Research-Based Principles for Designing Mathematics Instruction

Colloquium Presentation for
RUSMP Summer Program 2005

Overview

- Introduction to the National Academies
- Three core principles of learning
- Lenses for thinking about classroom environments

The National Academies: What do we do?

- Provide technical advice to government agencies and the public on key issues of national interest

- Convene groups of experts on a topic

- Produce research-based reports

The National Academies: Who are we?

- The umbrella organization for:
  - National Academy of Sciences (est. 1863)
  - **National Research Council (est. 1916)**
  - National Academy of Engineering (est. 1963)
  - The Institute of Medicine (est. 1970)
- NOT a government agency – independent non-profit

National Research Council

- 5 Major divisions
  - **Behavioral and social sciences and education**
  - Earth and life studies
  - Engineering and physical sciences
  - Policy and global affairs
  - Transportation research board

DBASSE

- Four Centers
  - Center for Study of Behavior and Development
  - Center for Economic, Governance, and International Studies
  - **Center for Education**
  - Committee on National Statistics

Center for Education

- Mathematical Sciences Education Board
- Board on Science Education
- Board on Testing and Assessment

Reports Related to Mathematics Education

- Adding It Up (2001)
- Helping Children Learn Mathematics (2002)
- **How Students Learn: Mathematics in the Classroom (2005)**

Three Core Principles of Learning and Teaching

1. Importance of engaging students' preconceptions
   – Allow multiple strategies
   – Encourage math talk
   – Design bridging instructional activities

2. Understanding requires factual knowledge <u>and</u> conceptual frameworks

3. Importance of supporting student self-monitoring
   – Emphasis on debugging
   – Internal and external dialogue
   – Seeking and giving help

Principle 1: Prior understandings

- Understanding is constructed on a foundation of existing understanding and experiences.
- Prior understanding can support further learning
- Prior understanding can also lead to the development of conceptions that act as barriers to learning

Principle 1: Prior understandings

- Misconception 1 -- Mathematics is about learning to compute
  - What, approximately, is the sum of 8/9 and 12/13?

Principles 1: Prior understandings

- Misconception 1 -- Mathematics is about learning to compute
- Misconception 2 -- Math is about “following rules” to guarantee correct answers.
- Misconception 3 -- Some people have the ability to “do math” and some don’t

Principles 1: Prior understandings

- Students also bring prior conceptions related to specific mathematical ideas

Example: When working with whole numbers, students have seen each quantity represented by a single numeral. In rational number the same quantity can be represented in many different ways that do not “look like” each other. Also, a single rational number can have several distinct meanings.

Principle 1: Prior understandings

- ¾ can mean:
  - A part-whole relation – 3 of 4 equal shares
  - A quotient – 4 children sharing 3 pies
  - A measure – ¾ of an inch
  - A multiplicative operator – reduces the size of another quantity

Principle 1: Prior understandings

- To understand rational numbers, students must move from their understanding of whole number to an understanding of a new set of representations, procedures & concepts.

Principle 1: Prior understandings

- Some preconceptions are useful starting points for instruction
  - Example: We have everyday experience with functions when we pay for gasoline by the gallon or fruit by the pound. Students have some informal understanding of these quantitative relationships.

Principle 1: Prior understandings

- Challenges for Instruction
  - How to teach mathematics so that students appreciate it is not about computation and following rules, but about solving important and relevant quantitative problems?
  - How to link formal mathematics training with students' informal knowledge and problem-solving capacities?

Principle 1: Prior understandings

- Instructional Strategies
  - Allowing multiple strategies
  - Encouraging math talk
  - Designing bridging instructional activities

Allowing multiple strategies

Maria	Peter	Manuel
	11 14	11 14
$1\ 2^{1}4$	~~1~~ ~~2~~ 4	~~1~~ ~~2~~ 4
$-\ {}^{1}5\ 6$	- 5 6	$-\ {}^{1}5\ 6$
---------	--------	--------
6 8	6 8	5 8

Allowing multiple strategies

- Drawbacks to showing the students the "right way" to subtract
  - Students may come to think there is one correct method that needs to be memorized
  - They will begin to see no connection between their own reasoning and "doing math"
  - When the nature of the problems changes slightly, students may feel completely lost

Allowing multiple strategies

Maria: Six is bigger than 4, so I can’t subtract here in the ones. So, I have to get more ones. But I have to be fair when I get more ones, so I add ten to both my numbers. I add a ten here in the top of the ones place to change the 4 to a 14, and I add a ten here in the bottom in the tens place. So I write another ten by my 5.

$$1\ 2^{1}4$$

$$-\ {}^{1}5\ 6$$

$$6\ 8$$

Allowing multiple strategies

Peter:

I like to ungroup my top number when I don't have enough to subtract everywhere. So here I ungrouped to make 14 ones, so I had 1 ten left here. [goes on to explain his subtraction]

```
 11 14
 1̶ 2̶ 4̶
-  5 6
---------
   6 8
```

Encouraging Math Talk

- Talk about mathematical thinking helps to make students’ thinking visible
- Can help everyone in a classroom understand a concept
- Especially helpful for drawing out and working with preconceptions

Encouraging math talk

Maria: So now I count up from 6 to 14, and I get 8 ones. [Demonstrates holding up a finger for each word.] And I know my doubles, so 6 plus 6 is 12, so I have 6 tens left.

Jorge: I don’t see the other 6 in your tens. I only see one 6 in your answer.

Maria: The other 6 is from my adding my 1 ten to the 5 tens to get 6 tens. I didn’t write it down.

Andy: But you’re changing the problem. How do you get the right answer.

Maria: If I make both numbers bigger by the same amount, the difference will stay the same. Remember we looked at that on drawings last week and on the meter stick.

Michelle: Why did you count up?

Maria: Counting down is too hard, and my mother taught me to count up to subtract in first grade.

$$
\begin{array}{r}
1\;2\,{}^{1}4 \\
-\;{}^{1}5\;6 \\
\hline
6\;8
\end{array}
$$

Encouraging math talk

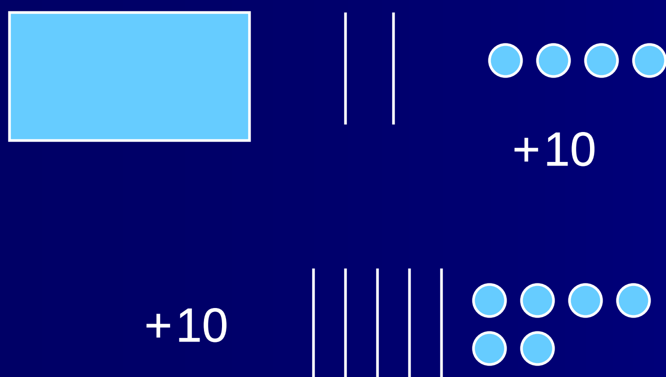
Teacher: How many of you remember how confused we were when we first saw Maria's method last week? Some of us could not figure out what she was doing even though 3 other students did it the same way. What did we do?

Rafael: We made drawings with our ten-sticks and dots to see what those numbers meant. And we figured out they were both tens. And we went home to see if any of our parents could explain it to us, but we had to figure it out ourselves and it took us 2 days.

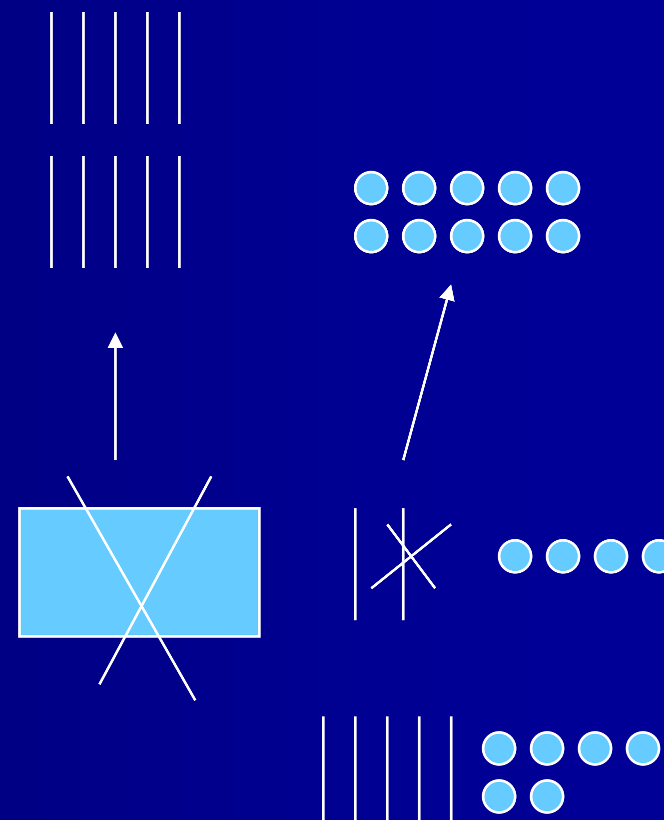
Teacher: Yes, I was asking other teachers, too. We worked on other methods too, but we kept trying to understand what this method was and why it worked.

Encouraging math talk: Using drawings

Maria's method:



Peter's method:



Designing Bridging Instructional Activities

- Activities that link students' everyday, experiential, informal understanding to form school concepts

  – Example: Using a walkathon scenario to introduce functions

Designing bridging activities

- A walkathon is a good context because:
  - Students are familiar with money and distance as variable quantities
  - They understand the contingency relationship between the variables
  - They are interested in and motivated by the rate at which money is earned

Designing bridging activities

- The walkathon scenario was used to move students through more and more complex work with functions
  - In the first lesson students record in tables the money earned for each kilometer walked and plot each pair of values for a variety of situations. They then construct an equation.
  - In later lessons, students learn about the y-intercept through introduction of a starting bonus, and about non-linear functions through introduction of decreases or increases for each kilometer walked.

Principle 2: Facts and conceptual frameworks

- Factual knowledge must be placed in a conceptual framework to be well understood
- Concepts are given meaning by multiple representations that are rich in factual detail

Principle 2: Facts and conceptual frameworks

- Core conceptual understandings:
  - To understand whole number, students must master concept of quantity
  - To understand rational numbers, students must master concept of proportion and relative number
  - To understand function, students must master the concept of dependent in quantitative relationships

Principle 2: Facts and conceptual frameworks

- Examples of facts without a conceptual framework
  - Rote counting to 10 without an understanding of quantity
- Examples of multiple representations
  - Different experiences with percents: everyday examples, beakers filled with water, walking various distances down a sidewalk

Principle 3: Supporting self-monitoring

- Metacognition – people’s knowledge about themselves as learners, or “information processors”
- Focus on helping students develop the ability to take control of their own learning

Principle 3: Supporting self-monitoring

- An emphasis on debugging
- Internal & external dialogue as support for metacognition
- Seeking and giving help

Emphasis on Debugging

- Shift from a focus on answers being just right or wrong to a focus on debugging a wrong answer
  - Find where the error is, why it is an error, and consider ways to correct it

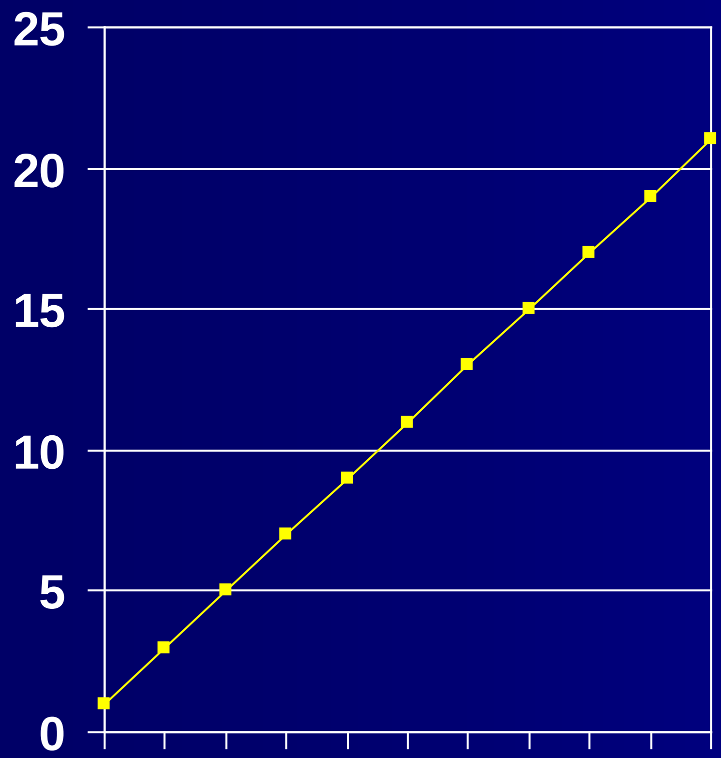
Emphasis on debugging

Example #1:

```
    1
    1
    2 6 8
+    1 5 6
-----------
    5 1 4
```

Emphasis on debugging

What shape would the graph of the function $y = X^2 + 1$ have? Draw it below.



X	$Y=X^2 + 1$
0	1
1	3
2	5
3	7
4	9
5	11
6	13
7	15
8	17
9	19
10	21

Emphasis on debugging

- Help students move toward debugging their own work
  - Model debugging for students as you work through problems
  - Give students opportunities to explain their thinking to you and to peers
  - Give students opportunities to help each other

Internal and external dialogue

- Teachers can model clear descriptions and supportive questioning or helping techniques

- Some students may solve problems at the board while others work at their seats

- Selected students can describe their solution methods and peers can ask questions to clarify and describe their solution methods

Seeking and giving help

- Create a classroom environment where students feel safe seeking help and support their skills in giving help

Seeking and giving help

Teacher: Manuel, don't erase your problem. I know you think it is probably wrong because you got a different answer, but remember how making a mistake helps everyone learn – because other students make that same mistake and you helped us talk about it. Do you want to draw a picture and think about your method while we do the next problem, or do you want someone to help you?

Manuel: Can Rafael help me?

Teacher: Yes, but what kind of helping should Rafael do?

Manuel: He should just help me with what I need help on and not do it for me.

Teacher: Ok, Rafael, go up and help Manuel that way while we go on to the next problem. I think it would help you to draw quick tens and ones to see what your numbers mean. But leave your solution so we can all see where the problem is. That helps us all get good at finding our mistakes. Do we all make mistakes?

Class: Yes.

Teacher: Can we all get help from each other?

Class: Yes.

Teacher: So mistakes are just a part of learning. We learn from our mistakes. Manuel is going to be brave and share his mistakes with us so we can all learn from it.

The 3 Principles

1. Engage students' preconceptions
   - Allow multiple strategies
   - Encourage math talk
   - Design bridging instructional activities

2. Understanding requires factual knowledge and conceptual frameworks

3. A metacognitive approach enables student self-monitoring
   - Emphasis on debugging
   - Internal and external dialogue
   - Seeking and giving help

Lenses for Looking at Classrooms

- **Learner-centered**: Attends to preconceptions; begins instruction with what students think and know
- **Knowledge-centered**: Focuses on what is taught, why it is taught, and what mastery looks like
- **Assessment-centered**: Emphasizes the need to provide frequent opportunities to make students' thinking and learning visible
- **Community-centered**: Encourages a culture of questioning, respect, and risk-taking

Lenses for looking at classrooms

- Lenses help you focus on different dimensions of the classroom
- Prompt you to think about how to balance the emphasis you give to each dimension

Lenses for looking at classrooms

- For example, over emphasis on learner-centered can lead to:
  - Too much focus on student invented methods with no evaluation of methods
  - Meandering student discussions without focus
  - A string of un-connected real-world experiences that lead nowhere
  - Too much time on introductory activities with no time for consolidating learning

Lenses for looking at classrooms

- Each lens also helps you to evaluate different aspects of your classroom
  - Example: How often and in how many ways am I incorporating assessment in my classroom?

- Considering two lenses together can prompt new questions about your classroom
  - Example: Am I building in enough opportunities for students to evaluate their own work?

National Academies

http://www.nas.edu/

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