## Math at Top Speed

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## Today we will talk about mathematics

## Definition of Mathematics

"The establishment of truth, (theorems), by rigorous deduction from appropriately chosen axioms and definitions."

- Rigorous mathematical arguments first appeared in Ancient Greek mathematics, most notably Euclid's elements.


# Mathematics is the Language of Science 

- Galileo: "The universe cannot be read until we have learned the language of mathematics."
- Einstein: " It is remarkable that mathematics, completely removed from reality, can be used so effectively to model and describe so much of reality."
- George Box: "All mathematical models are inaccurate, some are useful."


## Anatomy of the Mathematical Process

- Mathematical Modeling: The formulation of a given real-world problem, often given in word form, as a mathematical problem.
- Mathematical Analysis: Application of existing (or new) mathematical theory to find the solution of the problem at hand or demonstrate that the problem has no solution.
- Mathematical Computation: Use computational methods to calculate or approximate a solution of the mathematical problem.


## Math Awareness

- Historically not all science has required a knowledge of mathematics. For example Charles Darwin knew essentially no mathematics.
- Today big data science is quickly moving away from traditional mathematics and moving towards combining the tools from mathematical computation, computer science, and statistics; disciplines that did not exist 100 years ago.


## The Fair Lane Assignment Problem in BMX Bicycle Racing



## BMX (Bicycle Motorcross)

An exciting sport for boys and girls which promotes strong family interaction.

## The BMX Race

The Ultimate: The Pros
The Beginners: The 5 Year Olds


## BMX

- A short race encountering numerous jump and turn obstacles
- 3 Moto System
- Lanes assigned by random draw
- Sometimes draw not so random
- Must win a moto to advance to next round


## My Fatherly Duty:

Make my son Richard as competitive as possible
Richard's characteristics:

- Very quick and coordinated
- Not excessively fast or strong


## Our approach resuiding piniopiless:

## Great gate starts <br> Smart riding

## Our Implementation:

Train on backyard gate.
Study the track and learn lines.

## Training on the Backyard Gate



## Backyard Training Leads to Success




## Studying the Track





# The Advantage of a World-Class Mechanic 



## Discussions with the Coach






## The Support Group





## Parental Frustration



## The Curse of Lane 8

- Bad luck
- Lane 8
- Longer distance
- Easily held outside

Good luck

- Lane 3

Curse of Lane 8


## In a Highly Competitive National Meet, Is It Possible to Win Out of Lane 8?

Watch<br>Russel and Richard



# However, It Is Possible 

## Watch <br> Julie and Richard



## A Mathematical Challenge

Develop a BMX lane assignment process that is as fair as possible to all riders and can be implemented on any given track. In this process a rider should never be assigned three lane eights (or even two).

## Breakthrough Idea: Priority System

Work with lane priorities, instead of lane themselves:
1: represents the best lane
2: represents the second best lane

8: represents the worst lane
From an optimal priority assignment and a ranking of lanes at a particular track, we can obtain an optimal lane assignment.

## Priority System Example

| Priorities | Track\#1 | Track\#2 |
| :---: | :---: | :---: |
| 1 | 3 | 4 |
| 2 | 4 | 5 |
| 3 | 2 | 6 |
| 4 | 5 | 1 |
| 5 | 1 | 3 |
| 6 | 6 | 2 |
| 7 | 7 | 7 |
| 8 | 8 | 8 |

## Breakthrough Idea

At start of day, have a random draw for triples from the "fairest" set of eight triples (and not 3 random draws for singles).

- Insures some sense of fairness No triples should have 3 lane 8's


## An Example of Triples Giving Lane Assignments

|  | Moto 1 | Moto 2 | Moto 3 |
| :--- | :---: | :---: | :---: |
| Rider 1 | 5 | 2 | 6 |
| Rider 2 | 2 | 4 | 7 |
| Rider 3 | 3 | 7 | 4 |
| Rider 4 | 7 | 1 | 5 |
| Rider 5 | 6 | 6 | 2 |
| Rider 6 | 4 | 8 | 3 |
| Rider 7 | 8 | 5 | 1 |
| Rider 8 | 1 | 3 | 8 |

# Isn't the Curse of Lane 8 Rare? 

Total number of distinct sets of 8 triples with a triple of $(8,8,8)$ :

$$
\text { Prob. }=(1 / 64)=1.56 \%
$$

Total number of distinct set of 8 triples with at least two 8's in any given triple:

$$
\text { Prob. }=34.38 \%
$$

## Phase I: <br> Building a Mathematical Model

## Math Preliminaries

If a rider is assigned priority triple $\left(P_{1}, P_{2}, P_{3}\right)$, then his/her

$$
\text { Priority Sum }=P_{1}+P_{2}+P_{3}
$$

Equal Opportunity to Advance $\Leftrightarrow$ Equal Weighted Priority Sum
What we want to do is find the set of eight triples where the difference between the maximum priority sum of a rider and the minimum priority sum of a rider is as small as possible.

## More on Lane Assignments

Only 1 rider per lane in each moto.
Feasible Set
Infeasible Set
$(1,4,5)$
$(2,3,7)$
$(3,6,4)$
$(4,1,2)$
$(5,7,3)$
$(6,2,8)$
$(7,8,1)$
$(8,5,6)$
$(1,4,5)$
$(2,3,7)$
$(3,6,4)$
$(4,1,2)$
$(5,7,3)$
$(6,2,8)$
$(7,7,1)$
$(8,5,6)$

## Our Mathematical Model: A Discrete Optimization Problem

## Minimize

$$
\text { unfairness }=\max _{1 \leq i \leq 8} \text { priority } \text { sum }_{i}-\min _{1 \leq i \leq 8} \text { priority sum }{ }_{i}
$$

## over all feasible sets of 8 triples

## Phase II: A Solution Technique

The set we are searching over is too large. We cannot search over this set. Hence we must find a clever way to limit our search.

## A Solution Technique

- Total priority in a moto is:
- $(1+2+3+4+5+6+7+8)=36$
- Thus the total priority sum for all 3 motos is:
- $3 \times 36=108$
- So to be fair, every one of the 8 racers should get a priority sum of:
- $108 \div 8=13.5$


## We Have a Problem

Adding 3 integers gives another integer.
Priority values are integers.
13.5 is NOT an integer. Therefore the best we can do is get the priority sum of each rider as close to 13.5 possible.

So we want a set of 8 triples with priority sums equal to 13 and 14.

## An Observation

$x+y=8 \leftarrow$ the total number of riders has to be equal to 8 .
$13 x+14 y=108 \leftarrow$ the sum of riders with priority sum of 13 and 14 must be equal to the total Priority Sum, which is 108.

## The unique Solution is $x=4$ and $y=4$

Thus if an optimal solution with unfairness measure equal to 1 exists, 4 of the triples will have Priority Sum 13 and 4 will have Priority Sum 14.

## Our Selective Search Algorithm

In less then 1 second we found 7,812 optimal solutions, i.e. solutions with unfairness measure equal to 1 (which is the best that can be done).

## An Optimal Solution

## Priorities Sum

## Lanes

## Sum

| 1 | 7 | 6 | $(14)$ | 3 | 7 | 6 | $(16)$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 2 | 4 | 7 | $(13)$ | 4 | 5 | 7 | $(16)$ |
| 3 | 8 | 2 | $(13)$ | 2 | 8 | 4 | $(14)$ |
| 4 | 5 | 4 | $(13)$ | 5 | 1 | 5 | $(11)$ |
| 5 | 1 | 8 | $(14)$ | 1 | 3 | 8 | $(12)$ |
| 6 | 3 | 5 | $(14)$ | 6 | 2 | 1 | $(9)$ |
| 7 | 6 | 1 | $(14)$ | 7 | 6 | 3 | $(16)$ |
| 8 | 2 | 3 | $(13)$ | 8 | 4 | 2 | $(14)$ |

## Wait!

- We have too many solutions.

This is an indication that we have no put enough information into the model.

## Modifying the Original Model: An Observation

If all riders are equally skilled, then the probability of winning the

$$
\text { First moto }=\frac{1}{8}
$$

$$
\text { Second } \operatorname{moto}=\frac{1}{7}
$$

$$
\text { Third moto }=\frac{1}{6}
$$

Therefore a good lane in the third moto is more valuable than a good lane in the second moto, which in turn is more valuable than a good lane in the first moto.

If a rider is assigned priority triple $\left(P_{1}, P_{2}, P_{3}\right)$, then his/her

Weighted Priority Sum $=\frac{1}{8} P_{1}+\frac{1}{7} P_{2}+\frac{1}{6} P_{3}$
Objective: Still the same except now we use the weighted priority sum.

## Success

| Priorities |  | Sum |  | Lanes |  | Sum |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 7 | 5 | (13) | 3 | 7 | 1 | (11) |
| 5 | 6 | (13) | 4 | 1 | 6 | (11) |
| 3 | 7 | (14) | 2 | 2 | 7 | (11) |
| 1 | 8 | (13) | 5 | 3 | 8 | (16) |
| 8 | 1 | (14) | 1 | 8 | 3 | (12) |
| 6 | 2 | (14) | 6 | 6 | 4 | ( 16) |
| 4 | 3 | (14) | 7 | 5 | 2 | (14) |
| 2 | 4 | (13) | 8 | 4 | 5 | (17) |

# Not only do the riders care, but ... 

# Houston Chronicle 

## Cycling/Recreation

## Professor uses math to uncover flaw in BMX racing

BMDX racing isn't Girr, and a
Rice University mathematician
can prove it. can prove it. Math has been a lifelong purprolessor of computational and


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Notebook
Steve

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three times in a row Tripia
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three timess in a row" Tapla
said -So started thinking
about ways to improve the sys
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## BP MS 150 training schedule

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that mathematics can be very that mathemstics can be very
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Tapin's talks with students
and at national mathematics meetings serve a dual purpose.
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same time promete Bur sline time promote BMXX or-
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It has become a valuable tool for me", he saide "This sombe-
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BP MS 150 bise toson for th BP MS Mardi Gras themed fest
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Katy Freeway, from 4 to 7 pm The event Wios riders a chance to regsoter for the tour, turn in
pledge money, or enlist as a

 Steve Slevert covers goling for poars Thursiope. Cail him at 632 978.4359 or sond ormals to oy ollingeowtinat

## Using Math to Construct a Psychedelic Video of My 1970 Chevelle Show Car



## "Heavy Metal" <br> (Three Times National Show Car Champion)





## The Entire Family Helps Prepare the Cars







## Mathematical Challenge

Use Mathematics to<br>Create a Psychedelic Video that Will Be a Part of the Car Exhibit

## Enter Rice Student Josef Sifuentes



## Objectives

- To use mathematics to create a video to show with the car that is "cool" and visually exciting.
- To create a piece of artwork that captures the feel and spirit of Heavy Metal music, muscle cars, and social rebellion from the late 60's and early 70's.
- To demonstrate another outlet of creativity that combines mathematics and an interesting real world activity.


## Outreach Objectives

By combining art and mathematics through cars we can spark an interest in youth who do not appreciate the role that mathematics can play in exciting everyday activities.

## Artwork of Found Materials

- A 1970 Chevelle
- Collage of photos and muscle car ads from the 70's
- The Navier-Stokes Partial Differential Equations
- Singular Value Decomposition


## XNS Fluid Flow Simulation

## Incompressible Navier-Stokes Partial Differential Equations

$$
\rho\left(\frac{\partial u}{\partial t}+u \cdot \nabla u-f\right)-\mu \nabla^{2} u+\nabla p=0
$$

$$
\nabla \cdot u=0
$$

## Finite Element Method

The Navier Stokes are applied at a finite number of nodes. The values at each nodes determine the values within each element.


## Flow Around the Chevelle Contour



## Make Contour of Chevelle the Boundary Conditions

(flow inside Chevelle)


Manipulate Mesh Spacing to Create New Special Effects


## Fun with Singular Value Decomposition



## Rank $=100$



## Rank $=50$



## Rank $=20$



## Rank $=5$



## Heavy Metal and the Seventies

- Incorporate sociological themes from the late 60's and 70's
- Rebellion!!!
- Heavy Metal music
- Muscle cars


## Rebellion



## Heavy Metal Music



## Muscle Cars



1970 Chevelle SS 396.
Its gettung tougher and tougher to resist. A tean and agite suspension. F70 $\times 14$ white-fettered wide oval treads. $7 \cdots$-wide
mase-type wheels. And power disc brakes.
 Hood awaits your order.
fow also
floor-mount also -order your choike of a
Turoo Hydra-matice. Putting you first, keeps us first.

Chivalust On The Move.

In ten seconds, your resistance will self-destruct.

The Movie

## mandwan



## THANK YOU



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www.tapiacenter.rice.edu

