Farey Fractions and Reduce compound fractions to single ones mixed Relational Understanding

denominator; then the sum of the numerators written a

Addition of Vulgar fractions

Arthur Howard

the common denomina A Presentation by sum of the fractions

Add 74, 4 Providence Classical School 74-39 5 from Spring, TX

at the $7 \times 5 \times 56 \times 1 = 2184$ $7 \times 5 \times 56 = 1960$ $7 \times 5 \times 56 = 1960$ $7 \times 5 \times 56 \times 1 = 200$

Saturday, September 29, 2012

Addition of Vulgar fractions What I plan to do this morning:

Reduce compound fractions to single ones; mixed

numbers to improper fractions; gractions of different in • Show you something surprising about adding fractions. fractions.

- Talk about relational understanding.

 - What it is. Why it is important.
- Share an eye-opening article with you.

What is the sum of to of 45, 3 of 3 and 94 19 80 Answer

200) H219 -15180

5×56×1= 200

Addition of Vulgar fractions A Little Pretest

Reduce compound fractions to single ones; mixed

numbers to improped fractions; fractions of different in-



required.

the common denominator will be the sum of the fractions

What is the sum of to of 45, 4 f 3 and 94 19 80 Inswer

39 15 and 7

 $39 \times 56 \times 1 = 2184$ $15 \times 5 \times 1 = 75$ $7 \times 5 \times 56 = 1960$

5×56×1= 200

100) 4219 -15230

Examples.

Add 7 4, 4 of 3 - 1 + to then?

A Little Pretest

Addition of Vilgar fractions

Reduce compound fractions to single ones; mixed

numbers to improper fractions; gractions of different in

Question 2: and all of them to a common

A family travels from Houston to San Antonio. Because of traffic, they only average 40 mph going there. I sand y together On the return trip the traffic is much better, so they average 50 mph. 15280 What is their average speed for the entire trip? 12 Dac Insover

A Little Pretest

Addition of Vulgar fractions

Aduce compound fractions to single ones; mixed

numbers to improped fractions; fractions of different in



required ,

the common denominator will be the sum of the fractions

What is the sum of to of 45, 34 of 3 and 94 19 59 commen

 $39 \times 56 \times 1 = 2184$ $15 \times 5 \times 1 = 75$ $7 \times 5 \times 56 = 1960$

5×56×1= 200

100) 4219 -15280

Add 1/4, 4 of 3 and 1 to Jeck -74 - 39 5 18 26 - 3 39 15 and 7 39×56×1-21

Addition of Vulgar fractions Answer: Reduce compound fractions to single ones; mixed numbers to improper fractions; fractions of different in-Question the same; and all of them to a common mominator; then the sum of the numerators written over the common denominator will be the sum of the fractions required, Examples. 2 Add 7 4, 4 of 3 and 1 tog the -7 4 - 39 5 + 256 and 3 - 7 - 35 + 5 and 7 7 5 - 3 7 + 256 and 3 - 7 - 35 + 5 and 7 39× 56×1-2104 7× 5×56=1960 100 4219 100) 4219 -15280 5×56×1= 200 What is the sum of to of 45, 4 of 3 and 94 19 59 commen

Addition of Vilgar fractions Rule. Acduce compound fractions to siggle ones; mixed numbers to improper fractions; fractions of different intagen to those of the same; and all of them to a common denominator; then the sum of the numerators written over the common denoming or will be the sum of the fractions required, Example. Add 1/4, for gan y together. 7 4 - 39 5 13 15 and 7 7 . 39 15 and 7 $\begin{array}{c} 56 \times 1 = 2184 \\ 5 \times 1 = 960 \\ 7 \times 5 \times 56 = 1960 \\ 100 & 4219 \\ 100 & 4219 \\ 100 & 200 \\ \end{array}$ 5×56×1= 200 What is the sum of to of 45, 4 f 3 and 94 19 80 Inswer

Addition of Vulgar fractions Mule. Reduce compound fractions to single ones; mixed numbers to improper fractions; fractions of different integers to those of the same; and all of them to a common denominator; then the sum of the numerators written over the common denomin for will be the sum of the fractions required. Examples. Add 7 4, 7 of 2 and 7 together. 0 74-39 5 13 15 and 7 7. 39 15 and 7 $\begin{array}{c} 56 \times 1 = 2104 \\ 556 \times 1 = 2104 \\ 7556 = 1960 \\ 100 & 4219 = 15130 \end{array}$ 5×56×1= 200 What is the sum of to of 45, 3 of 3 and 94 12 59 commen

Addition of Vulgar fractions Reduce comparing the mixed numbers to interpret practices of different in to a common denominator; then the sum of the numerators written over the common denominator will be the sum of the fractions required, What is the sum of to of 45, 34 f 3 and 94 12 59 comments

Addition of Vulgar fractions 1 1 3. 2e. 5 Reduce compound grace 6006 to Grafe ones; mixed numbers to improper gractions; grace on of different in agen to those of the amc; a ad + bc them to a common denominator; then the sum of the nu ators noritten over the common denominator will bake sum of the fractions required. We add the *parts*, but the *whole* stays the same. Add 1 4, F of 3 and 7 together. 7 - 39 5 - 5 and you - 15 and 7

What is the sum of to and frand gt 12 59 and gt 19 80 to sover

4×56=1960

5×56×1= 200

100) 4219 -15280

4

Addition of Vulgar fractions Reduce compound fractions & single ones; mixed numbers to improper gractions; gractions of different in tagens to those of the same can a + c 7 them to a common denominator; then the the the common denomindor dilbet de sum of the fractions required. Parts and wholes are both added. Add 17 4, Fof 3 and y together. Parts are the same, but the whole is larger. 5 ×56 =1960 100) 4219 -15280 ×56×1= 200 What is the sur and gt 12 Dave Insover

"Crazy" Addition?

Addition of Kulgar fractions

Makes sense when sets of things are combined.

Jamples. 3

Reduce compound fractions to single ones; mixed

Operation must be done with original data.

Add 7 4, 5 of 3 and 10 together 8 7 4 - 39 5 13 6 2 - 8 7 5 - 39 5 6 and 7 - 7. 39 15 and 7

required,

Key concept: ACCUMULATION

Farey Fractions

Addition of Vulgar fractions

1 2 1 2 5x56×1=

What is the sum of to of 45, 4 f 3 and 94 12 59 Answer 4773 5 2

Reduce compound fractions to single ones; m numbers to impropet fractions; grad John Farey, Sr. (1766 – 1826) denominator; then the sum of the me British geologistinator will be the Sequence of reduced fractions Called this "sum" the mediant Rational Mean to found your.



1 4219 -15,80

Addition of Vulgar fractions Jose Altuve 0.303 hits/at bat and at 0.299 hits/at bat Examples. Add 1 4, Fof 3 and 7 together. 7 4 - 39 5 - 13 15 and 7 - 7. 5×56×96 hits 96 hits 0 hits What is th 4 at bats 321 at bats 317 at bats

Mixtures

Addition of Vilgar fractions

5 liters of a 40% acid solution.

 $\frac{2 \text{ L of acid}}{5 \text{ L of solution}} \oplus \frac{10 \text{ L of acid}}{20 \text{ L of solution}} = \frac{12 \text{ L of acid}}{25 \text{ L of solution}}$

What is the s48% 5 Solution 4 12 39 - Inswer

Add 1 4, Fof 3 and 7 together.

Answer:

Vulgar fractio

Addition of

Reduce compound fractions to single ones; mixed Question 2: open fractions; fractions of different in A family travels from Houston to San Antonio. sum of the practions They average 40 mph going there. They average 50 mph coming back. What is their average speed for the entire trip? Assume the distance each way to be 200 miles. 200mi @ 40 mph is $\frac{200 \text{ mi}}{5 \text{ hr}}$ 200mi @ 50 mph is $\frac{3 \text{ hr}}{200 \text{ mi}} = \frac{400 \text{ mi}}{4 \text{ hr}} = 44.4 \text{ mph}$ Reshart is the 200 mi 5 hr

Puzzler:

Addition of Vilgar fractions

A car is to travel two miles on a straight track. It averages 30 mph for the first mile. How fast will it need to go in the second mile to average 60 mph for the entire length?

 $30 \text{ mph} = \frac{30 \text{ mi}}{60 \text{ min}} = \frac{1 \text{ mi}}{2 \text{ min}}$

 $\frac{2 \text{ mi}}{2 \text{ min}} = \frac{1 \text{ mi}}{2 \text{ min}} \frac{1 \text{ mi}}{0 \text{ min}}$



Addition of Vilgar fractions Does Anybody Else Know This Stuff? Reduce compound fractions to single ones; mixed numbers to improper fractions; fractions of different in-• Mathematicians and all of them to a common

• Science Teachers Examples. Accumulated Average Average "builds" throughout term Weights grades based on size What is the sum of to of 45, 4 f 3 and 94 12 80 Answer

the sum of the

numerators novitten over

Addition of Vulgar fractions Accumulated Average Reduce compound fractions to single ones; mixed Each assignment has its own total point value. $\Delta verage = \frac{\text{total points accumulated}}{2}$ Average = total points possible

F and y -

What is the sum of to of 45, 3 of 3 and 9 4 12-

 $\oplus \frac{45}{45} \oplus \frac{56}{60} \oplus \frac{10}{15} = \frac{185}{200} = 92$

Rational Mean of the scores and

 $\mathbf{28}$

30

Why Weren't We Taught This?

- Reduce compound fractions to single ones; mixed numbers to improper fractions; fractions of different in
- Curriculum largely fixed by 15th Century.
- Rational Mean is recent. be the sum of the fractions
- Pythagorean Means:
 - Arithmetic Mean (Statistics) $\frac{a+b}{2}$
 - Geometric Mean (Finance) $\sqrt{ab^2}$
 - Harmonic Mean (Statistics, Electronics, Music)

2dab

• Rational Mean (Finance) $\frac{a+c}{b+d} = \frac{a+c}{200} = \frac{a+b}{a-b}$ What is the sum of to of 45,3 f 3 and 94 12 39 chosen

Are These Means Related?

Addition of Kulgar fractions

Reduce compound fractions to single ones; mixed numbers to improper fractions; fractions of different in **Rational vs Harmonic** and 2(40)(50) and 4000 and denominator; then the sum of 2(40)(50) 4000 and 44.4 When distances are equal. 1/40+50 of 190 fractions

Rational vs Arithmetic $50 \text{ mi} \oplus 40 \text{ mi} = 90 \text{ mi} \\ 1 \text{ hr} \oplus 1 \text{ hr} = 2 \text{ hr}$ When *times* are equal. When *denominators* are equal. $3 \oplus 8 = 81/3$ $3 \oplus 8 = 81/3$

What is the sum of To of 45, 3 of 3 and 94 19 80 tosover

The Answers:

Addition of Valgar fractions

Reduce compound fractions to single ones; mixed

numbers to improped fractions; fractions of different in-



required,

the common denominator will be the sum of the fractions.

Are you sure 200 15290 15290

 $39 \times 56 \times 1 = 2184$ $15 \times 5 \times 1 = 75$ $7 \times 5 \times 56 = 1960$

Ecamples.

Add 7 4, 4 of 3 and 7 to the 3 2 45 and 7 7 4 39 5 7 8 26 3 39 × 56 × 1 - 21

Addition of Vulgar fractions Mule. Reduce compound fractions to single ones; mixed numbers to improper fractions; fractions of different in tagen to those of the same; and all of them to a common denominator; then the sam of the numerators written over the common denominator will be the sum of the fractions Examples. Add 1/4, 5 of 3 a 7 together 7 4-39 5-13-15 and 7-7 . 39 15 and 7 5×56×1= 200 What is the sum of to of 45, 4 of 3 and 94 19 80 Inswer

Addition of Vulgar fractions Mule. Reduce compound fractions to single ones; mixed numbers to improper fractions; fractions of different in tegen to that the to the to the the terminent denominator; then the sum of the numerators written over the common denominator will be the sum of the fractions required. Examples. Add 7 4, 5 of 3 and 7 Hogethes 7 3 8 8 6 0 7 . 0 9 0 6 0 7 What is the sum of to of 45, 4 of 3 and 94 19 59 commen

Additionof gar fractions Mule. Reduc mixed number event integens lo common denomin ritten over the comm actions required. Add y 15280 What is the sum of to of 45, 4 of 3 and 94 12 80 Insover

Addition of Valgar fractions Answer: Reduce compound fractions to single ones; mixed numbers to improper fractions; fractions of different in-Question 3: and all of them to a common the common denominator will be the sum of the fractions required. Add 1/4, fof 3 and 7 to Jeth 74-39 5 13 25 - 3 39 15 and 7 39×56×1-21 $39 \times 56 \times 1 = 2184$ $15 \times 5 \times 1 = 75$ $7 \times 5 \times 56 = 1960$ 100) 4219 -15280 5×56×1= 200 What is the sum of to of 45, 34 f 3 and 94 19 59 Answer

Implications

Reduce compound fractions to single ones; mixed

Addition of Vulgar fractions

Combining fractions of things is much more complicated than we thought.

BOTH occur in everyday life.

required,

Does that mean I have to teach *TWO* procedures for adding fractions?

What is the sum of go Bigg 3 and 94 19 80 tosover

7 3 39 5 13 15 and 7 7 . 39 15 and 7

Procedures

Reduce compound fractions to sing Prof. Richard R. Skemp

15240

5×56×1= 200

• Memorized without understanding

numbers to improper fractions; fraction

- Don't know when to use them
- Misremember rule together.

Addition of Vin

• Rules that get the answer

- No idea of error found 7=7. 29 15 and 7
- Don't know what to do about it

The Psychology of Learning Mathematics

Understanding

Addition of Kulgar fractions

Reduce compound fractions to single ones; mixed numbers to improper fractions; fractions of different in-• Two types of understanding of them to a common – Instrumental (Procedural) Understanding "Rules without reason" – Relational (Conceptual) Understanding
 "Knowing what to do, and why" _15 and 7 - Most math instruction is procedural 7X 5X56=1960 200) H219 -15280

What is the sum of to of 45, 3 of 3 and 94 19 80 Answer

5×56×1= 200



Addition of Valgar fractions

Often much easier to teach
Immediate rewards
Less knowledge involved, so answer comes more quickly
Requires a new rule for each situation

39×56×1=2184 15× 5×1= 75 7× 5×56=1960 100) 4219 -15280 5×56×1= 200

What is the sum of to of 45, 3 of 3 and 94 19 80 tosover

Conceptual Understanding

Addition of Vulgar fractions

Reduce compound fractions to single ones; mixed

numbers to improper quactions; gradions of different in ag- Takes longer to teach. and all of them to a common

- Students sometimes resist it.

– Sometimes involves more content.

Add 7 4, Fof 3 and 7 together.

7 4 - 39 5 13 15 and 7 7 . 39 15 and 7

Examples.

 $39 \times 56 \times 1 - 2184$ $15 \times 5 \times 1 = 75$ $7 \times 5 \times 56 = 1960$ 200) 4219 -15280 5×56×1= 200

What is the sum of to of 45, 4 of 3 and 94 19 80 mover

Addition of Vilgar fractions Example Reduce compound fractions to single ones; mixed numbers to impropet fractions; fractions of different in denominator; then the sum of the numerators written over the common denominator will be the sum of the fractions required, Add 7 4, 7 of -7 4 - 39 15 and 7 XI = 2104 XI = 75 X56 = 1960100) 4219 -15280 5×56×1= 200 What is the sum of to of 45, 4 of 3 and 94 19 80 Inswer

Addition of Vilgar fractions Extension Reduce compound fractions to single ones; mixed numbers to improper fractions; gractions of different in tegens to those of the same; and all of them to a common denominator; the tors noritten over the common des of the fractions required. Add 7 4 Fof 3 and 7 together. 74-39 15 and 7 $\begin{array}{l} \chi_1 = 2185 \\ \chi_1 = 75 \end{array}$ X56 =1960 200) H219 -15180 1= 280 What is the sum of to of 45, 4 of 3 and 94 19 80 Insover

Addition of Vulgar fractions Further Extension Reduce compound fractions to single ones; mixed numbers to improper fractions; fractions of different intagen to those of the same; and all of them to a common dimension distance deline deline and the commendation we be the most fractions Add 1 4, Fof 3 and 7 togethe 7 4 39 5 13 56 and 1 7 37 F6 and 7 $39 \times 56 \times 1 = 2184$ $15 \times 5 \times 1 = 75$ 1 X 5 X56 = 1960 100) 4219 -15280 5×56×1= 200 What is the sum of to of 45, 4 f 3 and 94 12 80 consover

Conceptual Understanding

Reduce compound fractions to single ones; mixed

Addition of Kulgar fractions

numbers to improper fractions; fractions of different in Takes longer to teach. and all of them to a common

- Students sometimes resist it. the common denominator will be the sum of the fractions.
 - Sometimes involves more content.
 - Examples. – Easier to remember.
 - More adaptable to new tasks.
 - Is effective as a goal in itself. $39 \times 56 \times 1 = 2$
 - Leads to willingness to study further. 9-15,30 5×56×1= 280

What is the sum of to of 45, 3 of 3 and 94 12 Black mover

56×1-2184

Failure to teach Conceptually

Addition of Valgar fractions

Reduce compound fractions to single ones; mixed

- Use procedures inappropriately them to a common
- . Can't transfer inator will be the sum of the fractions

7 4 - 39 5 13 15 and 7 7 . 39 15 and 7

 $39 \times 56 \times 1 - 2184$ $15 \times 5 \times 1 = 75$ $7 \times 5 \times 56 = 1960$

- Baffled by word problems
- · Math phobia and y together.

"I never could do math."

"The negative attitude to mathematics, unhappily so common, even among otherwise highly-educated people, is surely the greatest measure for our failure and a real danger to our society. 39 5 13 15 and 7=7. 39 15 and 7

Addition of Vulgar fractions

Sir Herman Bondi, Sir Herman Bondi, Austian/British Mathematician and Cosmologist

What is the sum of to of 45, 4 f 3 and 94 12 80 chosmen

A public unable to reason with figures is an electorate unable to discriminate between rational and reckless claims in public policy.

Addition of Vilgar fractions Rule.

Lynn Arthur Steen: Numeracy

 $39 \times 56 \times 1 = 2184$ $15 \times 5 \times 1 = 75$ $7 \times 5 \times 56 = 1960$ 100) 4219 -15280 5×56×1= 200

What is the sum of to of 45, 4 f 3 and 94 12 80 mover

Addition of Valgar fractions What Should We Do?

- Reduce compound fractions to single ones; mixed numbers to improper fractions; fractions of different in
- Start with real-world situations. dem to a common
- Use manipulatives to teach concepts. // fractions
- Ask students to explain their thinking.
- Connect physical, pictorial and abstract.
- Different contexts, but the same concepts. 39×56×1-2187 15× 5×1= 75 7× 5×56=1960 100) 4219-15280

What is the sum of to of 45, 3 of 3 and 94 12 80 tosover

5×56×1= 280



Enavaahcine the see problem is, the first number shore sents the minut The tronstofun bechethats intaithed fraction defya Eage of found les nitatris is the Thracticop of an inter its the frantionofTanestheyr. bag of marbles Howismedy inches of rain fell in thetewowdays go gethoo ured into a bowl, what fraction of the marbles in the bowl will be red? 5×56×1

ulgar fractions

15, 4 f 3 and 94 19 Blackmover

Asking Questions

Reduce compound fractions to single ones; mixed

Vulgar fractio

Addition of

What "action" do you see in this problem?

Does it sound like something we've done before?

Can you use objects or draw me a picture? Is this sometimes/always/never true? - 15 and T How do you know that? Do you see a pattern? 152 40 Can you explain that in your own words? 2 5 gue Inso What kind of answer should we expect? Is this answer reasonable?

Data Analysis

Addition of Vulgar fractions

30

Reduce compound Aractions to single ones; mixed					
numl	Object	Circumference	Diameter	Ratio	ment in-
tegens i	Jar Lid	32.6	10.2	3.20	common
denom	Bicycle Wheel	210	66	3.18	tten over
the cor required	Coffee Lid	68.6	20.2	3.40	actions
	Mixing Bowl	94.1	30	3.14	
	Saucer	57.5	17.9	3.21	
Add	Dinner Plate	82	26.5	3.09	
What kind of "average" should we use here?					
Median? 3.19.56-19.66 Arithmetic Mean? 3.09 - 200					
What is Geometric Mean? 3.20 + 12 29 the					
Harmonic Mean? 3.20					
Rational Mean? 3.19					

Addition of Vilgar fractions Our Task Adduce compound fractions to single ones; mixed numbers to improper fractions of different in agen To create a classroom where mon denominator; then the sum of the memorators written over the common denominator will be the sum of the fractions rowing spatial provide the sum of the fractions a spatial provide the sum of the sum of the fractions Campiles.

Give an education that will last a lifetime!

 $39 \times 56 \times 1 = 2184 \\ 15 \times 5 \times 1 = 75 \\ 7 \times 5 \times 56 = 1960 \\ 180) 4219 = 15280 \\ 5 \times 56 \times 1 = 280$

What is the sum of to of 45, 4 of 3 and 94 12 80 Answer

Addition of Vilgar fractions Mule. Reduce compound fractions to single ones; mixed numbers to improper fractions; fractions of different intagen to those of the same; and all of them to a common denominator; then the sum of the numerators written over Thank you for your time. Add 1/4, Fof 3 and 7 together. 7 4 - 39 5 13 15 and 7 7 . 39 15 and 7 $39 \times 56 \times 1 = 2184$ $15 \times 5 \times 1 = 75$ 7X 5X56=1960 200) 4219 -15280 5×56×1= 200 What is the sum of to of 45, 3 of 3 and 9 4 19 80 Insover