## Math at Top Speed

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2015 RUSMP Fall Networking Conference

# Today we will talk about mathematics

## **Definition of Mathematics**

"The establishment of truth, (theorems), by rigorous deduction from appropriately chosen axioms and definitions."

 Rigorous mathematical arguments first appeared in Ancient Greek mathematics, most notably Euclid's elements.

# Mathematics is the Language of Science

- Galileo: "The universe cannot be read until we have learned the language of mathematics."
- Einstein: "It is remarkable that mathematics, completely removed from reality, can be used so effectively to model and describe so much of reality."
- George Box: "All mathematical models are inaccurate, some are useful."

## **Anatomy of the Mathematical Process**

- Mathematical Modeling: The formulation of a given real-world problem, often given in word form, as a mathematical problem.
- Mathematical Analysis: Application of existing (or new) mathematical theory to find the solution of the problem at hand or demonstrate that the problem has no solution.
- Mathematical Computation: Use computational methods to calculate or approximate a solution of the mathematical problem.

## **Math Awareness**

- Historically not all science has required a knowledge of mathematics. For example Charles Darwin knew essentially no mathematics.
- Today big data science is quickly moving away from traditional mathematics and moving towards combining the tools from mathematical computation, computer science, and statistics; disciplines that did not exist 100 years ago.

### The Fair Lane Assignment Problem in BMX Bicycle Racing



# **BMX (Bicycle Motorcross)**

#### An exciting sport for boys and girls which promotes strong family interaction.

### **The BMX Race**

## The Ultimate: The Pros The Beginners: The 5 Year Olds



# BMX

- A short race encountering numerous jump and turn obstacles
- B 3 Moto System
  - Lanes assigned by random draw
  - Sometimes draw not so random
- Must win a moto to advance to next round

# **My Fatherly Duty:**

- Make my son Richard as competitive as possible
- **Richard's characteristics:**
- Very quick and coordinated
  Not excessively fast or strong

## Our approach (\*Guiding principles):

- Great gate startsSmart riding
  - **Our Implementation:**
- Train on backyard gate.
  Study the track and learn lines.

## **Training on the Backyard Gate**



## **Backyard Training Leads to Success**





# **Studying the Track**







## The Advantage of a World-Class Mechanic



### **Discussions with the Coach**









# **The Support Group**







## **Parental Frustration**



## The Curse of Lane 8

Bad luck Lane 8 Longer distance Easily held outside Good luck • Lane 3


#### **Curse of Lane 8**



## In a Highly Competitive National Meet, Is It Possible to Win Out of Lane 8?

#### It Is Essentially Impossible!

#### Watch Russel and Richard



#### However, It Is Possible

#### Watch Julie and Richard



#### **A Mathematical Challenge**

**Develop a BMX lane assignment** process that is as fair as possible to all riders and can be implemented on any given track. In this process a rider should never be assigned three lane eights (or even two).

#### Breakthrough Idea: Priority System

Work with lane priorities, instead of lane themselves:

- 1: represents the best lane
- 2: represents the second best lane
- 8: represents the worst lane

From an optimal priority assignment and a ranking of lanes at a particular track, we can obtain an optimal lane assignment.

### **Priority System Example**

<b>Priorities</b>	Track#1	Track#2
1	3	4
2	4	5
3	2	6
4	5	1
5	1	3
6	6	2
7	7	7
8	8	8

#### **Breakthrough Idea**

At start of day, have a random draw for triples from the "fairest" set of eight triples (and not 3 random draws for singles).

Insures some sense of fairness
No triples should have 3 lane 8's

#### An Example of Triples Giving Lane Assignments

	Moto 1	Moto 2	Moto 3
Rider 1	5	2	6
Rider 2	2	4	7
Rider 3	3	7	4
Rider 4	7	1	5
Rider 5	6	6	2
Rider 6	4	8	3
Rider 7	8	5	1
Rider 8	1	3	8

#### Isn't the Curse of Lane 8 Rare?

Total number of distinct sets of 8 triples with a triple of (8,8,8):

Prob. = 
$$(1/64) = 1.56\%$$

Total number of distinct set of 8 triples with at least two 8's in any given triple: Prob. = 34.38%

#### Phase I: Building a Mathematical Model

#### **Math Preliminaries**

If a rider is assigned priority triple  $(P_1, P_2, P_3)$ , then his/her

Priority Sum =  $P_1 + P_2 + P_3$ 

Equal Opportunity to Advance  $\Leftrightarrow$  Equal Weighted Priority Sum

What we want to do is find the set of eight triples where the difference between the maximum priority sum of a rider and the minimum priority sum of a rider is as small as possible.

#### **More on Lane Assignments**

Only 1 rider per lane in each moto. **Feasible Set** Infeasible Set (1,4,5)(1,4,5)(2,3,7)(2,3,7)(3, 6, 4)(3, 6, 4)(4, 1, 2)(4, 1, 2)(5,7,3)(5, 7, 3)(6,2,8)(6,2,8)(7, 8, 1)(7, 7, 1)(8,5,6)(8,5,6)

#### Our Mathematical Model: A Discrete Optimization Problem

#### Minimize

unfairness = max priority  $sum_i - \min_{1 \le i \le 8} priority sum_i$ 

#### over all feasible sets of 8 triples

### **Phase II: A Solution Technique**

The set we are searching over is too large. We cannot search over this set. Hence we must find a clever way to limit our search.

#### **A Solution Technique**

Total priority in a moto is:

(1+2+3+4+5+6+7+8)=36

Thus the total priority sum for all 3 motos is:

- 3 × 36 = 108

So to be fair, every one of the 8 racers should get a priority sum of:

- 108 ÷ 8 = 13.5

#### We Have a Problem

Adding 3 integers gives another integer.

Priority values are integers.

13.5 is NOT an integer. Therefore the best we can do is get the priority sum of each rider as close to 13.5 possible.

So we want a set of 8 triples with priority sums equal to 13 and 14.

#### **An Observation**

 $x + y = 8 \leftarrow$  the total number of riders has to be equal to 8.

13x + 14y = 108 ← the sum of riders with priority sum of 13 and 14 must be equal to the total Priority Sum, which is 108.

The unique Solution is x = 4 and y = 4

Thus if an optimal solution with unfairness measure equal to 1 exists, 4 of the triples will have Priority Sum 13 and 4 will have Priority Sum 14.

#### **Our Selective Search Algorithm**

In less then 1 second we found 7,812 optimal solutions, i.e. solutions with unfairness measure equal to 1 (which is the best that can be done).

#### **An Optimal Solution**

P	Prioritie	<u>es</u>	<u>Sum</u>			Lanes		<u>Sum</u>
1	7	6	(14)		3	7	6	(16)
2	4	7	(13)		4	5	7	(16)
3	8	2	(13)		2	8	4	(14)
4	5	4	(13)	$\Leftrightarrow$	5	1	5	(11)
5	1	8	(14)		1	3	8	(12)
6	3	5	(14)		6	2	1	(9)
7	6	1	(14)		7	6	3	(16)
8	2	3	(13)		8	4	2	(14)

**58** 

#### Wait!

#### We have too many solutions.

#### This is an indication that we have no put enough information into the model.

#### Modifying the Original Model: An Observation

If all riders are equally skilled, then the probability of winning the First moto =  $\frac{1}{8}$ 

Second moto  $=\frac{1}{7}$ 

Third moto  $=\frac{1}{6}$ 

Therefore a good lane in the third moto is more valuable than a good lane in the second moto, which in turn is more valuable than a good lane in the first moto. If a rider is assigned priority triple  $(P_1, P_2, P_3)$ , then his/her

# Weighted Priority Sum = $\frac{1}{8}P_1 + \frac{1}{7}P_2 + \frac{1}{6}P_3$

Objective: Still the same except now we use the weighted priority sum.

#### **Success**

Our Search Algorithm Found the Following Unique Optimal Solution

	Priorities		<u>Sum</u>		<u>Lanes</u>		<u>Sum</u>
1	7	5	(13)	3	7	1	(11)
2	5	6	(13)	4	1	6	(11)
3	3	7	(14)	2	2	7	(11)
4	1	8	(13)	5	3	8	(16)
5	8	1	(14)	1	8	3	(12)
6	6	2	(14)	6	6	4	(16)
7	4	3	(14)	7	5	2	(14)
8	2	4	(13)	8	4	5	(17)

#### Not only do the riders care, but ...

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#### Cycling/Recreation

#### Professor uses math to uncover flaw in BMX racing

BMX racing isn't fair, and a Rice University mathematician can prove it.

Math has been a lifelong pursuit for Richard Tapia, Ph.D., a professor of computational and applied



Cycling Notebook Steve Slevert

types of racing. As his son, Richard, Jr., became active in BMX rac-

mathemat-

ics at Rice

an affinity

for various

ing, it didn't take many trips to the

track for Tapia to realize that the deck was stacked against riders who were saddled with lane eight.

"In BMX, the lane that racers draw is a significant factor in determining whether they win or lose," Tapia said. "In fact, racers in lane eight have very

little chance of winning because the lane is so far from the first turn. I call it the 'curse of lane

eight.' " Tapia witnessed the curse firsthand during an event in which his son, competing in three separate heats - or motos in BMX vernacular, drew lane eight three consecutive He also has times. Each time, he finished in

the middle of the pack. "There's something unfair about a situation in which someone can pull lane eight three times in a row," Tapia said. "So I started thinking about ways to improve the system."

As you would expect from a professor whose livelihood revolves around numbers and formulas, he turned to math for the answer.

"I wanted to devise eight triples so they are fairest to the riders according to some crite-ria," Tapia said. "This is a collection of three numbers that would be assigned to riders for each of the three motos." Tapia initially scratched out

#### **BP MS 150 training schedule**

The latest training schedule for the April 12-13 BP MS 150 as sug-gested by USA Fit, Luke's Locker and Memorial Athletic Club: Week of March 2

 Sunday — Novice, 20 miles easy; intermediate, 25 miles easy; experienced, 25 miles easy to moderate.

Monday — Of.

 Tuesday — Novice, 45 minutes to hour indoor cycling with strength intervals or spin class: intermediate. 45 minutes to hour indoor cycling with strength intervals or spin class; experienced, 60 minutes indoor cy-

some triples - such as one, five, seven and three, four, two - to run by Richard, Jr. After that first round of feedback. Tapia began using computer programs, formulas and algo-rithms to identify eight optimal triples from millions of combinations.

Once he determined the best possible combinations, Tapia

cling with strength intervals of spin class Wednesday — Off. Thursday — Novice, 45 minutes

moderate; intermediate, 60 minutes moderate; experienced, 75 minutes moderate Friday — Off.

 Saturday — Novice, 30 minutes; intermediate, 40 minutes; experienced, 45 minutes. Sunday -- Novice, 40 miles or

Space Race 60 miles: intermediate. Space Race 60 miles: experienced, Space Race 60 or 100 miles.

had an intriguing and powerful example of how math can be applied to everyday life.

"I use this example in presen-tations I have with students and teachers across the country," Tapia said. "I can show them that mathematics can be very practical in their lives. It isn't just used in a vacuum without relevancy to the real world."

Tapia's talks with students and at national mathematics meetings serve a dual purpose. They expound on the benefits and impact of math while at the same time promote BMX cv-

cling. "They have a personal feel to them," Tapia said. "My presen-tation shows my family at the BMX track, Richard racing and how much fun BMX racing can be. Then I pose the question about the fairness of lane assignments and demonstrate how mathematics can be used to make the lane-assignment system better."

Tapia is now in the process of documenting his "triples study" and intends to have the work published in a mathematics journal

Actually utilizing the triples system at BMX tracks across the country might prove to be more challenging

"Tve talked with a couple of track officials who are interested, but as with many other breakthroughs, there is some reluctance to change," Tapia

said.

Tapia remains undeterred by the initial resistance and plans to continue to use this study as a demonstration of math's realworld impact.

"It has become a valuable tool for me," he said. "It's something people can relate to."

MS 150 kick-off event ---More than 1,500 cyclists are ex-pected for a kick-off party Fri-day to officially break the seal on the training season for the BP MS 150 blice tour.

The Mardi Gras-themed festival will be held at Jillian's, 7620 Katy Freeway, from 4 to 7 pm. The event gives riders a chance to register for the tour, turn in piedge money, or enlist as a Pedal Partner - giving riders a face to fight for during their ride to help stop the devastat-ing effects of multiple scierosis.

Steve Slevert covers cycling for the Chronicle. His notebook appears Thursdays. Call him at 832-978-4359 or send e-mails to cycling@ev1.net.

## Using Math to Construct a Psychedelic Video of My 1970 Chevelle Show Car





# **"Heavy Metal"** (Three Times National Show Car Champion)





#### BEST IN SHOW

GRANT PETERSON

It furnity how many liftle old ladies have been unknowingly stakked over the years by men who cover their hustworthy, clear, one-owner cars. I've done it, my Data has done it, my heads have done it, and Richard Tigo has also done it. In Richard's case, their life and add lady in question proved to be well acquarted with her '70 Chevela and gave. Recruit the rahast'' before he could pops the question of sucharang her cas. Much to his charant the rahast'' before he could pops the question of sucharang her cas. Much to his charant, he went home empty-handed, but he didn't underestimate the power of visualization.

A few years later, this same Chwelle pulled up in hort of Richard's home in Houston, Tecas, Surprised, and trying to make heads or talk of the situation, his thend Bill Kolley emerged from the car and expland that the just bought it from an entate asel What are the

The Tapia's Award-Winning '70 Chevelle



42 www.b.petche-y.com



#### The Entire Family Helps Prepare the Cars

![](_page_71_Picture_0.jpeg)








## **Mathematical Challenge**

# Use Mathematics to Create a Psychedelic Video that Will Be a Part of the Car Exhibit

### **Enter Rice Student Josef Sifuentes**



# **Objectives**

- To use mathematics to create a video to show with the car that is "cool" and visually exciting.
- To create a piece of artwork that captures the feel and spirit of Heavy Metal music, muscle cars, and social rebellion from the late 60's and early 70's.
- To demonstrate another outlet of creativity that combines mathematics and an interesting real world activity.

# **Outreach Objectives**

By combining art and mathematics through cars we can spark an interest in youth who do not appreciate the role that mathematics can play in exciting everyday activities.

# **Artwork of Found Materials**

- A 1970 Chevelle
- Collage of photos and muscle car ads from the 70's
- The Navier-Stokes Partial Differential Equations
- Singular Value Decomposition

## **XNS Fluid Flow Simulation**

Incompressible Navier-Stokes Partial Differential Equations

$$\rho\left(\frac{\partial u}{\partial t} + u \cdot \nabla u - f\right) - \mu \nabla^2 u + \nabla p = 0$$

 $\boldsymbol{\nabla} \boldsymbol{\cdot} \boldsymbol{\boldsymbol{\mathcal{U}}} = \boldsymbol{\boldsymbol{0}}$ 

## **Finite Element Method**

The Navier Stokes are applied at a finite number of nodes. The values at each nodes determine the values within each element.



### Flow Around the Chevelle Contour



### Make Contour of Chevelle the Boundary Conditions

### (flow inside Chevelle)



### Manipulate Mesh Spacing to Create New Special Effects



### Fun with Singular Value Decomposition



#### Rank = 450

Rank =100 (full rank between 450 and 460 for 3 RGB matrices)



Rank =50 (full rank between 450 and 460 for 3 RGB matrices)



Rank =20 (full rank between 450 and 460 for 3 RGB matrices)



Rank =5 (full rank between 450 and 460 for 3 RGB matrices)



# **Heavy Metal and the Seventies**

- Incorporate sociological themes from the late 60's and 70's
  - Rebellion!!!
  - Heavy Metal music
  - Muscle cars

### Rebellion



## **Heavy Metal Music**



### **Muscle Cars**



1970 Chevelle SS 396. It's getting tougher and tougher to resist. The standard V8 has been kicked up to 350 hp.

A new air-gulping Cowl Induction Hood awaits your order.

You can also order your choice of a floor-mounted 4-speed or the 3-range Turbo Hydra-matic.

Under that lean and hungry look is a lean and agile suspension. F70 x 14 white-lettered wide oval treads. 7"-wide mag-type wheels. And power disc brakes. Your mission is to infiltrate your Chevy dealer's and escape with this car. It will go willingly.



Putting you first, keeps us first.



### **The Movie**

## HEAVY METAL CHEVELLE

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## **THANK YOU**



Est. 1998

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