Problem-Solving in the 19th Century

The Roots of the Standards



Our Heritage...



OF THE SINGLE RULE OF THREE.

Q. I OW many parts are there in the Rule of Three? A. Two: Single, or Simple, and Double, or Compound. Q. By what is the Single Rule of Three known? A. By Three Terms, which are always given in the

question to find a fourth.

Q. Are any of the terms given to be reduced from one Denomination to another?

A. If any of the given Terms be of several denominations, they must be reduced into the lowest Denomination mentioned.

Q. What do you observe concerning the 1st & 3d Terms?

A. They must be of the same name and kind.

| | EXAMPLE | s for Pra | retiee. | |
|-------------------|---------|-----------|----------------------|---------|
| L. Yds. | Gals. | Tons. | Hhds. | 15. |
| 4 43 | 764 | 3746 | 47476 | 461748 |
| 7. 27 | 347 | 7436 | 73712 | 761780 |
| 3. 39 | 387 | \$406 | 31819 | 476332 |
| 2 13 | 736 | 7398 | 41243 | 126722 |
| 3 37 | 397 | 3373 | 71208 | 310748 |
| 7 46 | 473 | 4731 | 70956 | 571388 |
| 6 23 | 382 | 2264 | 81465 | 704714 |
| 4 59 | 769 | 4731 | 31269 | 812624 |
| 7 94 | 367 | 7169 | 74196 | 781462, |
| rings consecution | | | Ramon and the second | |
| Miles. | Liec | agues. | Y | ears. |
| 4734736 | • 464 | 31734 | 347. | 312484 |
| 3474312 | 712 | 61374 | 168 | 126312 |
| 2546325 | 926 | 52724 | 718 | 125191 |
| 7369138 | 863 | 37266 | 731 | 618191 |
| 3143618 | 741 | 47312 | 312 | 134716 |
| 4733216 | 473 | 12614 | 873 | 263298 |
| 2473347 | 274 | 77573 | 3120 | 514712 |
| 3712612 | 312 | 16126 | 9770 | 547829 |
| 5723384 | 398 | 74129 | 312 | 814795 |
| | | | | |



SIMPLE ADDITION.

ADDITION

Is the putting together two or more numbers, or fums, to make them one total, or whole fum.

SIMPLE ADDITION.

Is the adding of feveral integers or whole numbers together, which are all of one kind, or fort; as 7 pounds, 12 pounds, and 20 pounds, being added together, their aggregate, or fum total is 39 pounds.

RULE.

HAVING placed units under units, tens under tens, &c. draw a line underneath, and begin with the units : after adding up every figure in that column, confider how many tens are contained in their fum, and, placing the excefs under the units, carry fo many, as you have tens, to the next column, of tens :—proceed in the fame manner through every column, or row, and fet down the whole amount of the laft row.*

'ritz

SCHOLAR'S ARITHMETIC :

02,

Federal Accountant.

CONTAINING,

L. Costnos Antennerse, the Roles and Hickory.

II. Excertise and Answers with starts fram, fifthing for their operation by the SCHOLAR.

III. To each Rule a Supercontrast comprehending 1. QUESTIONS on the nature of the Rule, ite ufs, and the manuer of ite operations. 2. Energies.

IV. FIDIALL MOREY, with rules for all the variant spiralizes in it — torolast Federal to Old Lawful, and Old Lawful to Federal Money.

V. INTERNET off in Federal Money, with Compound Multiplication, Compound Division, and Practice swraphs in Old Lowfol and in Federal Money, the fame gapture long pat, in figurate column, on the fame pape, in each lind of many, by which their two modes of account learns, castle fame pape, in each lind of many, by which their two modes of account learns, castle fame pape, in each document for gammed by realizing in Federal Money offly different.

VI. Demonstrate rooms by regressings, of the Roofin and autors of the various flops in the entrothins of the Square and Code Roots, and to be found in any other transfe or Arithmatic.

VII. Forms of Nam, Dode, Bonde and other inframents of meriding.

Then whole in a form and method altogether new, for the rafe of the Mafler and granter progress of the Schular.

> SECOND EDITION, CAMPULAT COMPLETED AND CARGON WITH ADDITION

> > BY DANIEL ADAMS, M. S.

PERSONALD ACCORDING TO ACT OF CONCLEME.

PRINTED AT LEONINETIE, Meffachafdit, By ADAMS & WILDER, For E. & S. LARKIN, J. WEST, WEST & GREENLEAF, Combil, Bution, and for the AUTHOR.



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| Rept | e | re | mpel. | | | |
| 4 | 3 | 34 | 12 | 27 | 330 | 743 |
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| 6 | 4 | 21 | 14 | 10 | 702 | 912 |
| 5 | 2 | 12 | 16 | 11 | 315 | 563 |
| 7 | 1 | 5 | 1 | 13 | 400 | 152 |
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| id. | qr. | na. | E. E. | gr. | na. | : E. | Fl. | gr. | na. |
|-----|-----|-----|-------|-----|-----|----------|-----|-----|-----|
| 79 | 2 | 1 | 86 | 4 | 2 | | 14 | 2 | 1 |
| 25 | 1 | 3 | 44 | 3 | 0 | | 25 | 1 | 0 |
| 14 | 3 | 2 . | 21 | 0 | 2 | | 14 | 0 | 3 |
| 46 | 2 | 1 | 5 | 0 | 3 | Sales in | 25 | 0 | 0 |

4. Add 15 yards, 3 quarters, 2 nails; 45 yards, 2 quarters; 1 yard, 3 nails; and 125 yards.
5. Add 14 English ells, 3 quarters; 25 English ells, 2 quarters, 3 nails: and 3 quarters, 1 nail.

| Sec. 2 | adam . | | | Am | LEBO | | | 1000 | |
|--------|--------|----|-----|----|------|-------------|------|------|----|
| £ | 5. | d. | £ | 8. | d | N. Sale and | £ | 8. | d. |
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THE SINGLE RULE OF TIRES.

74

RULE.

Write down, for the third term, that number which is of the same name or kind with the answer.

Consider, from the nature of the question, whether the answer should be greater or less than this third term. If it is to be greater, set the greater of the two remaining numbers on the left hand, for the second term, and the other for the first; but if *less*, set the less of those two numbers for the second, and the other for the first.

When the question is thus stated, if the first and second terms be not of the same denomination, reduce one or both of them till they are; and if the third term consist of several denominations, reduce it to its lowest denomination; then,

Multiply the second and third terms together, and divide the product by the first term: the quotient will be the answer.

Note.—The product of the second and third terms is of the same denomination as the third term; and the learner may be reminded, that the quotient and remainder are of the same denomination as the number divided.

See examples 14, 15 and 16, under rule 1, and 7, 8, under rule 3, Compound Division.

Rule of Three

If 3 pounds of sugar cost 25 cents, what will 18 pounds cost at the same rate?

Rule of Three

If 3 pounds of sugar cost 25 cents, what will 18 pounds cost at the same rate?

3 : 18 : : 25 : ?

Rule of Three

If 3 pounds of sugar cost 25 cents, what will 18 pounds cost at the same rate?

3 : 18 : 25 : ? $18 \times 25 = 450$ $450 \div 3 = 150$

Your Turn

If 750 men require 22500 rations of bread for a month, what will a garrison of 1200 require?

Your Turn

If 750 men require 22500 rations of bread for a month, what will a garrison of 1200 require?

750 : 1200 : : 22500 :

Your Turn

If 750 men require 22500 rations of bread for a month, what will a garrison of 1200 require?

750 : 1200 : 22500 : $1200 \times 22500 = 27000000$ $27000000 \div 750 = 36000$

Practice



Practice

What will 50 yards of tape cost at $\frac{1}{2}$ of a cent per yard?

Practice

What will 50 yards of tape cost at $\frac{1}{2}$ of a cent per yard?

$$1 : 50 : :\frac{1}{2}:$$

If 8 men, in 16 days, can earn 96 dollars, how much can 12 men earn in 26 days?

If 8 men, in 16 days, can earn 96 dollars, how much can 12 men earn in 26 days?

If 8 men, in 16 days, can earn 96 dollars, how much can 12 men earn in 26 days?

$\begin{array}{cccc} 8 & 12 \\ 596 \\ 16 & 26 \end{array}$ $\begin{array}{cccc} 996 \\ 12 \times 26 = 312 \times 96 = 29952 \end{array}$

If 8 men, in 16 days, can earn 96 dollars, how much can 12 men earn in 26 days?

$8 \quad 12 \\ \$96 \\ 16 \quad 26 \end{bmatrix} \$96 \\ 12 \times 26 = 312 \times 96 = 29952 \\ 8 \times 16 = 128 \quad 29952 \div 128 = 234$

POSITION.

POSITION is a rule, which, by false, or supposed numbers, taken at pleasure, discovers the true ones required. It is divided into two parts; SINGLE and DOUBLE.

SINGLE POSITION.

SINGLE POSITION teaches to refolve those questions, whose refults are proportional to their suppositions: such are those which require the multiplication or division of the number sought by any proposed number; or when it is to be increased or diminished by itself a certain proposed number of times.

A schoolmaster, being asked how many scholars he had, replied, "If I had as many more as I now have, one half as many more, one third and one fourth as many more, I should have 296." How many had he?

A schoolmaster, being asked how many scholars he had, replied, "If I had as many more as I now have, one half as many more, one third and one fourth as many more, I should have 296." How many had he?

24 + 24 + 12 + 8 + 6 = 74

A schoolmaster, being asked how many scholars he had, replied, "If I had as many more as I now have, one half as many more, one third and one fourth as many more, I should have 296." How many had he?

24 + 24 + 12 + 8 + 6 = 7474 : 296 : : 24 : ?

A schoolmaster, being asked how many scholars he had, replied, "If I had as many more as I now have, one half as many more, one third and one fourth as many more, I should have 296." How many had he?

- 24 + 24 + 12 + 8 + 6 = 74
- 74 : 296 : : 24 : ?

 $296 \times 24 = 7104$ $7104 \div 74 = 96$

A schoolmaster, being asked how many scholars he had, replied, "If I had as many more as I now have, one half as many more, one third and one fourth as many more, I should have 296." How many had he?

96 + 96 + 48 + 32 + 24 = 296 \checkmark

Double Position

Double Position

A Lady bought damask for a gown, at 8*s*. per yard, and lining for it, at 3*s*. per yard; the gown and the lining contained 15 yards, and the price of the whole was £3 10*s*.; How many yards were there of each?

1st Supposition: 6 yds damask, 9 yds lining

Double Position

A Lady bought damask for a gown, at 8*s*. per yard, and lining for it, at 3*s*. per yard; the gown and the lining contained 15 yards, and the price of the whole was £3 10*s*.; How many yards were there of each? Note: £3 10s = 70s.

1st Supposition: 6 yds damask, 9 yds lining $6 \times 8 + 9 \times 3 = 48 + 27 = 75$

This is 5 too much.
A Lady bought damask for a gown, at 8*s*. per yard, and lining for it, at 3*s*. per yard; the gown and the lining contained 15 yards, and the price of the whole was £3 10*s*.; How many yards were there of each? Note: £3 10s = 70s.

2nd Supposition: 4 yds damask, 11 yds lining

A Lady bought damask for a gown, at 8*s*. per yard, and lining for it, at 3*s*. per yard; the gown and the lining contained 15 yards, and the price of the whole was £3 10*s*.; How many yards were there of each? Note: £3 10s = 70s.

 2^{nd} Supposition: 4 yds damask, 11 yds lining $4 \times 8 + 11 \times 3 = 32 + 33 = 65$

This is 5 too little.

The two suppositions are written on the left of a large X. The resulting errors are written on the right.



Multiply the numbers on the opposite corners – the original "cross-multiplication."



Multiply the numbers on the opposite corners – the original "cross-multiplication."



 $(20+30) \div (5+5) = 50 \div 10 = 5$

Alligation

- Alligation Medial
- Alligation Alternate



A farmer mingled 19 bushels of wheat at 6s. per bushel, and 40 bushels of rye at 4s. per bushel, and 12 bushels of barley at 3s. per bushel together; I demand what a bushel of this mixture is worth?

A farmer mingled 19 bushels of wheat at 6s. Per bushel, and 40 bushels of rye at 4s. Per bushel, and 12 bushels of barley at 3s. Per bushel together; I demand what a bushel of this mixture is worth?

 $19 \times 6 = 114$ $40 \times 4 = 160$ $12 \times 3 = 36$

A farmer mingled 19 bushels of wheat at 6s. Per bushel, and 40 bushels of rye at 4s. Per bushel, and 12 bushels of barley at 3s. Per bushel together; I demand what a bushel of this mixture is worth?

 $19 \times 6 = 114$ $40 \times 4 = 160$ $12 \times 3 = 36$ 114 + 160 + 36 = 310 $310 \div 71 \approx 4.37$

A vintner has three kinds of wine, viz. one kind at 160 cents per gallon, another at 180 cents, and another at 240 cents; how much of each kind must he take to make a mixture, worth 190 cents per gallon?

Wine #1 160

Wine #2 180

Wine #3 240

Wine #1 160 -30

Wine #2 180 -10

Wine #3 240 +50



A man being determined to mix 10 bushels of wheat, at 4s. per bushel, with rye at 3s. with barley at 2s. and with oats at 1s. per bushel; I demand how much rye, barley and oats, must be mixed with the 10 bushels of wheat, that the whole may be sold at 28d per bushel?

| Grain | Price | Qty |
|--------|-------|-------|
| Wheat | 48d | 10 bu |
| Rye | 36d | ? |
| Barley | 24d | ? |
| Oats | 12d | ? |

| Grain | Price | Qty | Diff |
|--------|-------|-------|------|
| Wheat | 48d | 10 bu | +20 |
| Rye | 36d | ? | +12 |
| Barley | 24d | ? | -4 |
| Oats | 12d | ? | -16 |

| Grain | Price | Qty | Diff | Prod |
|--------|-------|-------|------|------|
| Wheat | 48d | 10 bu | +20 | +200 |
| Rye | 36d | 4 | +12 | +48 |
| Barley | 24d | 22 | -4 | -88 |
| Oats | 12d | 10 | -16 | -160 |

State of Problem Solving

- □ Rule-based instruction
- □ No support for conceptual understanding
- Emphasis on adult applications

Problem-solving Revolution



- □ Based on Colburn's classroom experience
- □ Used age-appropriate numbers and situations
- □ No rules
- Concepts developed inductivly
- □ Replaced all the older techniques

Pestalozzian Texts

THE

JUVENILE ARITHMETICS

32.00

SCHOLAR'S GUIDE;

ADDRESS TREAST AND PRACTICE THE CONFERENCE AND ADDRESS TO THE CAPACITY.

n Pi

YOUNG BEGINNERS;

UNTERING & DES PROPORTION OF PROPERTY.

110

PEDERAL MONTH'

AND THE WINELS RELYC RELEVERATED BY VORTHERS

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PESTALOZZI.

BE MARTIN AUTER, A. M.

Cincinnati:

PURISHED AND SOLD OF S. AND G. CUTLFORD.

18:24

W. M. AND D. PARRIMONTH, JR. PRINTERS

Colburn's Method

Remark. When any thing, or any number, 15 divided into three equal parts, one of those parts is called the *third* part of the thing or number. When it is divided into four equal parts, one part is called the *fourth* part, and so on.

17. If a yard of cloth be worth three dollars, and it be cut into three equal pieces, what will one of the pieces be worth? that is, what will one third of a yard be worth ?

18. What is a third of three ?

19. Suppose the yard of cloth to be cut as before, what will two pieces of it cost? that is, what will two thirds of a yard cost?

20. What is two thirds of three?

21. If three shillings will buy one bushel of corn, what part of a bushel will one shilling buy? What part of a bushel will two shillings buy?

22. One is what part of three ?

- Recognized as the best book for early instruction
- □ Continued to sell well into the 1890s
- □ Sold 100,000 per year nationally
- Methods still in use in 1950s
- Well-aligned to NCTM Standards

If 8 men can build a wall 15 rods in length in 10 days, how many men will it take to build a wall 45 rods in length in 5 days?

If 8 men can build a wall 15 rods in length in 10 days, how many men will it take to build a wall 45 rods in length in 5 days?

$$8 \times 3 \times 2 = 48$$
 men

Beyond Colburn

"The Scholar's Arithmetic, published in 1801, is synthetic. If that is a *fault* of the work, it is a fault of the *times* in which it appeared. The analytic or inductive method of teaching, as now applied to elementary teaching, is among the improvements of later years."

Beyond Colburn



Beyond Colburn



If three apples cost 6 cents, what will 4 apples cost?

If three apples cost 6 cents, what will 4 apples cost?

If 3 apples cost 6 cents, 1 apple will cost $\frac{1}{3}$ of 6, or 2 cents. 4 apples will cost 4 times as much, or 8 cents.

If three apples cost 6 cents, what will 4 apples cost?

If 3 apples cost 6 cents, 1 apple will cost $\frac{1}{3}$ of 6, or 2 cents. 4 apples will cost 4 times as much, or 8 cents.



If one ton of hay cost 8 dollars, what will $\frac{3}{4}$ of a ton cost?

If one ton of hay cost 8 dollars, what will $\frac{3}{4}$ of a ton cost?

If one ton of hay costs 8 dollars, $\frac{1}{4}$ of a ton will cost $\frac{1}{4}$ of 8, or 2 dollars. And $\frac{3}{4}$ of a ton will cost three times as much, or 6 dollars.

If one ton of hay costs 8 dollars, $\frac{1}{4}$ of a ton will cost $\frac{1}{4}$ of 8, or 2 dollars. And $\frac{3}{4}$ of a ton will cost three times as much, or 6 dollars.


If $\frac{3}{4}$ of a ton of hay costs 6 dollars, how much will one ton cost?

If $\frac{3}{4}$ of a ton of hay costs 6 dollars, how much will one ton cost?

If $\frac{3}{4}$ of a ton of hay costs 6 dollars, $\frac{1}{4}$ of a ton will cost $\frac{1}{3}$ as much, or 2 dollars. And if $\frac{1}{4}$ of a ton costs 2 dollars, $\frac{4}{4}$ of a ton, or one ton, will cost 4 times as much, or 8 dollars.

If $\frac{3}{4}$ of a ton of hay costs 6 dollars, $\frac{1}{4}$ of a ton will cost $\frac{1}{3}$ as much, or 2 dollars. And if $\frac{1}{4}$ of a ton costs 2 dollars, $\frac{4}{4}$ of a ton, or one ton, will cost 4 times as much, or 8 dollars.



If $\frac{3}{4}$ of a ton of hay costs 15 dollars, what will $\frac{4}{5}$ of a ton cost?

If $\frac{3}{4}$ of a ton of hay costs 15 dollars, $\frac{1}{4}$ of a ton will cost $\frac{1}{3}$ as much, or 5 dollars. Then $\frac{4}{4}$, or one ton, will cost 4 times as much, or 20 dollars. If one ton costs 20 dollars, $\frac{1}{5}$ of a ton will cost $\frac{1}{5}$ of 20, or 4 dollars. And if $\frac{1}{5}$ of a ton cost 4 dollars, $\frac{4}{5}$ of a ton will cost 4 times as much, or 16 dollars.



Relevance to Today

- Real Understanding of Fractions
- Mental Math Skills
- Critical Building Block for Estimation
- Analytic Thinking Skills of Algebra
- Mathematical Sense-Making

What Can I do?

- Teach with Manipulatives
- Use Age-appropriate problems
- Connect Division to Unit Fractions
- **Give Mental Calculations**
- **Think-Alouds with Choral Responses**
- Solutions with Defense of Answer

Examples:

• What is $\frac{3}{4}$ of 28?

Examples:

What is ³/₄ of 28?
What is 40% of 70?

Examples:

- What is $\frac{3}{4}$ of 28?
- $\square \text{ What is } \overline{40\%} \text{ of } 70?$
- □ If two apples cost 50 cents, what will 5 cost?

Example:

What is 80% of 50?*Solution*: What is 10% of 50? [5] Then 80% will be 8 times as much. [40]

Example:

320 is 80% of what number?

Solution: The answer is 400.

Defense: If 320 is 80% of the number, then 10% of it will be an eighth of that, or 40. Then the whole amount, or 100%, will be ten times as much, or 400.

19th Century Problem Solving

Began as a British legacy that

- □ Was based on memorizing rules and
- □ Treated children as little adults

19th Century Problem Solving

- Revolutionized by Colburn into a method that
- □ Used real objects to teach concepts, not rules
- Inductively developed the concepts
- Developed mental math skills and
- □ Real understanding of the concepts

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Questions?

