Mathematical Habits of Mind

Developing Mathematical Thinkers

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DISCUSSION

- Write 3-5 words or phrases that describe the kind of mathematical thinking you want students to be able to do.

In this session...

- College and Career Readiness
- Thinking like a Mathematician
- Mathematical Habits of Mind Through the Years
- Mathematical Habits of Mind at the Heart of the TEKS
- **Mathematical Habits of Instruction . . .**

  . . . to Help Every Student Learn to Think

The New Question

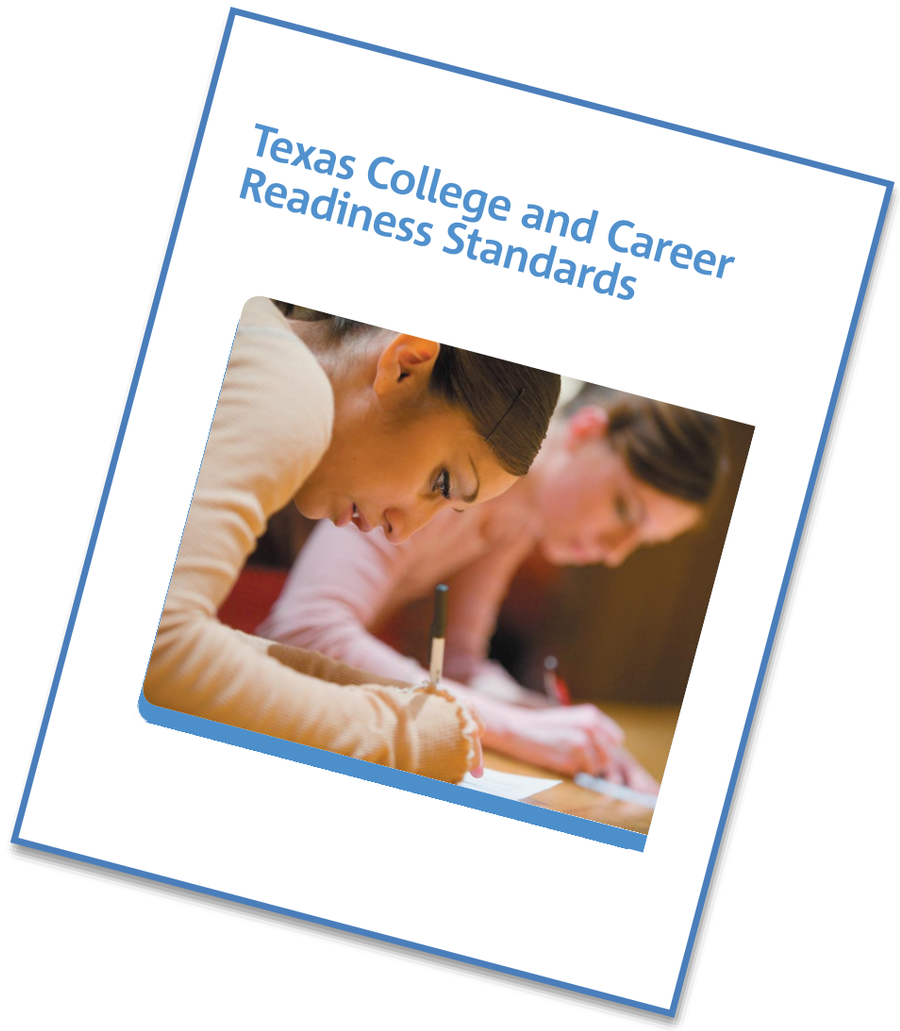
- ***College and Career Readiness***:

Is every student graduating from high school ready for success in college or a well-paying career?

College Readiness Standards

- ACT
- The College Board
- Common Core State Standards (CCSS)
- Texas College and Career Readiness Standards (CCRS)

TX College & Career Readiness Standards



- Intellectual curiosity
- Reasoning
- Problem solving
- Academic behaviors
- Work habits
- Academic integrity
- Reading / Writing / Research across the curriculum
- Use of data
- Technology

TX College & Career Readiness Standards

- "Students must develop **ways of thinking about mathematics**. These key cognitive skills elevate mathematics from an exercise in rote memorization to **a process of analysis and interpretation** that enables the learner to work with a range of complex questions, topics, and issues."
- "Mathematical thinking never occurs in a vacuum; it is always embedded in appropriate content."

[CCRS, p. 8]

Being a ‘Doer’ of Mathematics

“The only way to know mathematics
is to ***do*** mathematics.”

Paul Halmos, mathematician

Thinking Like a Mathematician

- Doing mathematics--figuring out hard problems
- Reasoning and explaining and arguing
- Zooming in and zooming out
- Exploration that's sometimes messy
- Trying things that sometimes work and sometimes don't
- Reflecting, considering, analyzing
- Lots of people have thought about the mathematical habits of mind we want students to learn

What math do all students need?

- The Big Three:
  - **Understanding math** (making sense of it)
  - **Doing math** (skills, facts, procedures)
  - **Using math** (thinking, reasoning, applying, solving a range of problems)
- The New Basics: deep transferable skills for *versatilizing*:
  - Problem solving, reasoning, research, communication, creativity

Mathematical Thinking

Overarching goal:

Students who can think mathematically . . .

Students who have developed and use mathematical habits of mind.

Mathematical Habits of Mind

- **Performing thought experiments**
- **Finding, articulating, and explaining patterns**
- Creating and using representations
- Generalizing from examples
- Articulating generality in precise language
- **Expecting mathematics to make sense**

Al Cuoco, E. Paul Goldenberg, June Mark.
"Organizing a Curriculum around Mathematical Habits of Mind."
Mathematics Teacher May 2010

Mathematical Habits of Mind

- Connective tissue for mathematical content
- Tools and strategies for using mathematical content
- Ways of thinking and figuring things out; making sense of and doing something with mathematics
- The heart and soul of the mathematics our students need

NCTM Process Standards

- Problem Solving
- Reasoning and Proof
- Communication
- Connections
- Representations

***Principles and Standards for School Mathematics*, NCTM 2000**
(expanded from *Curriculum and Evaluation Standards for School Mathematics*, NCTM, 1989)

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments; critique others' reasoning.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and make use of regularity in reasoning.

CCSS for Mathematics, pages 6-8

Mathematical Habits of Mind--Common Themes

- Solving problems (take apart, put together)
- Thinking and Reasoning
- Reflecting
- Discussing
- Justifying, Explaining, Communicating
- Generalizing (make generalizations; use properties)
- Connecting
- Zooming out and Zooming in
- Patience, persistence

TEKS Process Standards

(A) **apply mathematics** to problems arising in everyday life, society, and the workplace (CCRS ; CCSSM 1, 4; NCTM 1)

(B) use a **problem-solving model** that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution (CCRS ; CCSSM 1; NCTM 1)

(C) select **tools**, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems (CCRS ; CCSSM 1, 5; NCTM 1, 5)

(D) **communicate** mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate (CCRS ; CCSSM 2, 3, 6; NCTM 2, 3, 5)

(E) create and use **representations** to organize, record, and communicate mathematical ideas (CCRS ; CCSSM 3, 5; NCTM 3, 5)

(F) analyze mathematical relationships to **connect and communicate mathematical ideas** (CCRS ; CCSSM 3; NCTM 3, 4)

(G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication (CCRS ; CCSSM 3, 6; NCTM 2)

Problem Solving, Applications, Mathematical Modeling

Process standards (A), (B)

- **Persevere in solving problems**
- Explain to selves; analyze givens, constraints; consider analogous problems; represent the situation in different ways; search for regularity/trends; plan solution path (problem solving model)
- **Solve problems that apply the mathematics being learned**
- **Solve problems that may involve mathematics not yet learned**
- Use mathematical modeling to solve problems that may not be well defined; Interpret, reflect, fine-tune the model
- When solving problems, keep an eye on the big picture while also attending to the details (zooming in and out); shift perspective
- **Evaluate the reasonableness of results.**

Reasoning, Explaining, Justifying

Process Standard (G)

- **Includes quantitative reasoning**; make sense of quantities and relationships in problems
- Make conjectures and explore the truth of conjectures.
- Analyze situations; recognize and use counterexamples.
- **Justify conclusions and communicate/respond to arguments.**
- **Decide whether arguments of others make sense; ask useful questions to clarify or improve the arguments.**

Representing, Connecting, Communicating Precisely

Process Standards (D), (E), (F)

- Look for commonalities/relationships in similar problems
- Coherently represent a problem;
  decontextualize and contextualize
- **Communicate precisely; make clear, effective arguments;**
  use clear definitions
- **Put precision in perspective (finding balance . . .)**
- **Does this make sense?**
- Mathematical structure: Precise definitions, vocabulary

Tools

Process Standard (C)

- Pencil/paper, manipulatives/concrete models, compass, protractor, calculator, spreadsheet, CAS, software, . . .
- Find and use external information (Internet, research, etc.)
- **Mental math**
- **Make decisions and understand limitations**

TEKS Process Standards and Content

- Every TEKS content standard begins with '. . . apply the process standards with/to . . .'

- Overarching Mathematical Habits of Mind in the TEKS content standards: Making sense (reasonableness), Making Generalizations, and . . .

Discussion:

What factors keep some students from reaching their mathematical potential?

Factors to consider

- Student factors: Motivation, intelligence, beliefs
- Teacher factors: Beliefs, knowledge, and expectations
- Instructional factors
  - Nature of the task
  - Opportunities to struggle, think, figure things out
  - Expectations that they will succeed
  - An environment of trust, collaboration, and respect

Intelligence

- Fixed vs. malleable (can also motivate learning)
- Your mindset influences confidence, perseverance, and your willingness to take risks
- From brain research:
  The activities a person engages in can change their intelligence.
- Who determines the activities a student engages in?

Targeting beliefs with action

- Students' beliefs matter.
- Teachers' beliefs and actions matter.
- Modest interventions make a difference.

High Expectations means...

- Challenging our habits and beliefs
- Setting challenging standards for all students
- Doing whatever it takes for students to achieve the standards
- Never thinking in advance that you know where they're headed
- Making sure they all get to struggle and succeed

Premise:

What students need for their future is as much about how they ***think*** as it is about what they ***know,***

and helping students succeed is as much about ***how*** we teach as about ***what*** we teach.

mathreasoninginventory.com

Marilyn Burns, PI

Funded by Gates Foundation

https://www.mathreasoninginventory.com/Home/AssessmentsOverview

The difference between...

- Clue/key words vs. mathematical communication
- Learning how to do mathematical procedures vs. learning mathematical habits of mind

DISCUSSION

- How did the teacher find out what Marisa was thinking?
- Had Marisa likely had experience developing mathematical habits of mind?
- What examples of this kind of ‘clueless’ thinking can you identify for the grade(s) or course(s) you teach?

Answer-getting vs. learning mathematics

- USA:
  How can I teach my kids to get the answer to this problem?

- Japanese:
  How can I use this problem to teach the mathematics of this unit?
  - Devised methods for slowing down, postponing answer-getting

Phil Daro, 2012

The difference between Japan and the US

- "You quit teaching too soon and go on to the next thing."
- "We finish."
- Finishing happens when students have learned.
- And learning is incomplete if students aren't developing mathematical thinking.

Marisa didn't get to finish...

Upside-down teaching

- From: *"I - We - You"*

- To: *"You - We - I"*

Upside-down teaching

- Starting with a rich problem
- Students engaged in dealing with the problem
- Discussion, comparing, interacting
- Teacher helps students connect and notice what they've learned
- Then, exercises and homework

Mathematical Habits of Mind

Don't teach mathematical habits of mind--

use and infuse them

Mathematical Habits of Instruction

- Use a problem-centered, upside-down teaching model
- Use appropriate technology appropriately
- Learn to zoom out, zoom in, and go back and forth
- Help students learn to notice and use patterns and properties within and across mathematical topics and problems (mathematical structure)
- Use formative assessment to pay attention to learning

Achievement Gap

Untapped Potential

What if we raise the floor AND the ceiling?

Two Sides of Untapped Potential

- Bringing up all students to achieve their highest levels of mathematics and science--raising the floor
- Identifying the stars
- Raising the ceiling and letting them soar
- Untapped potential within each student, within groups of students, and at the school, district, state and national level--potential we haven't reached . . . YET.

Untapped Potential

Unlimited Potential

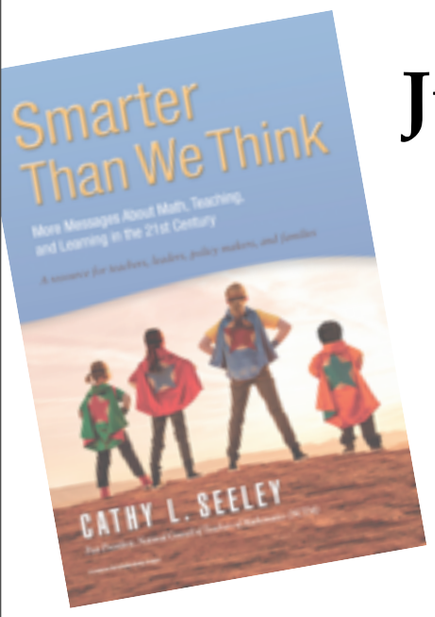
Even our *best* students...

...will benefit from a strong, diverse, engaging, relevant classroom.

Their future is in our hands

...and ours is in theirs

E-mail for a copy of the slides: cseeley@utexas.edu



Just published April 2014: *Smarter Than We Think*

Messages from today...

Smarter Than We Think, Upside-Down Teaching, Clueless, Mathematical Habits of Mind, Mathematical Habits of Instruction

Faster Isn't Smarter--

Messages About Math, Teaching, and Learning in the 21st Century

Seeley 2009

http://mathsolutions.com/fasterisntsmarter *(Download 5 messages)*

Constructive Struggling, Crystal's Calculator, Balance is Basic

Cathy's websites:

http://cathyseeley.com http://csinburkinafaso.com