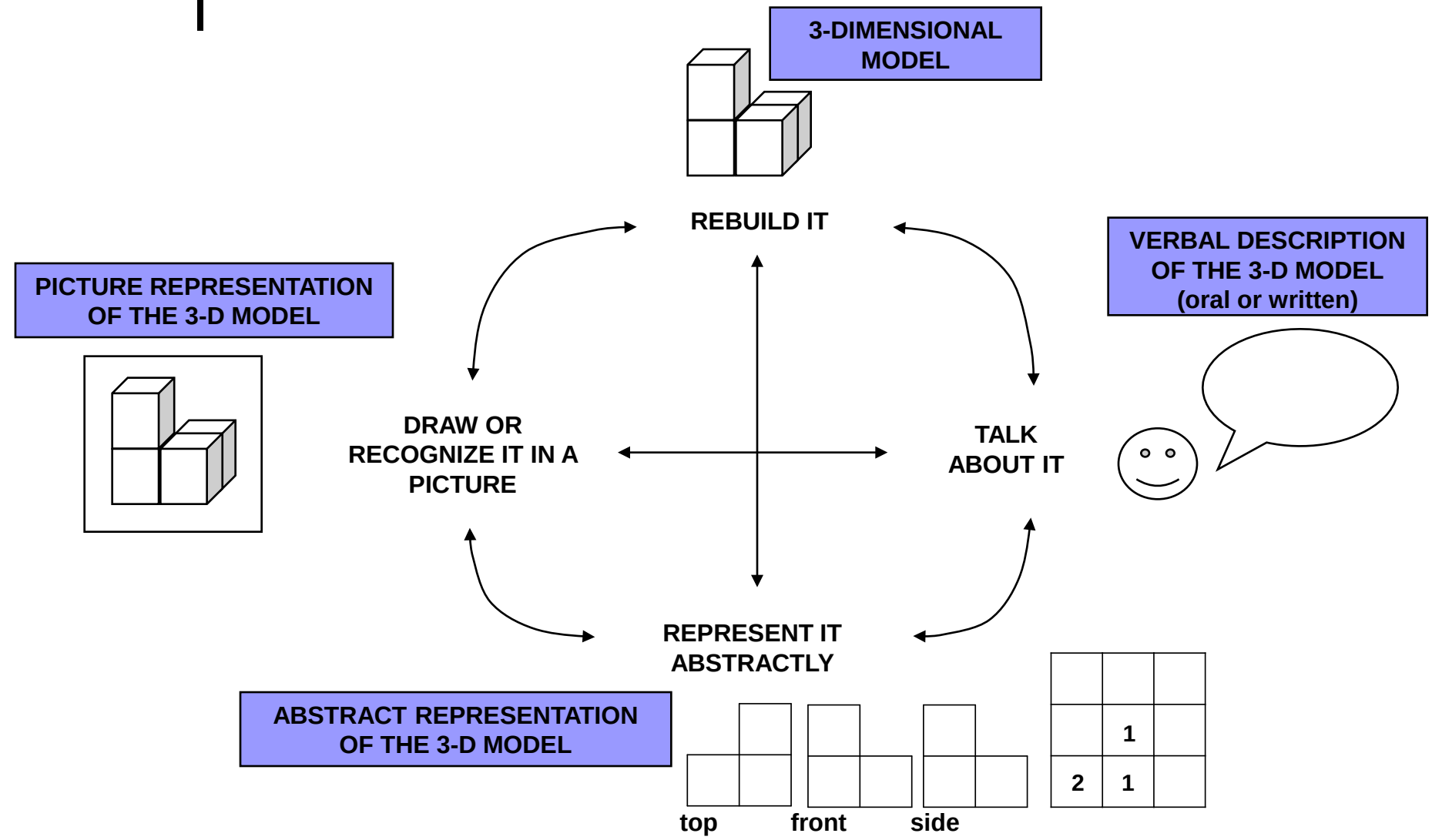
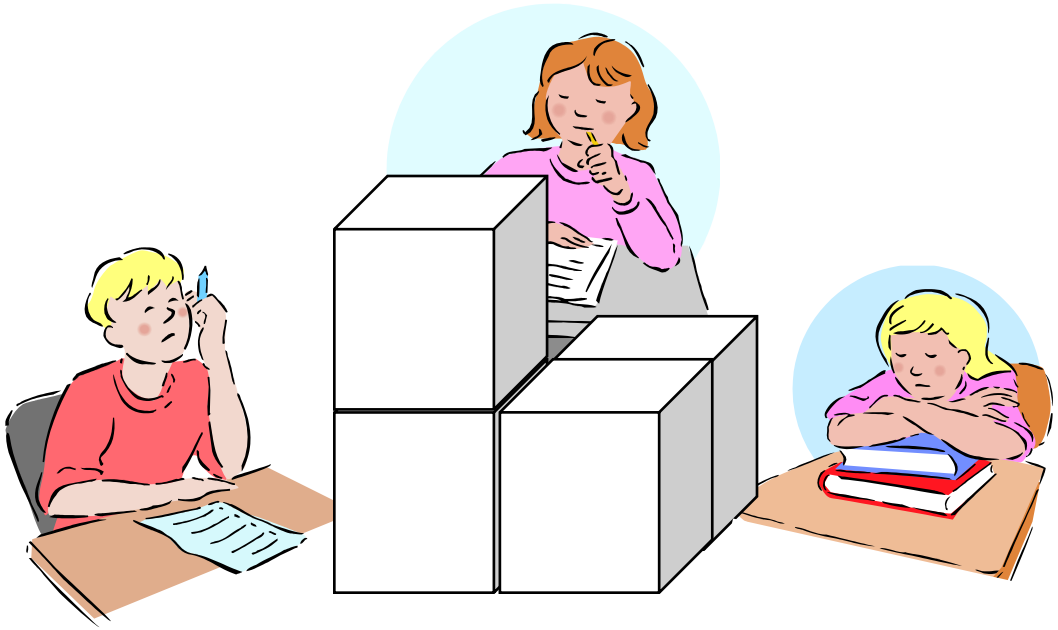
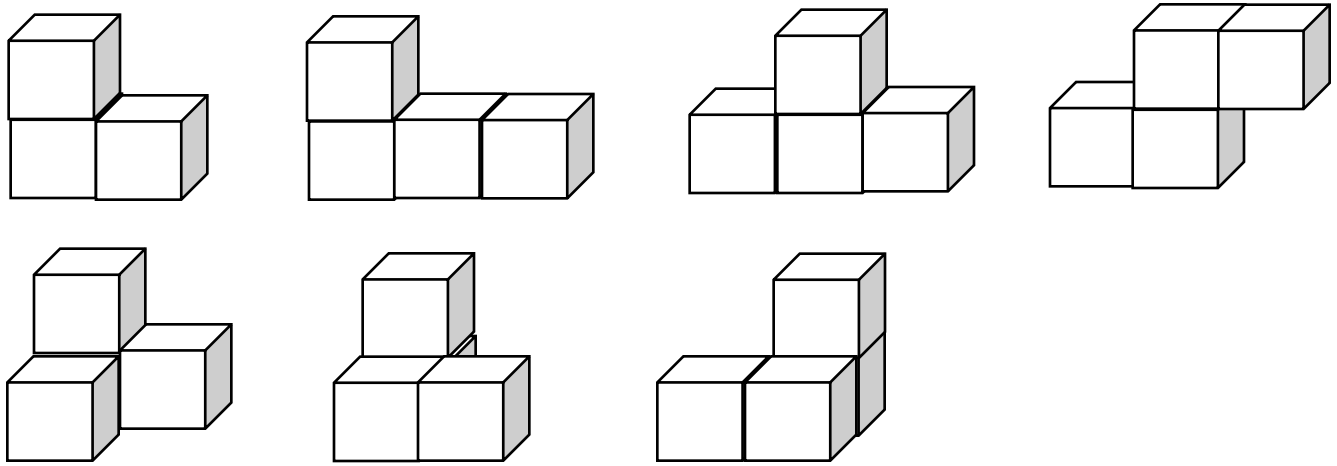


Framework for 3-Dimensional Visualization

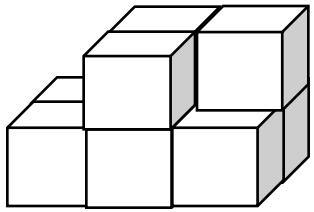


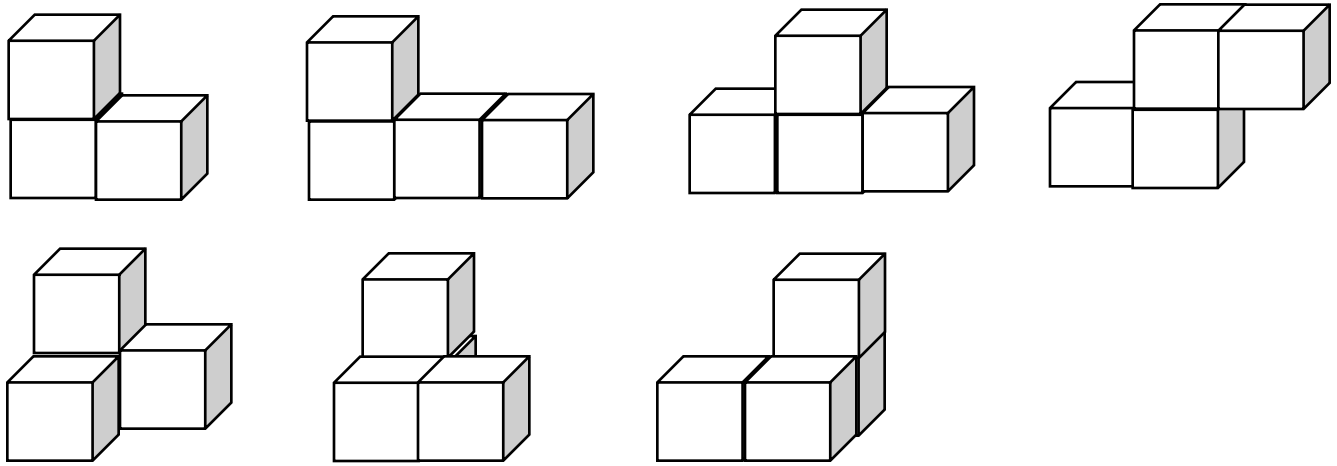


Describe/rebuild/draw the view that the person opposite from you sees.

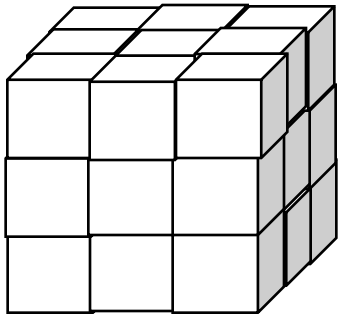


Find a pair of Soma shapes that combine to create the figure below. Is there more than one pair?

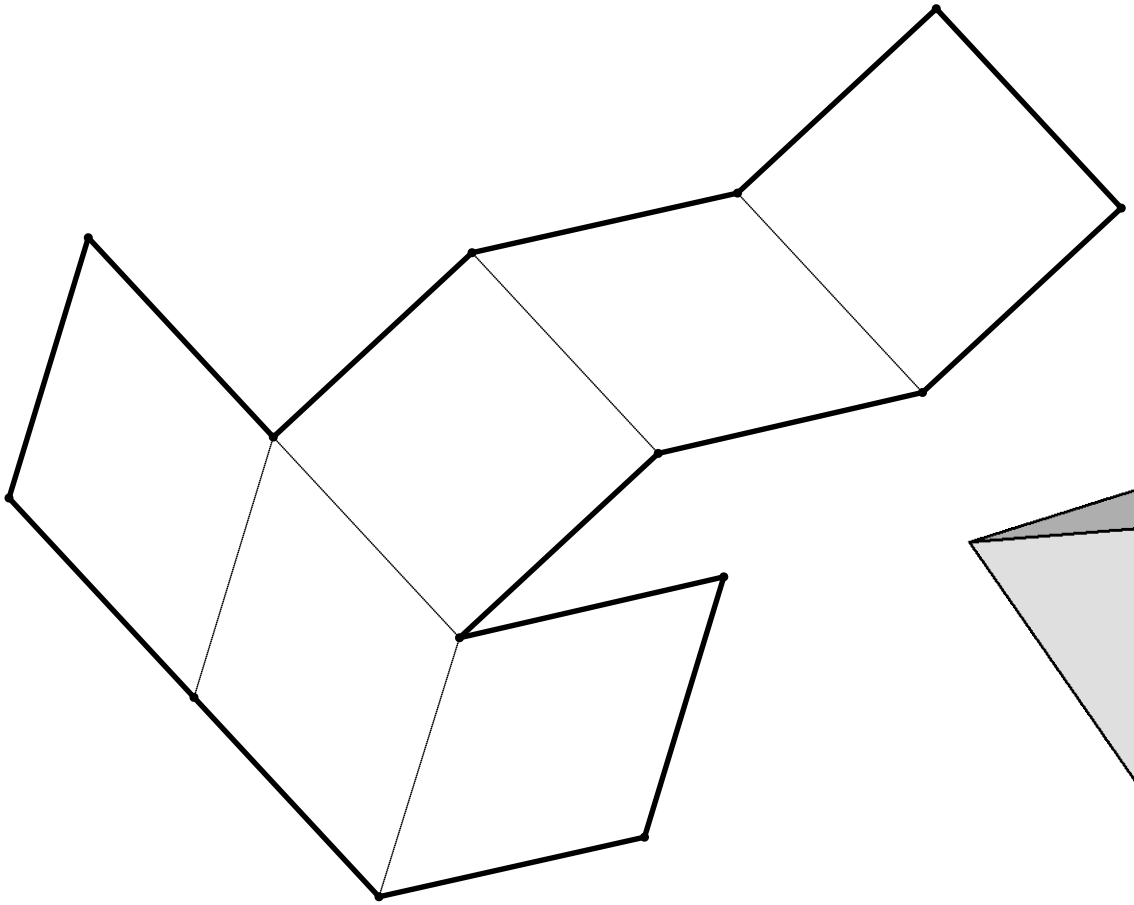


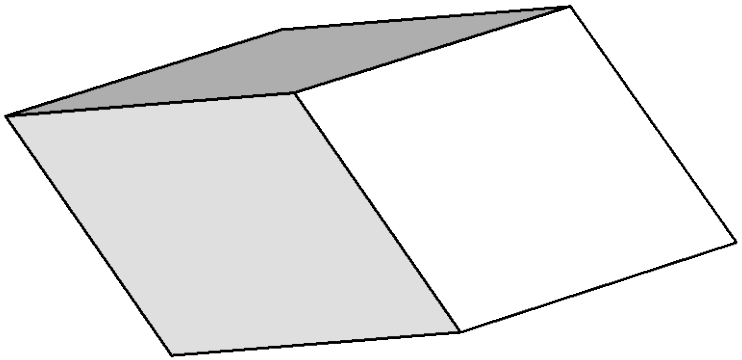


Write directions for combining the seven Soma shapes to form the 27-unit cube below.



Cut out the net and then fold along the dotted lines to form the solid shape. Describe it.





van Hiele Framework for 2-Dimensional Learning

Levels and Language

van Hiele Levels

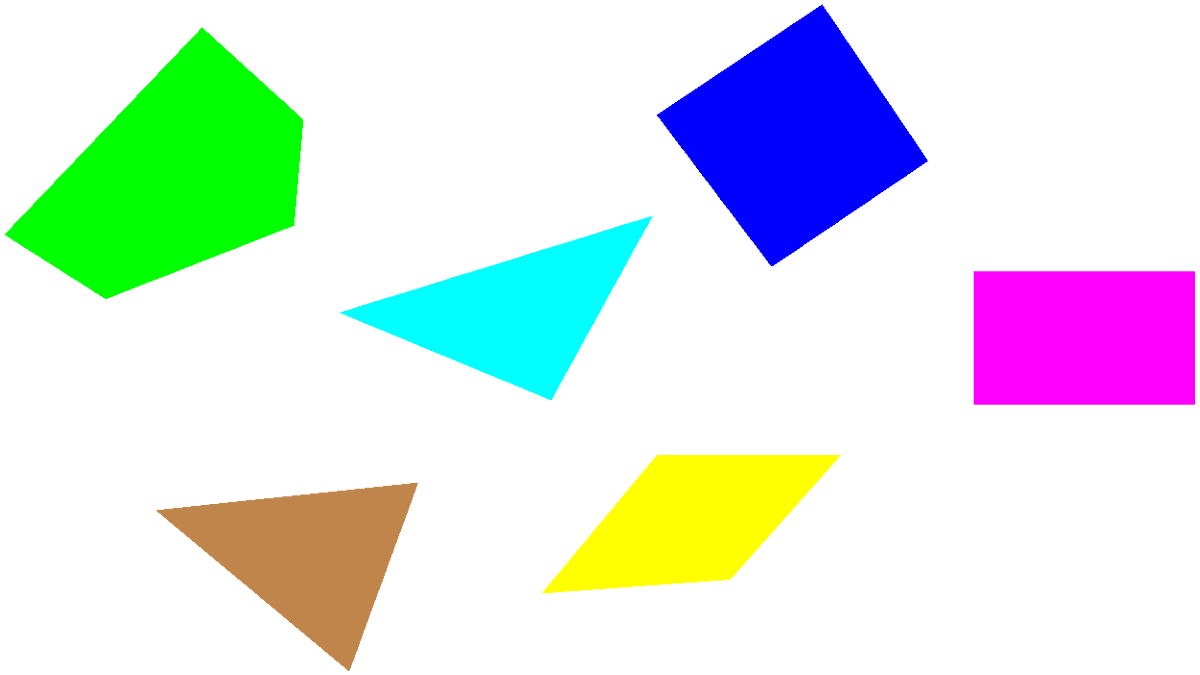
- Visual Level
- Descriptive Level
- Relational Level
- Deductive Level
- Rigor

Visual Level

Students

- identify, compare and sort shapes based on their appearance as a whole;
- use informal language;
- do NOT analyze shapes in terms of their components or properties.

Visual Level



Sort the shapes into two sets.

Describe your sorting rule.

Descriptive Level

Students

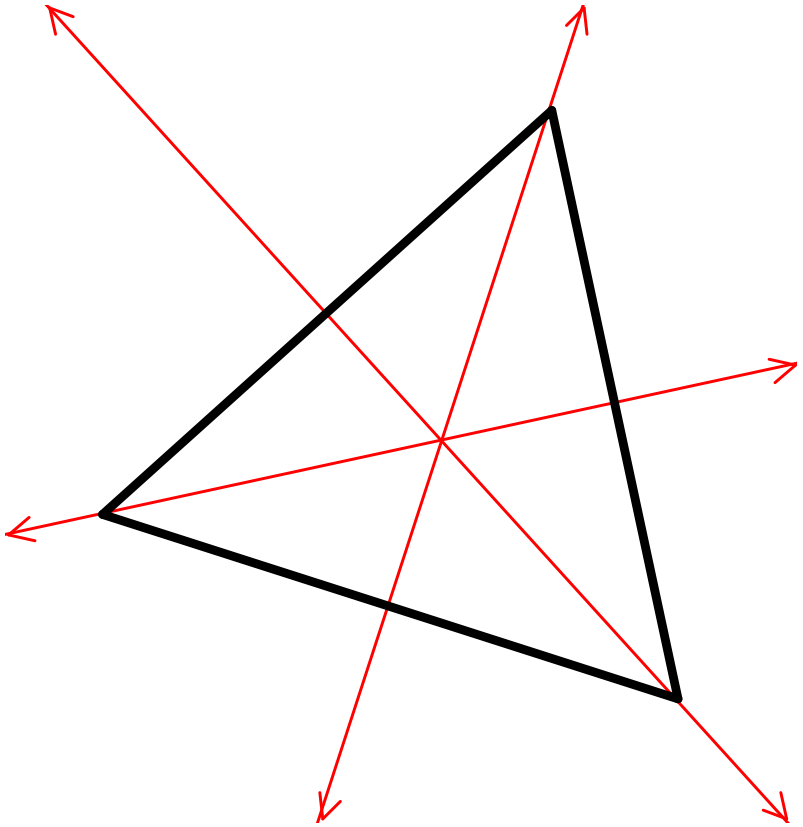
- recognize and describe shapes in terms of their properties;
- discover properties by observing, measuring, drawing and modeling;
- use formal language and symbols;
- list properties exhaustively rather than sufficiently;
- do NOT see purpose in deductive proof.

Descriptive Level

Construct an equilateral triangle.

Find its lines of symmetry.

Make a list of properties for equilateral triangles.



Relational Level

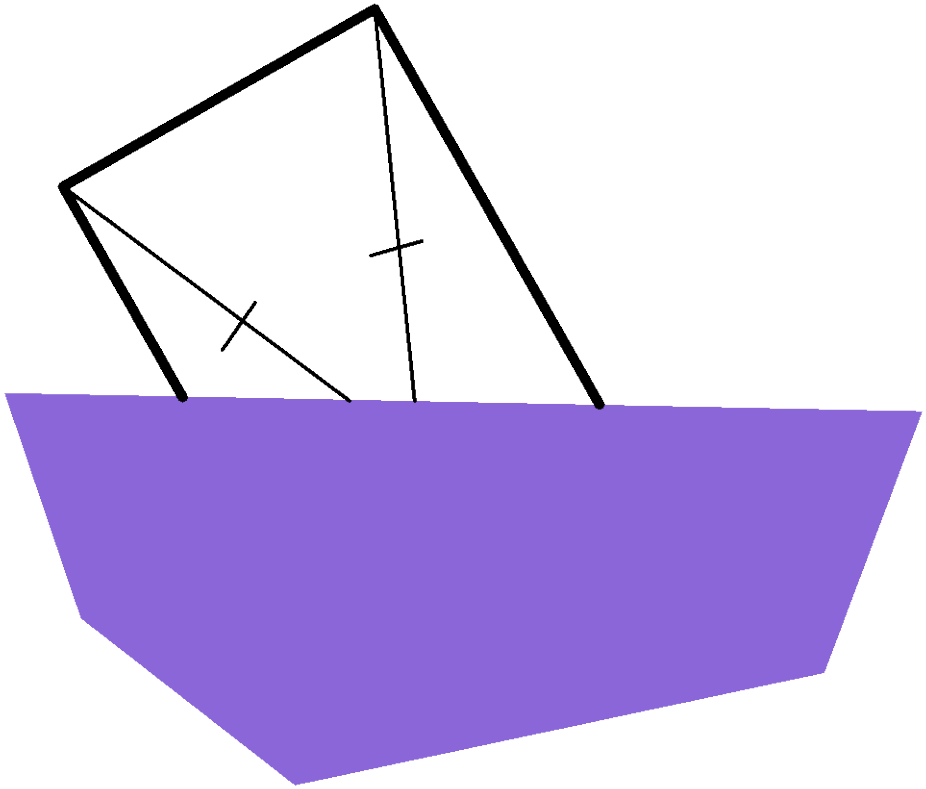
Students

- define figures using minimal (sufficient) sets of properties;
- discover new properties by deduction;
- follow and can supply parts of a deductive argument;
- do NOT grasp the meaning of an axiomatic system or see the interrelationships between networks of theorems.

Relational Level

A quadrilateral with marked diagonals is partially covered.

What kind of quadrilateral is it? How do you know?



Deductive (Axiomatic) Level

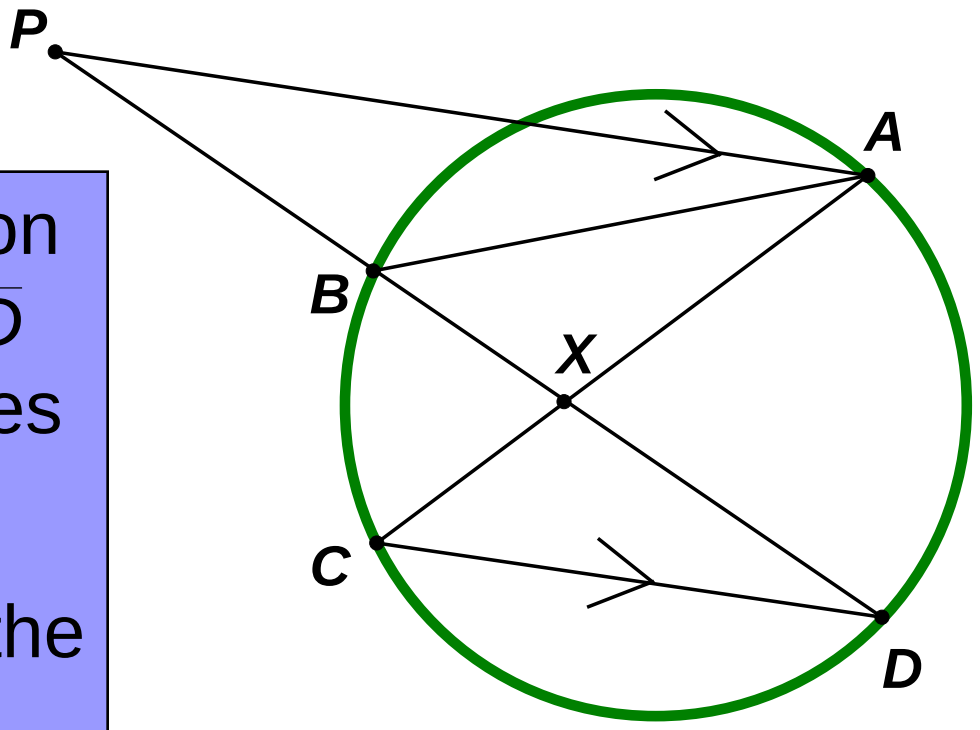
Students

- recognize and flexibly use the components of an axiomatic system (undefined terms, definitions, postulates, theorems);
- create, compare, contrast different proofs;
- do NOT compare axiomatic systems.

Deductive Level

A, B, C and D are points on a circle. $\overline{AC}$ intersects $\overline{BD}$ at X. $\overleftrightarrow{CD} \parallel \overleftrightarrow{AP}$, where P lies on $\overrightarrow{DX}$.

Prove $\overleftrightarrow{XA}$ is a tangent to the circle that circumscribes ΔABP.



Rigor

Students

- compare axiomatic systems;
- rigorously establish theorems in axiomatic systems in the absence of reference models.

van Hiele Levels

- Visual Level
- Descriptive Level
- Relational Level
- Deductive Level
- Rigor

van Hiele and Language

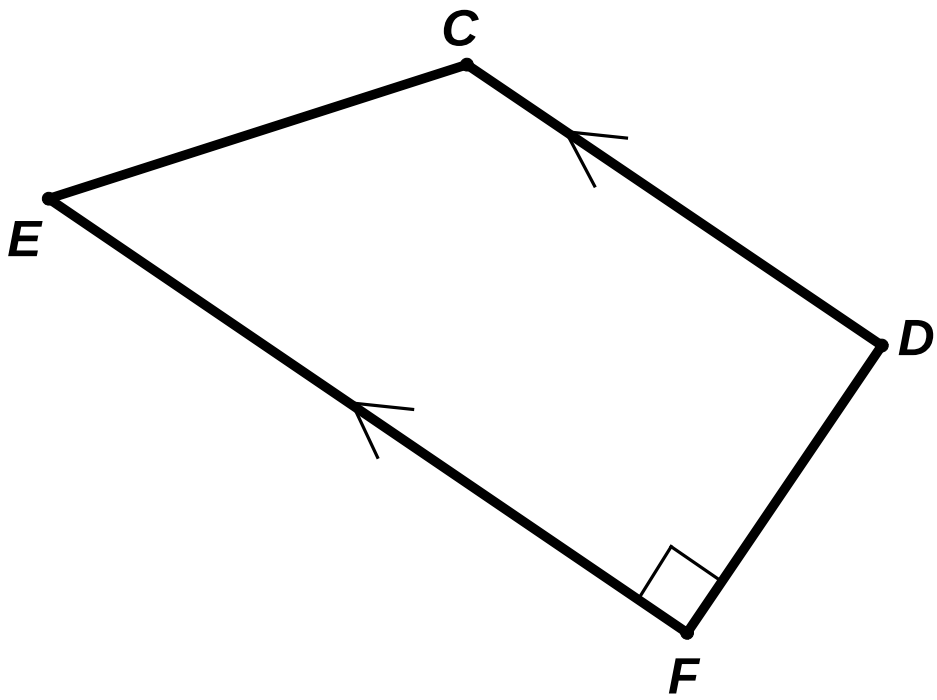
- Language is the basis for understanding and communicating.

$\overline{CD} \parallel \overline{EF}$

$\overline{CD}$ is parallel to $\overline{EF}$

$\overline{DF} \perp \overline{EF}$

$\overline{DF}$ is perpendicular to $\overline{EF}$



Definitions

- A definition of a concept is only possible if one knows, to some extent, the thing that is to be defined (van Hiele).

- How can you define a thing before you know what you have to define? Most definitions are not preconceived but the finished touch of the organizing activity. The child should not be deprived of this privilege…(Freudenthal).

What we do and what we do not do…

- It is customary to illustrate newly introduced technical language with a few examples.
- If the examples are deficient, the technical language will be deficient.
- We often neglect the importance of whole-group discussion which helps clear up misconceptions and cements understanding.

What we do and what we do not do…

- Sometimes we attempt to inform by explanation, but this is useless. Students should learn by doing, not be informed by explanation.

- The teacher must give guidance to the process of learning, allowing students to discuss their orientations and by having them find their way in the field of thinking.

Consequences

- Many textbooks are written with only the objective of the learning in place.
- Many teachers switch to, or even begin, their teaching with this objective using direct delivery approaches.
- Many teachers do not realize that their information cannot be understood by their pupils.

Children whose geometric thinking you nurture carefully will be better able to successfully study the kind of mathematics that Euclid created.

Pierre van Hiele

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