

# COOL MATHEMATICS SCENARIOS, CONCEPTS, TOOLS, ETC.

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RUSMP Fall Networking Conference  
September 11, 2010  
The Rice School, Houston, TX

RICE UNIVERSITY  
SCHOOL MATHEMATICS PROJECT

# Celebrating RUSMP Success

- Begun in 1987
- Triplets of primes  $(p, p + 6, p + 12)$  such that  $p + 18$  is composite are called sexy prime triplets.
  - Last year until [2013](#) with four distinct digits.

# What Else was Going on in 1987?

He was a 26 yr. old community organizer.



**President Obama**

# What Else was Going on in 1987?

Who was U.S. President? Hint: That year he gave the, “Mr. Gorbachev, tear down this wall” speech.



**Ronald Reagan**

# What Else was Going on in 1987?

Who was the governor of Texas (first Republican governor since Reconstruction)?



Bill Clements

# What Else was Going on in 1987?

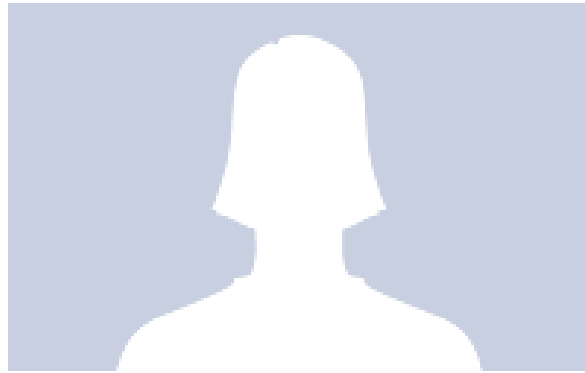
Who was the mayor of Houston?



Kathy Whitmire

# What Else was Going on in 1987?

Who was the superintendent of HISD?



**Joan Raymond**

# What Else was Going on in 1987?

25 Year old Rockets Star?



**Hakeem Olajuwon**



# What Else was Going on in 1987?

Oilers Quarterback?



**Warren Moon**

# What Else was Going on in 1987?

3 years before the development of this tool that has dramatically changed the teaching of algebra across the country



**TI-81 graphing calculator**

# What Else was Going on in 1987?

- In 1987, Texas instituted the Texas Educational Assessment of Minimum Skills (TEAMS) high school graduation exit test for 11<sup>th</sup> graders.
- In 1990, Texas replaced the TEAMS test with the Texas Assessment of Academic Skills (TAAS).
- In 1993, the Texas Education Agency (TEA) developed and integrated an accountability system for schools.
- In 1998, the Texas Essential Knowledge and Skills (TEKS) replaced the "Essential Elements" as the state's curriculum.
- ETC.



# RUSMP Outcomes

## **Participation**

- Over 7000 teacher participants; many have become leaders
- How many students?

## **Impact**

- Due to the Rice University Mathematics Leadership Institute (MLI), the number of teachers with Texas Master Mathematics Teacher (8-12) Certification has increased by more than 50%.
- Students of MLI teachers performed on a higher level on state and national standardized tests than students of other teachers
- Student performance was higher than district-wide performance on TAKS Mathematics in schools whose teachers participated in RUSMP

## **Reasons for Success**

- Leadership
- Quality Instruction

# Let's Do Math

- Consider our audience: K-12 teachers
- Need a problem with multiple entry and exit points with opportunities for exploration and development
- Schoenfeld calls problems “starting points for serious explorations, rather than tasks to be completed”
- Let's start with a problem that I originally saw at RUSMP as a participant in 1991. YIKES! Almost 20 years ago!
- Problem was used by the Math Forum [here](#).

Schoenfeld, A.H. "Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics." *Handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics*. Ed. D.A. Grouws. New York, NY: Macmillan, 1992. 334–370.

# “Starting Point”

## The Problem:

Form two (baseless) cylinders from a rectangular piece of paper, one by joining the long sides, one by joining the short sides. How do the surface areas of these cylinders compare? How do you think the volumes would compare? Fill to confirm.

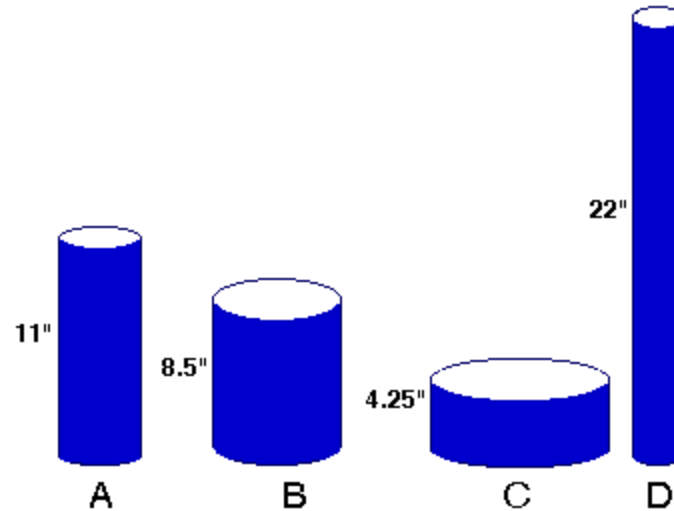


# Reflection Questions for Various Levels

- Let's go back and look at our original sheet of paper. From it, we made two different cylinders. What geometric shape is the sheet of paper? What are its dimensions?
- What are the dimensions of the resulting cylinders? That is, what are the heights and what are the circumferences?
- Are there any other cylinders that we can make from this same sheet of paper?

# More Cylinders

Now we have four cylinders. Which of them would hold the most?



[Spreadsheet](#)



# Conjecture

As the cylinders get taller and narrower, the volume seems to get less. To explore our conjecture, let's calculate many volumes of cylinders made from this sheet of paper.

[Spreadsheet](#)

# Explorations Using Algebra (1)

Consider the whole family of cylinders that you could make with a fixed lateral surface area of  $93.5 \text{ in}^2$ . How many would there be? Write an expression for the volume of the cylinders as a function of  $h$ .

$$V = \pi r^2 h$$

$$C = 2 \pi r \text{ and } C = 93.5/h, \text{ so}$$

$$2 \pi r = 93.5/h$$

$$r = 93.5 / 2 \pi h = 46.75 / \pi h$$

$$V = \pi (46.75 / \pi h)^2 h$$

$$V = 46.75^2 / \pi h$$

Volume to height is what kind of function?

# Explorations Using Algebra (2)

Write an expression for the volume of the cylinders as a function of  $r$ .

$$V = \pi r^2 h$$

$$C = 2 \pi r \text{ and } C = 93.5/h \text{ so}$$

$$2 \pi r = 93.5/h$$

$$2 \pi r h = 93.5$$

$$h = 93.5 / (2 \pi r)$$

$$V = \pi r^2 93.5 / (2 \pi r)$$

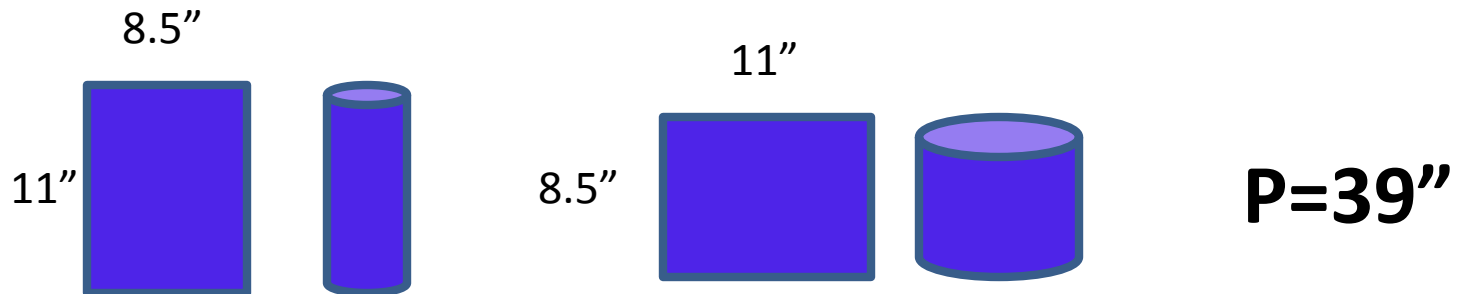
$$V = 93.5r/2$$

Volume to radius is what kind of function?

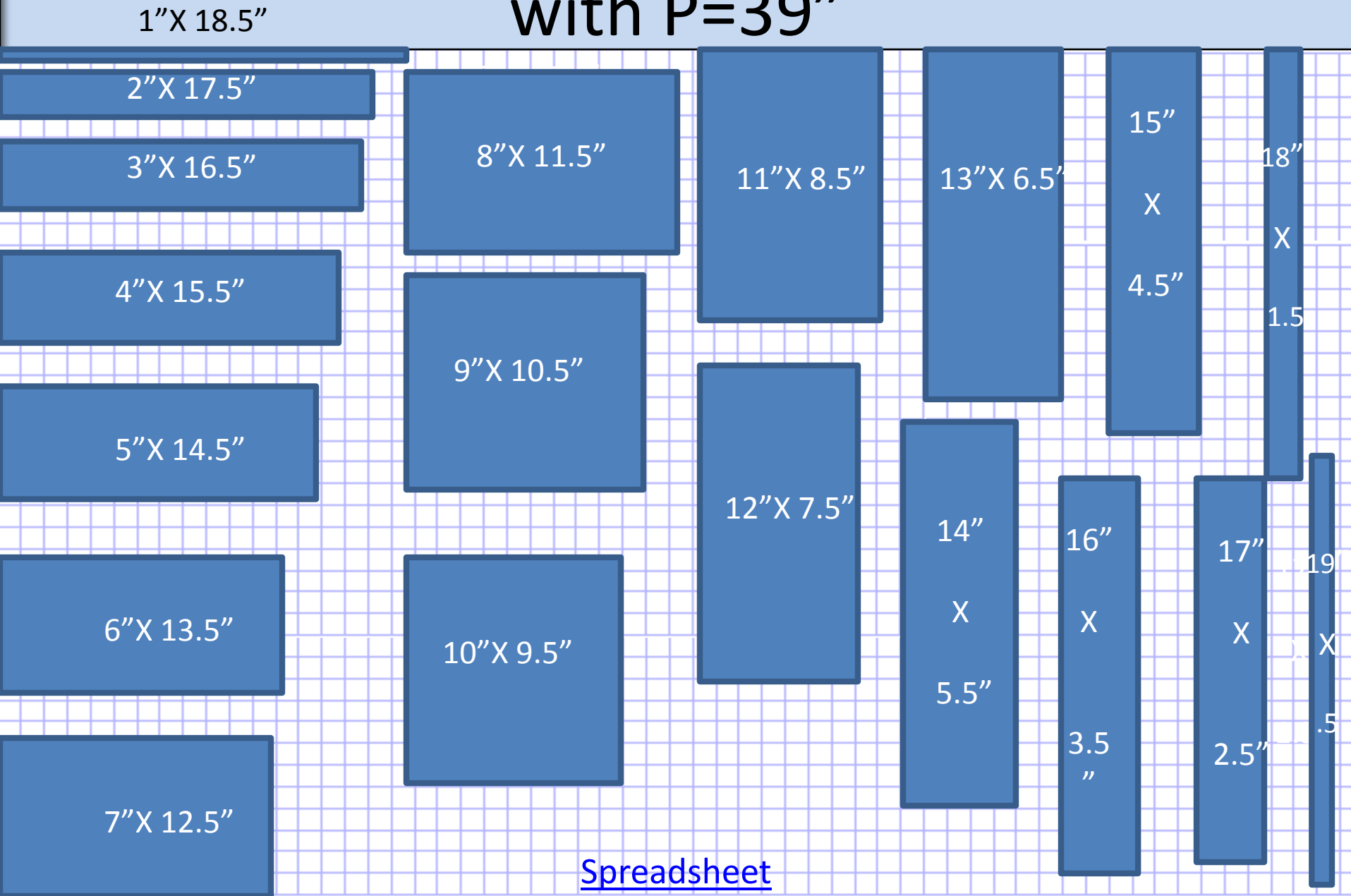
[Spreadsheet](#)

# More Explorations – Constant Perimeter

Let's go all the way back and consider again the original problem. This time instead of considering the family of rectangles that had an area of  $93.5\text{in}^2$ , let's consider the PERIMETER of the rectangles, and find all the cylinders with the same perimeter.



# Develop Understanding: Some Rectangles with P=39"



[Spreadsheet](#)

# Explorations Using Algebra (3)

Write an expression for the volume of these cylinders as a function of  $r$ .

$$V = \pi r^2 h$$

$$C = 2 \pi r \text{ and } C = (39 - 2h)/2 \text{ so}$$

$$2 \pi r = (39 - 2h)/2$$

$$2 \pi r = 19.5 - h$$

$$h = 19.5 - 2 \pi r$$

$$V = \pi r^2 (19.5 - 2 \pi r)$$

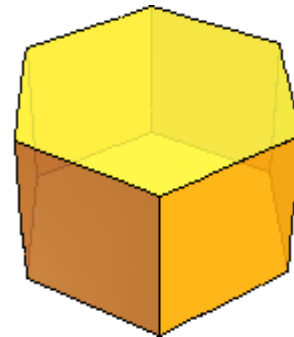
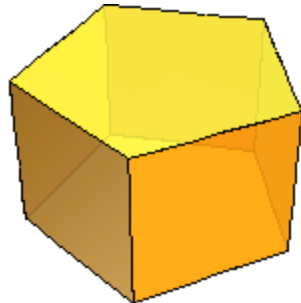
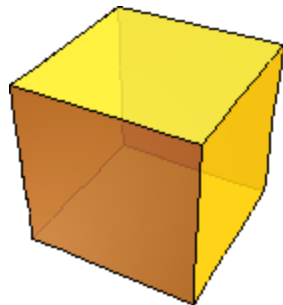
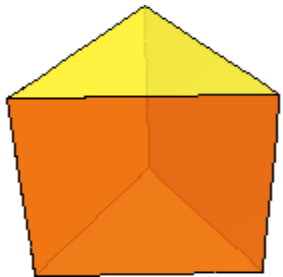
$$V = 19.5 \pi r^2 - 2 \pi^2 r^3$$

Volume to radius is what kind of function?

[Spreadsheet](#)

# Even More Explorations

Take that same sheet of paper and form this group of figures, with bases all regular polygons and circle, all with the same base perimeter and height. Which has the greatest volume?



# Isoperimetric Problem

This is a version of a very famous and historical problem in mathematics called the Isoperimetric Problem:

*Among all planar shapes with the same perimeter which figure has the largest area?*

and equivalently

*Among all planar shapes with the same area which figure has the shortest perimeter?*



# Isoperimetric Problem

Richard Tapia, a mathematician from Rice University calls the Isoperimetric Problem the “World's Most Influential Mathematics Problem”.

# Isoperimetric Quotient

The isoperimetric quotient of a closed curve is defined as the ratio of the curve area to the area of a circle with same perimeter as the curve.



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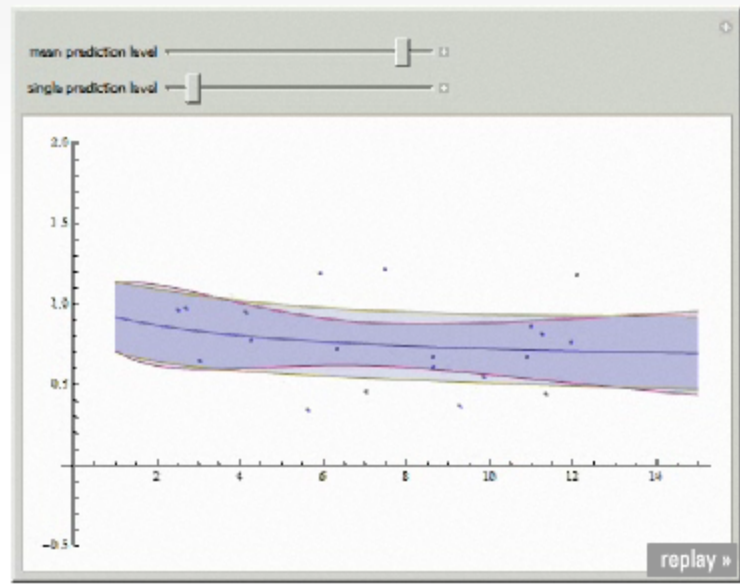
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Contributed by: Darren Glosemeyer

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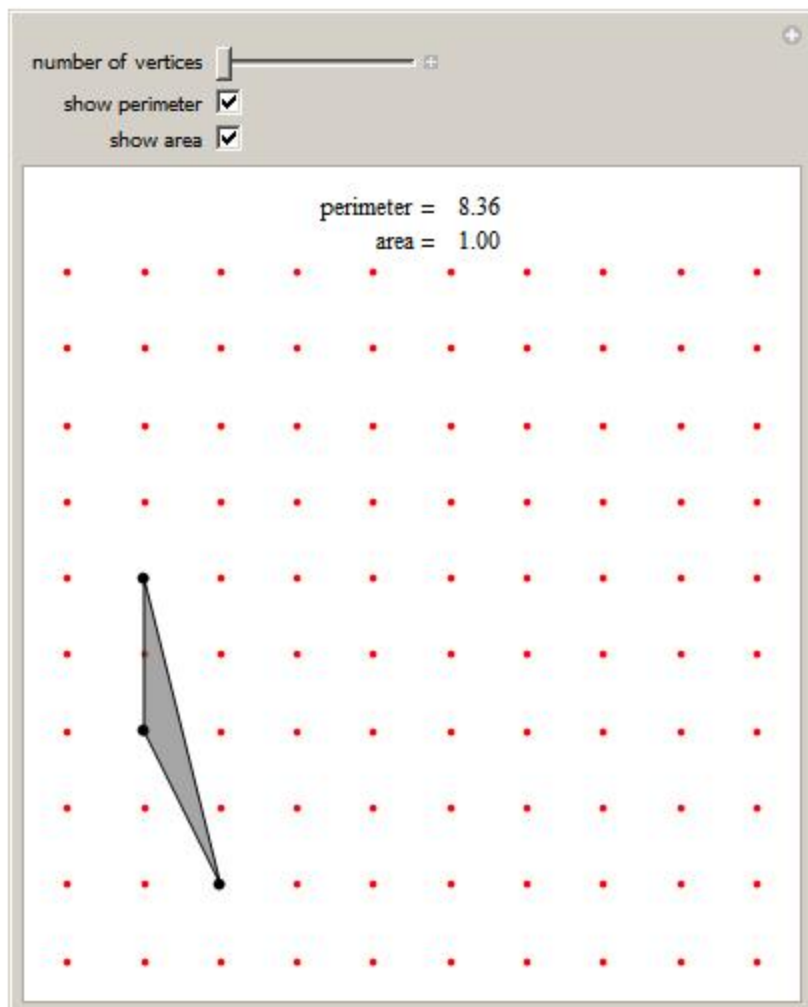
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