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Mathematical Literacy Assessment Design: A Dimensionality Analysis of Programme for International Student Assessment (PISA) Mathematics Framework

Adem Ekmekci
Rice University

Ph.D. from The University of Texas at Austin
August 5th, 2013

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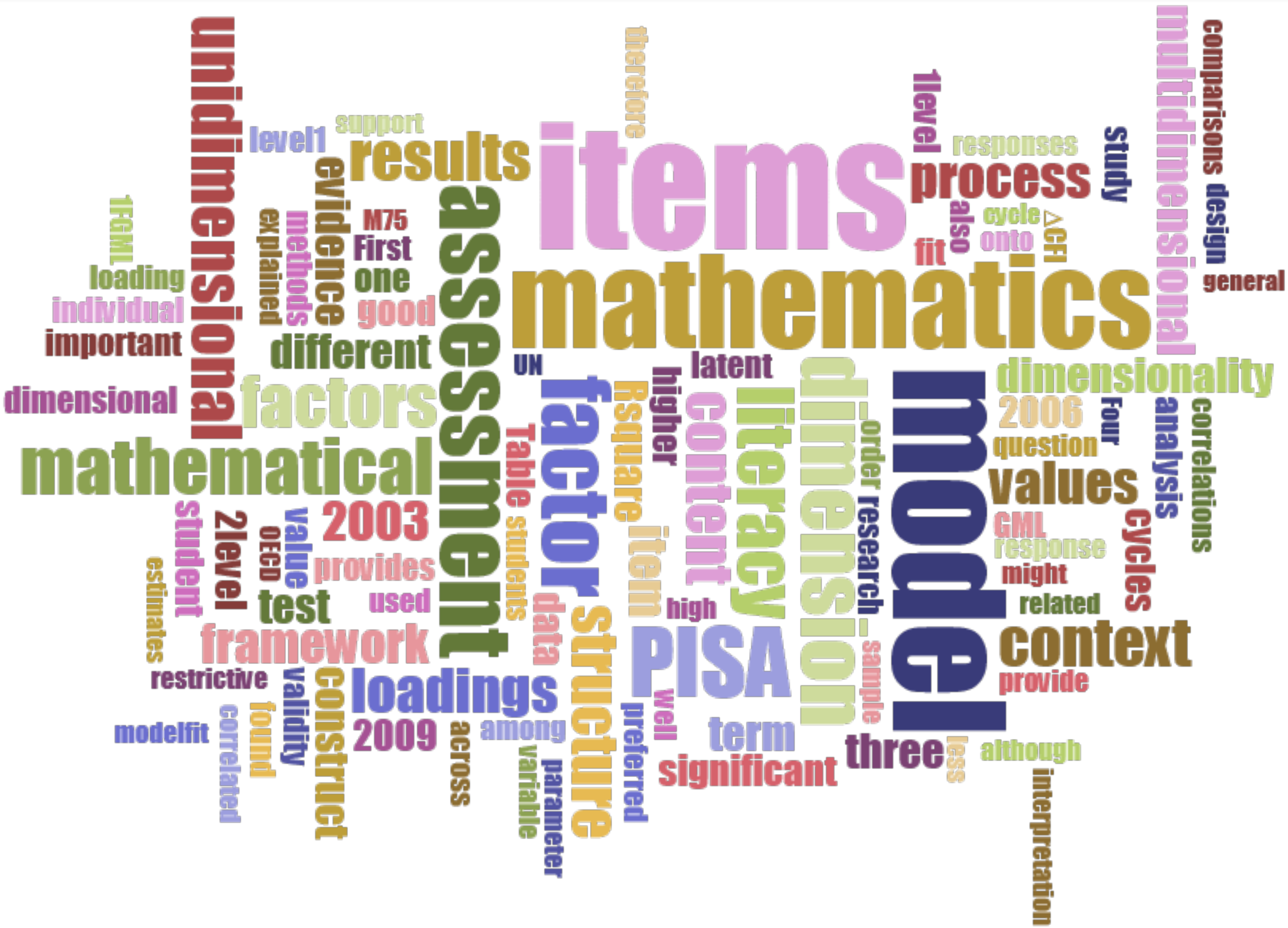

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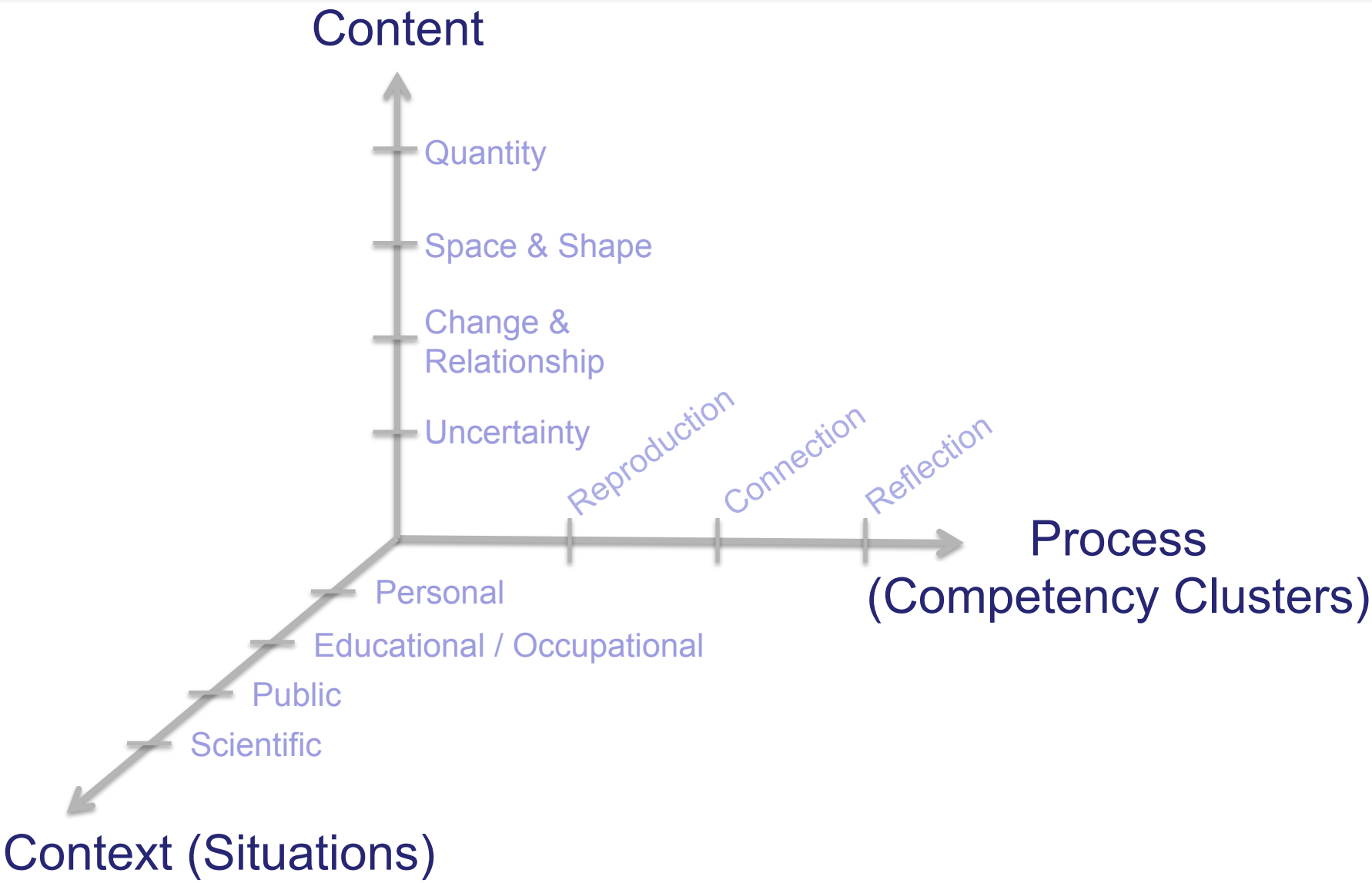
PISA

- Programme for International Student Assessment
- Assesses 15-year-olders' knowledge and competencies in *mathematics, science,* and *reading*
- OECD – Organisation for Economic Co-operation and Development
- Every 3 years starting in the year 2000 with reading as the main domain

PISA Math Framework



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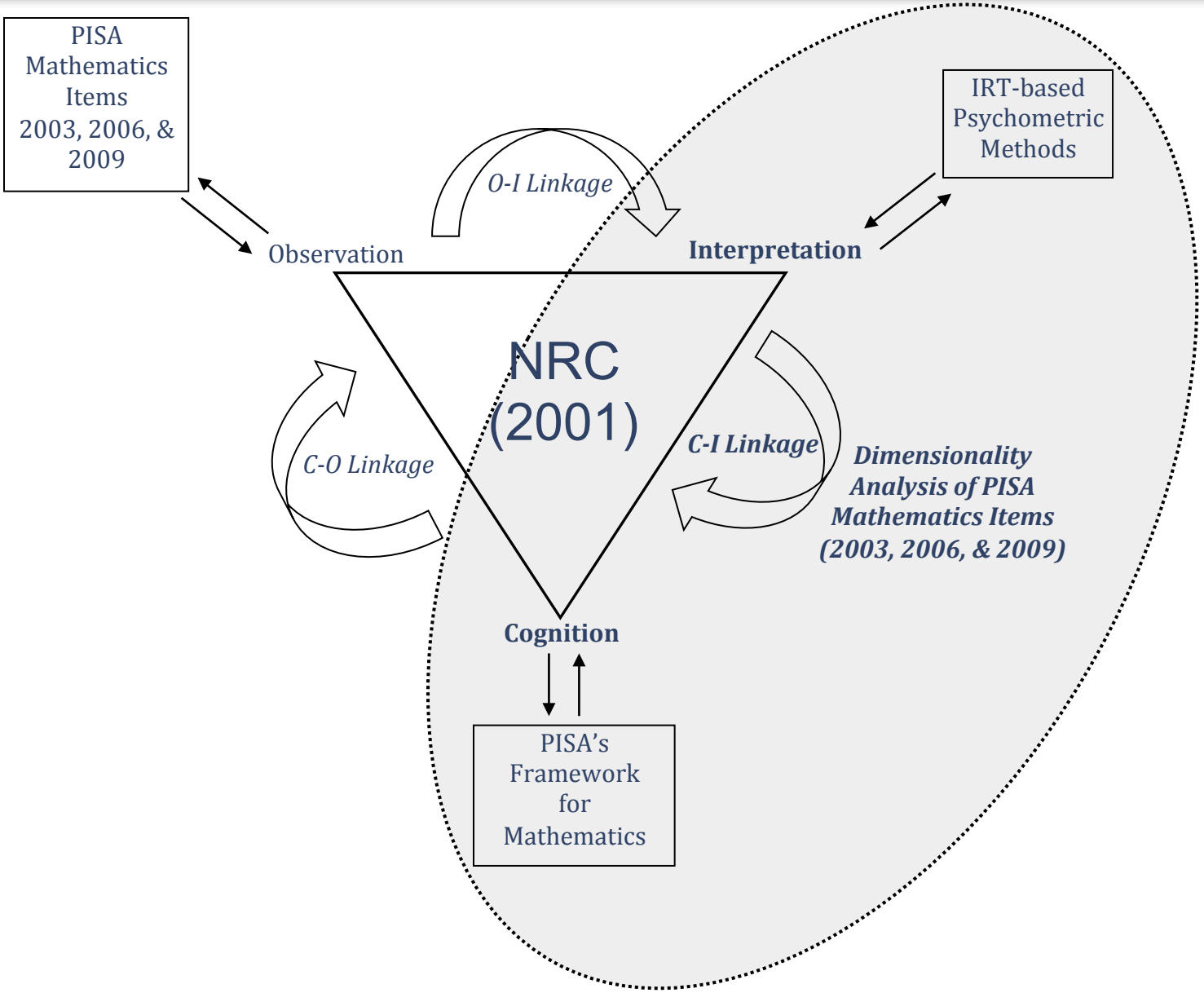
The Purpose

- To investigate the extent to which the multidimensional nature of PISA's mathematical literacy (ML) is reflected on the actual items.
- To investigate the extent to which the unidimensionality assumption is reflected on the actual items.
- To monitor the stability of these correspondence between PISA ML framework and the actual items over the three implementation cycles: 2003, 2006, 2009.

Conceptual Framework



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Participants

- About 200K students from 30 OECD countries for each of 2003, 2006, and 2009 cycles.
- Simple random sampling: **17,000 respondents**
- Student weights to ensure accurate representation of PISA population.

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Instrument

Dimensions →	**Content (84 – 48 – 35)**	**Process (84 – 48 – 35)**	**Context (84 – 48 – 35)**
Sub-dimensions →	Quantity (22 – 13 – 11)	Reproduction (26 – 11 – 9)	Personal (18 – 9 – 4)
	Space and Shape (20 – 11 – 8)	Connection (39 – 24 – 18)	Educational / Occupational (20 – 8 – 5)
	Change and Relationship (22 – 13 – 9)	Reflection (19 – 13 – 8)	Public (28 – 18 – 13)
	Uncertainty (20 – 11 – 7)		Scientific (18 – 13 – 13)

Structural Equation Modeling

- Confirmatory Factor Analytic (CFA) methods were employed.

Models:

- One unidimensional model
- Three 1-level models (Content, Process, Context)
- Three 2-level models (Content, Process, Context)

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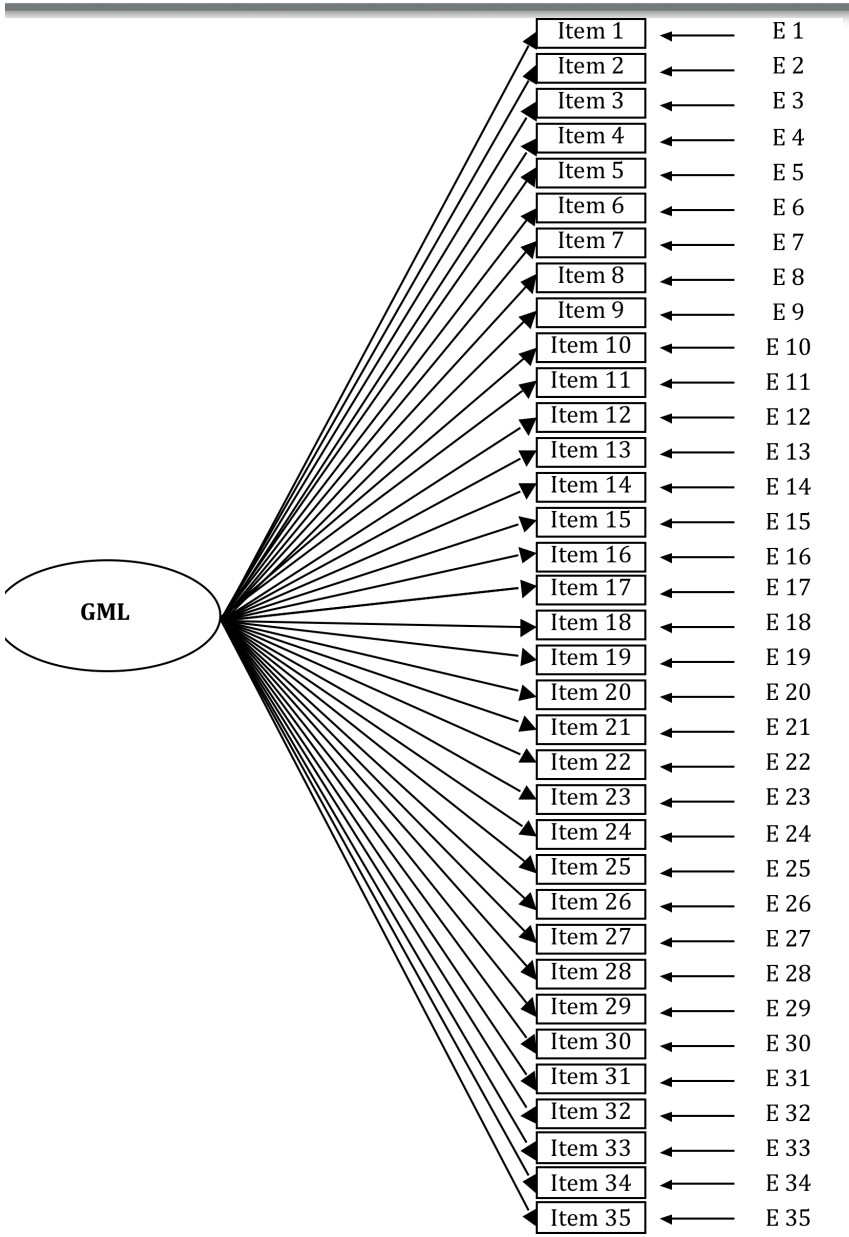
Models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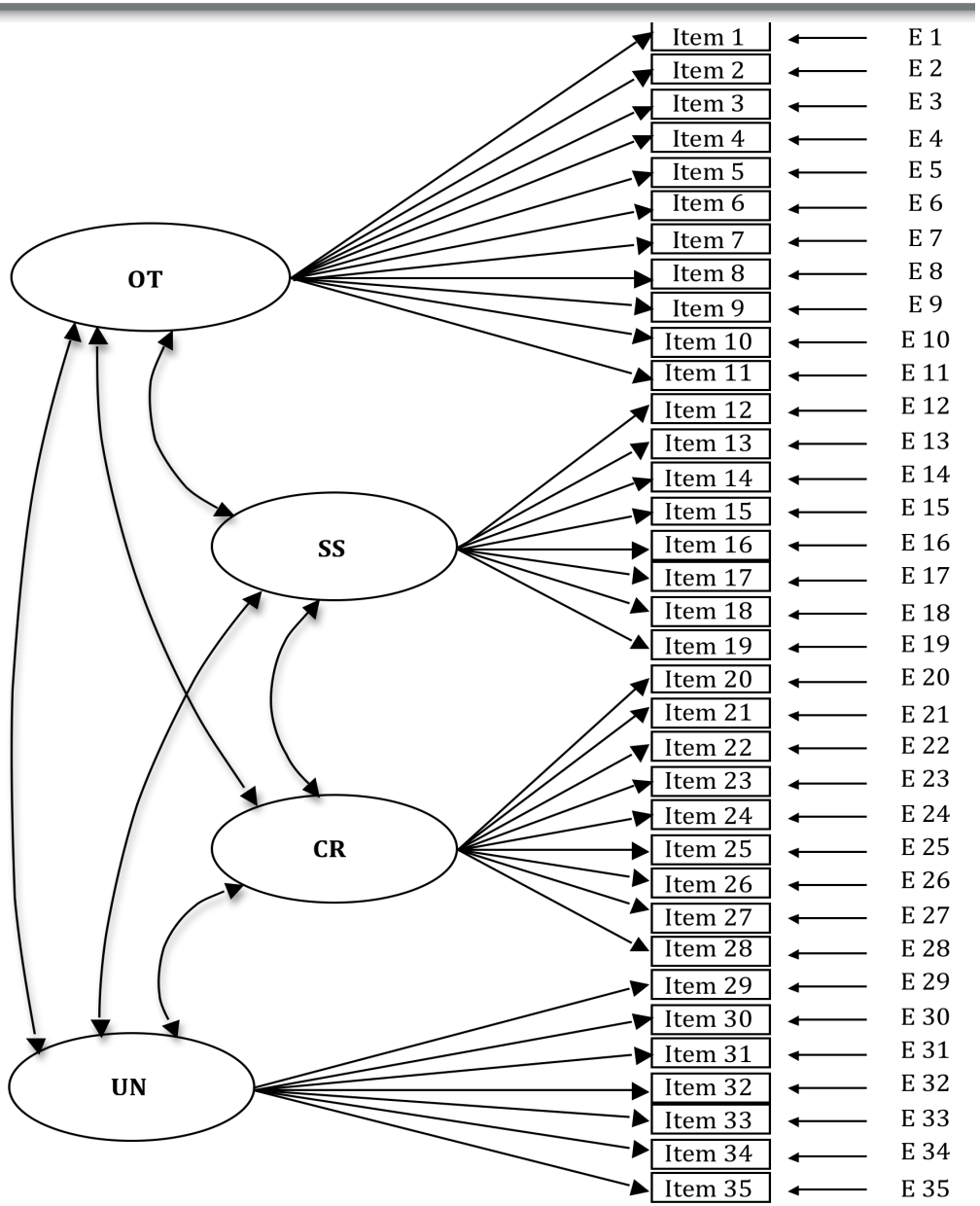
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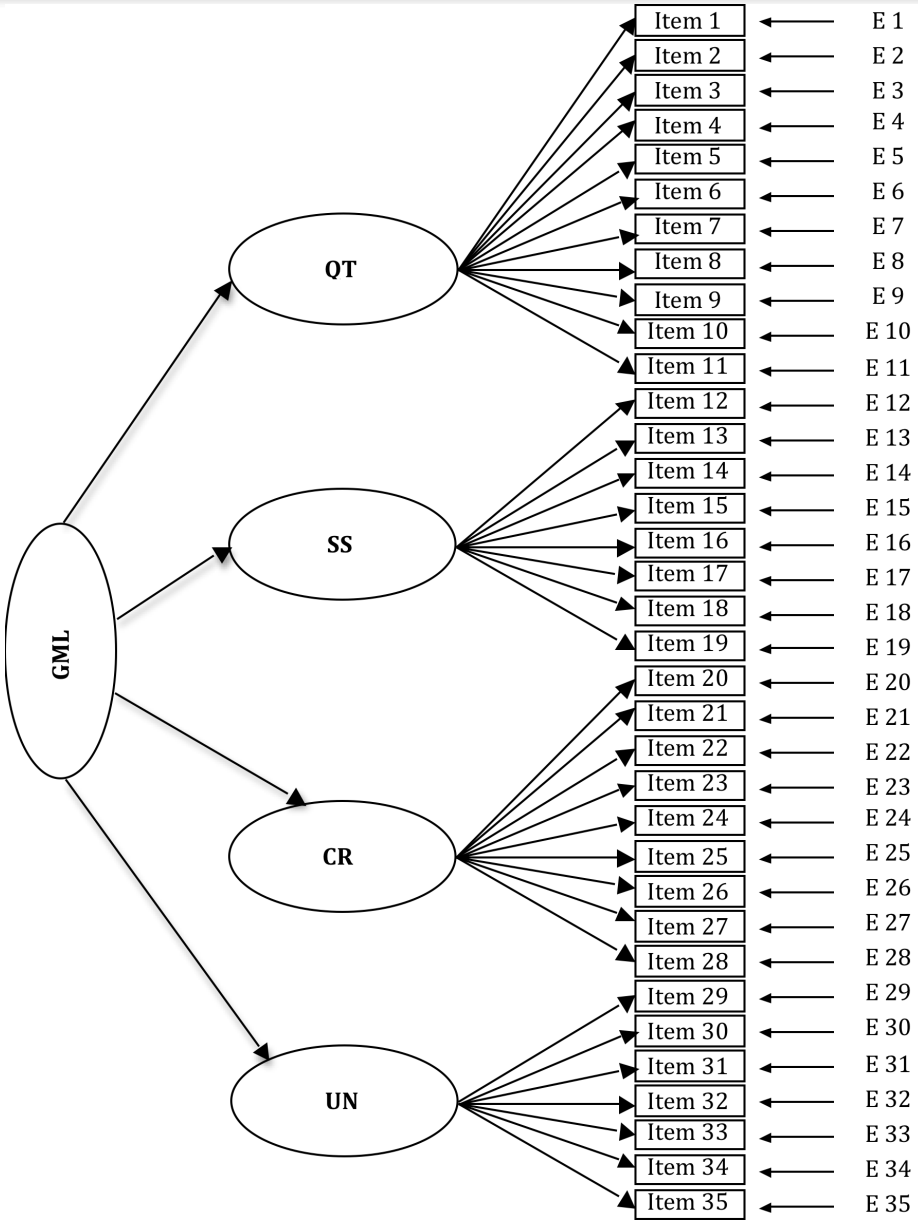
Models (cont.)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Research Questions

- What is the correspondence between the dimensional structure of the PISA mathematics items and PISA's ML framework in terms of the content, process, and context dimensions?
- What is the best representation for the dimensional structure of the PISA mathematics items for implementation cycles 2003, 2006, and 2009?
- How does the dimensional structure of the PISA mathematics items change over time?

Model Fit - *2003*

	Model 1: 1F-GML	Model 2: 4F-Content	Model 3: 3F-Process	Model 4: 4F-Context	Model 5: L2-Content	Model 6: L2-Process	Model 7: L2-Context
Chi-Square Test of Model Fit							
Value	3898.008	3859.488	3892.262	3890.017	3862.815	3894.486	3890.814
Degrees of freedom	3402	3396	3399	3396	3398	3401	3398
p-value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
CFI/TLI							
CFI	**0.973**	**0.975**	**0.973**	**0.973**	**0.975**	**0.973**	**0.973**
TLI	**0.972**	**0.974**	**0.972**	**0.972**	**0.974**	**0.972**	**0.972**
RMSEA (Root Mean Square Error of Approximation)							
Estimate	**0.003**	**0.002**	**0.002**	**0.002**	**0.002**	**0.002**	**0.002**
90 Percent C.I.	0.002-0.003	0.002-0.003	0.002-0.003	0.002-0.003	0.002-0.003	0.002-0.003	0.002-0.003
Probability RMSEA <= .05	1.000	1.000	1.000	1.000	1.000	1.000	1.000
WRMR (Weighted Root Mean Square Residual)	1.163	1.148	1.162	1.161	1.149	1.162	1.161

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Individual parameters 2003

Models	Item loadings: Low (<0.400)	Item loadings: High (>0.800)	Items with low R-square (<0.250)	Correlations b/w L-1 factors	L-1 loadings onto L-2
Model 1: 1F-GML	M75, M82, M83	M28, M45, M48, M80	M20, M24, M36, M40, M41, M57, M65, M66, M68, M75, M82, M83, M84	N/A	N/A
Model 2: 1-L Content	M75, M82, M83	M07, M27, M28, M45, M48, M63, M80	M20, M24, M36, M40, M41, M57, M65, M66, M68, M75 M82, M83	>0.905	N/A
Model 3: 1-L Process	M75, M82, M83	M28, M45, M48, M80	M20, M24, M36, M40, M41, M57, M65, M66, M68, M75, M82, M83, M84	>0.966	N/A
Model 4: 1-L Context	M75, M82, M83	M15, M28, M45, M48, M80	M20, M24, M36, M40, M41, M57, M65, M66, M68, M75, M82, M83, M84	>0.939	N/A
Model 5: 2-L Content	M75, M82, M83	M07, M27, M28, M45, M48, M63, M80	M20, M24, M36, M40, M41, M57, M65, M66, M68, M75 M82, M83	N/A	>0.914
Model 6: 2-L Process	M75, M82, M83	M28, M45, M48, M80	M20, M24, M36, M40, M41, M57, M65, M66, M68, M75, M82, M83, M84	N/A	>0.968
Model 7: 2-L Context	M75, M82, M83	M15, M28, M45, M48, M80	M20, M24, M36, M40, M41, M57, M65, M66, M68, M75, M82, M83, M84	N/A	>0.967

Model Comparisons – *2003*

Content

2-Level > 1-Level > 1F-GML

Process

1F-GML > 2-Level > 1-Level

Context

1F-GML > 2-Level > 1-Level

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2006 & 2009 Results

- Only slight changes in the model fit indices
- Slight changes in the individual item parameters
- Evidence for multidimensionality and unidimensionality
- Stability across cycles

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Conclusions

- There is evidence for both unidimensionality and multidimensionality
- Stronger evidence for unidimensionality
- Multidimensional nature of ML as described in the theoretical framework is not well-reflected in the mathematics items
- Weak connection between the cognition and interpretation components of PISA assessment design (NRC, 2001)

Conclusions (cont.)

- Multidimensional representation seems to be reflected better for content
- Multidimensional representation is not well reflected for the process and context dimensions
- Again, evidence for both unidimensionality and multidimensionality
- Consistency in individual item parameters across different models – all constructs could be behaving as one unifying construct.

Conclusions (cont.)

Stability across cycles in both

- Model-fits and model comparisons
- Individual parameter estimates

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Discussion

- One of the most robust tools to assess dimensionality
- Well-developed and respected assessment design
- Somewhat ambiguous results
- Strict vs. Essential unidimensionality (Stout, 1990; Tate, 2002)
- Need qualitative analysis of interesting items (*not released*)

Discussion (cont.)

- New psychometric models that allows assessing ML in a multidimensional way
- New ML frameworks that would incorporate other aspects of ML as documented in the literature such as social and democratic perspectives.

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THANK YOU

VERY MUCH!!!